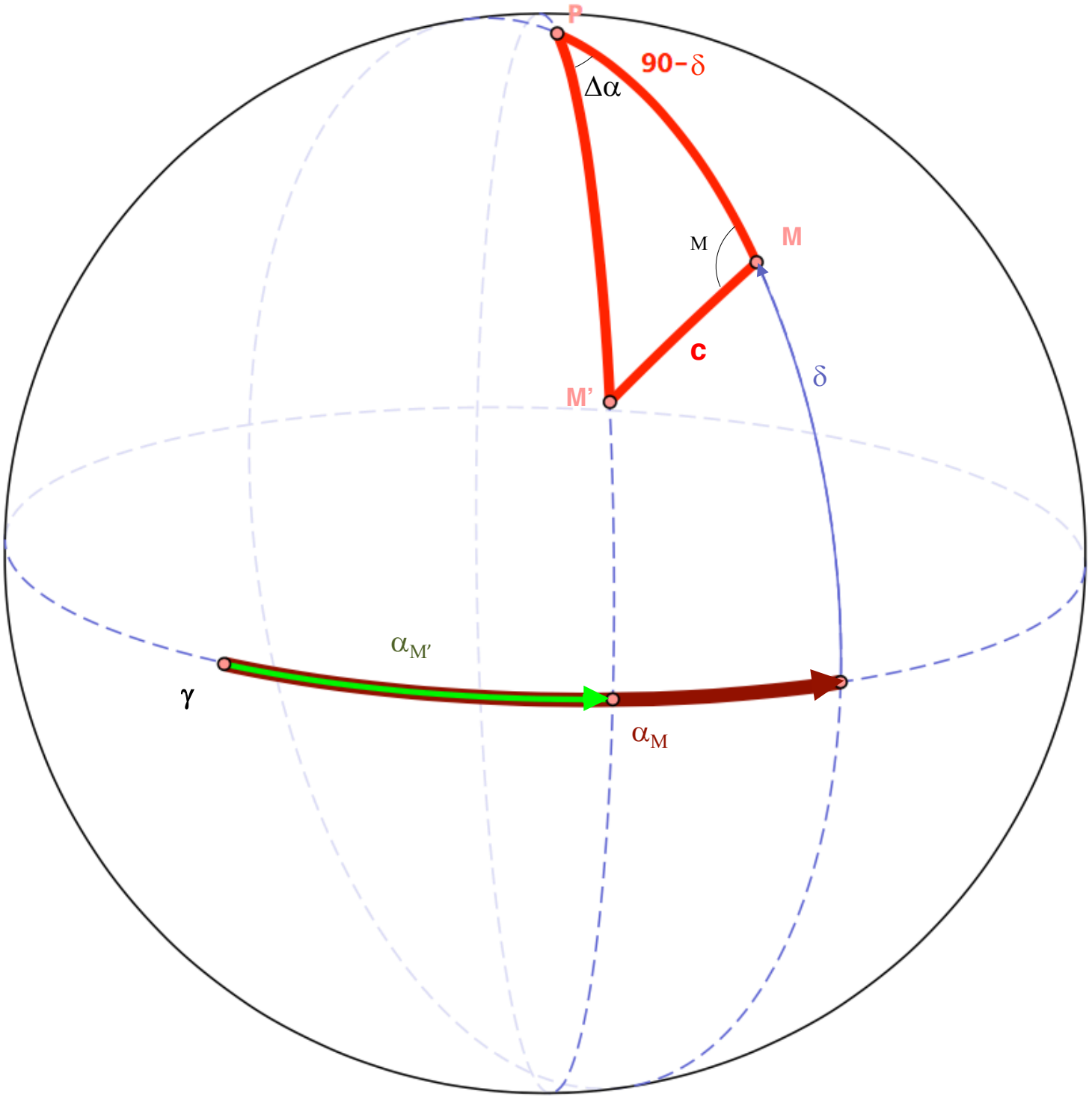
Kepler's Laws

Learning Objectives

- Emergence of modern astronomy
- Planetary motion
- The interplay between theory and observation
- The fundamental importance of observational accuracy


P
90−δ
Δα
M
M
c
M'
δ
γ
$\alpha_{M'}$
α_M

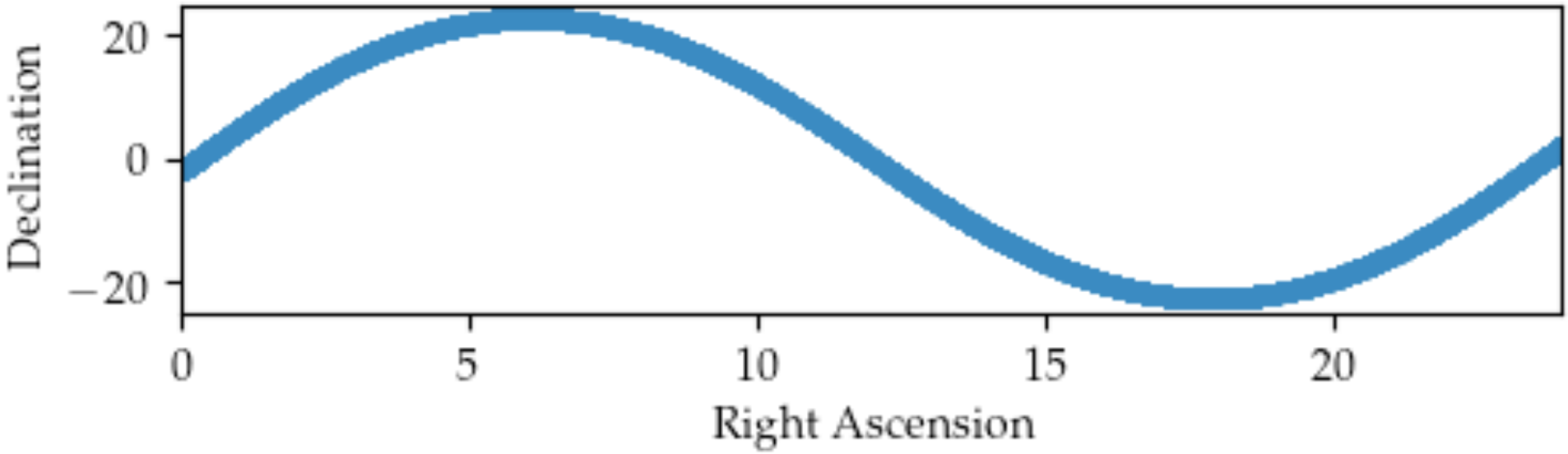
Elongation

angle between a Planet and the Sun

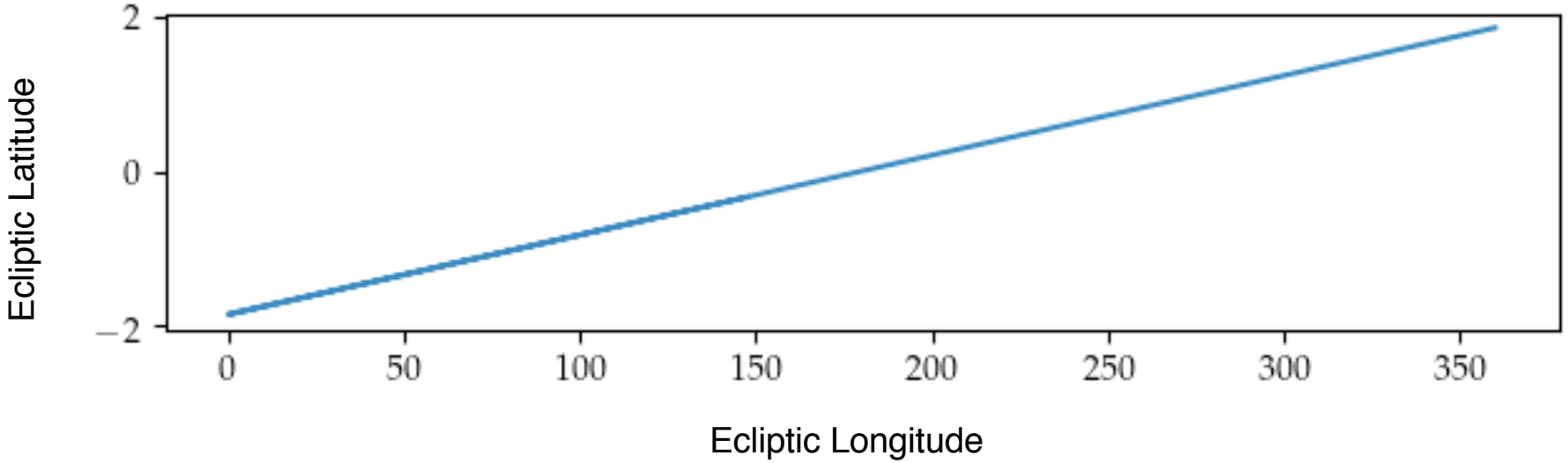
Path of Mars

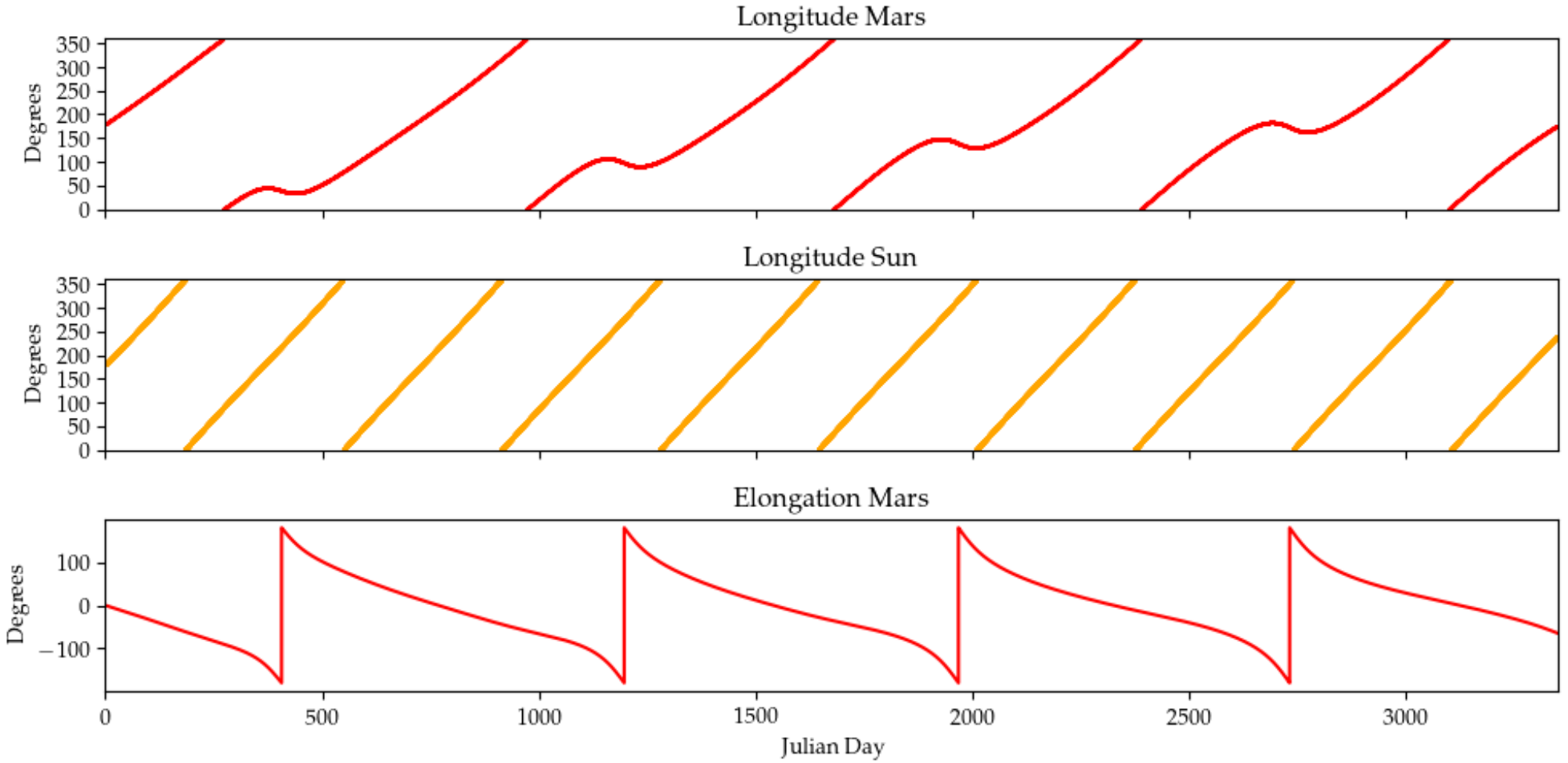
23h00m
22h30m
22h00m
-5°
-10°
-15°
-20°
φ Aqr
λ Aqr
θ Aqr
ψ3 Aqr
ECLIPTIC
σ Aqr
τ2 Aqr
ι Aqr
♂
δ Aqr
66 Aqr
88 Aqr
89 Aqr
2003-06-15

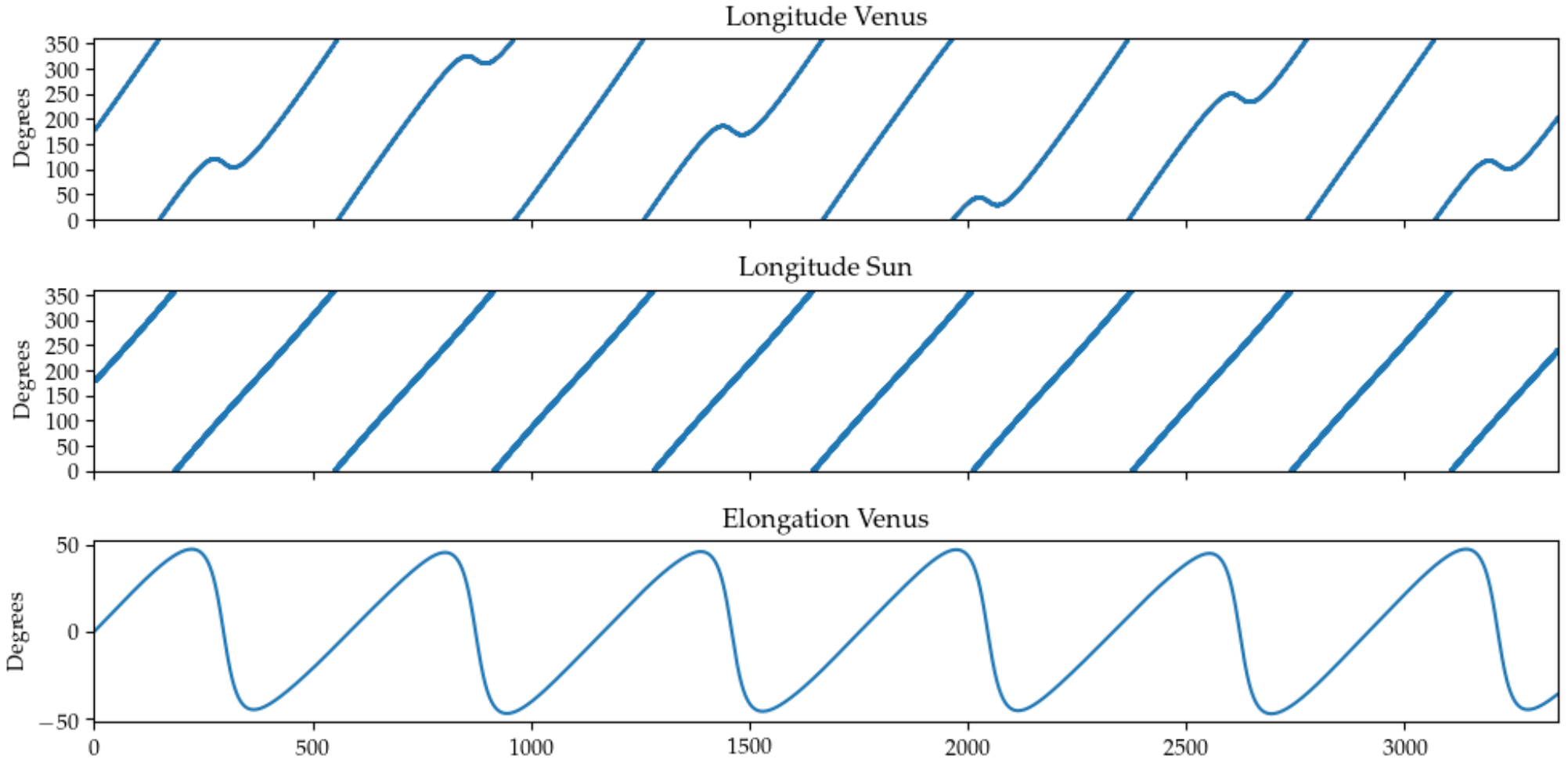
Mars Path in the Sky – Equatorial Coordinates

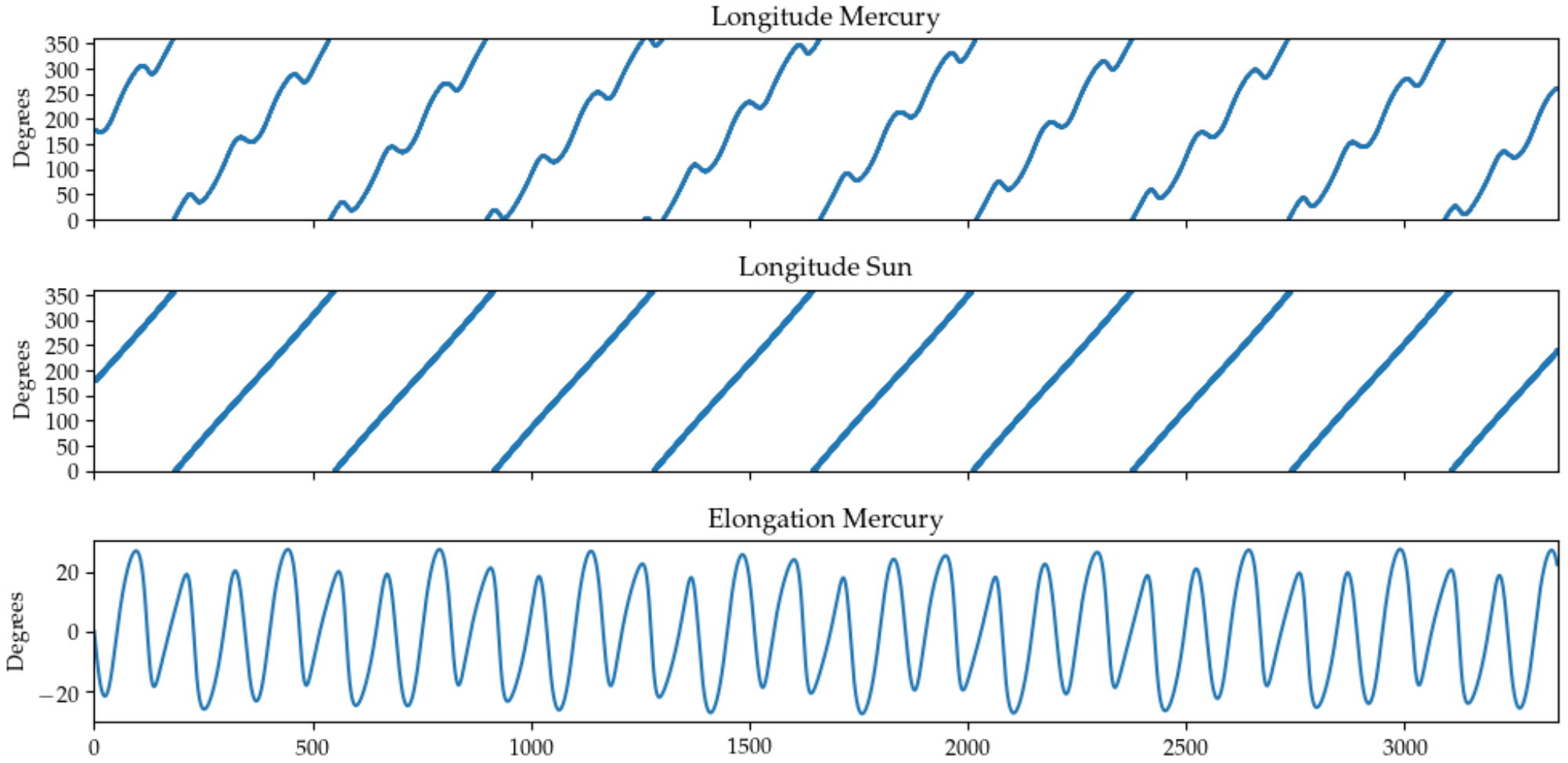


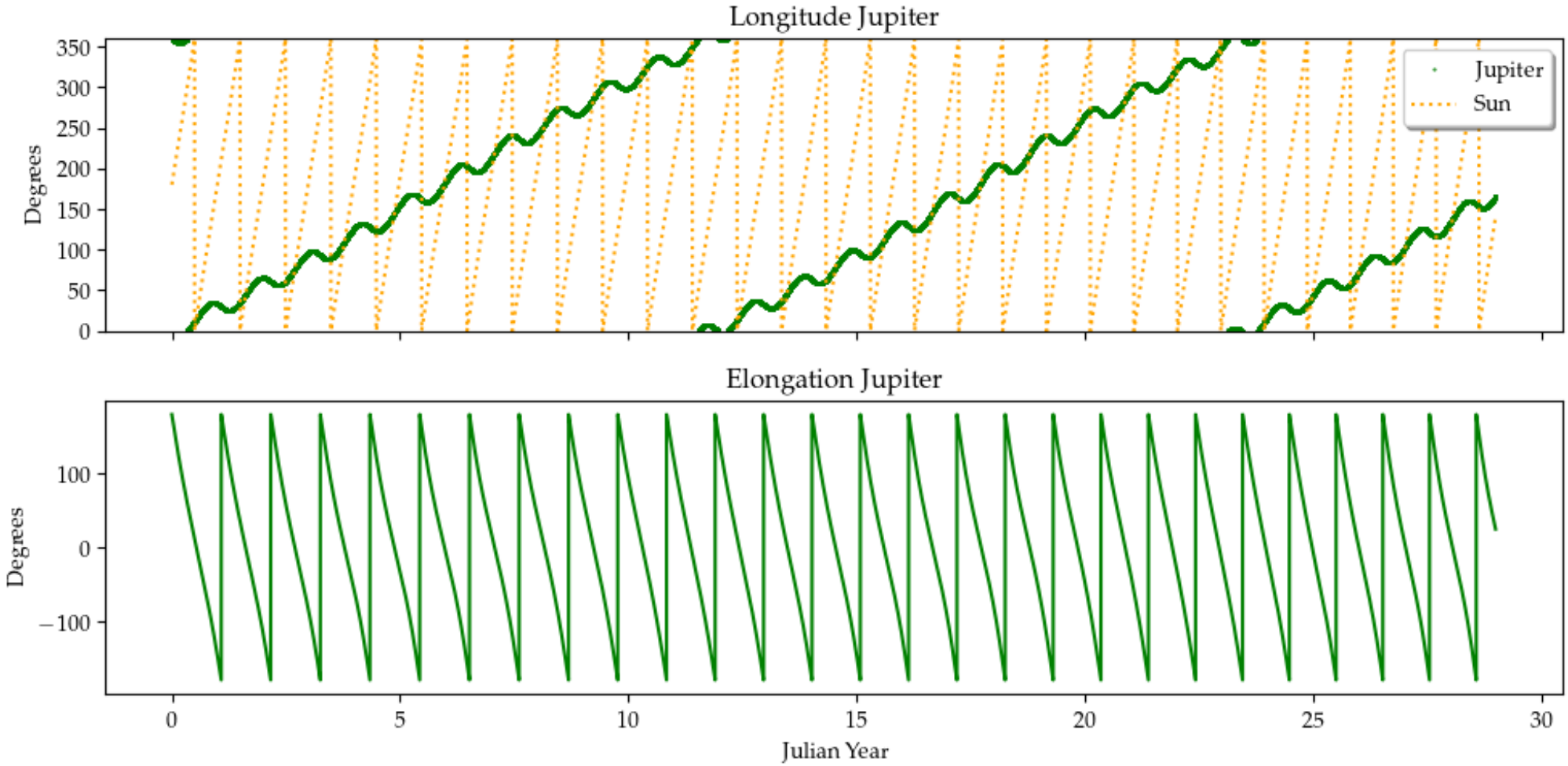
Mars Path in the Sky – Ecliptic Coordinates

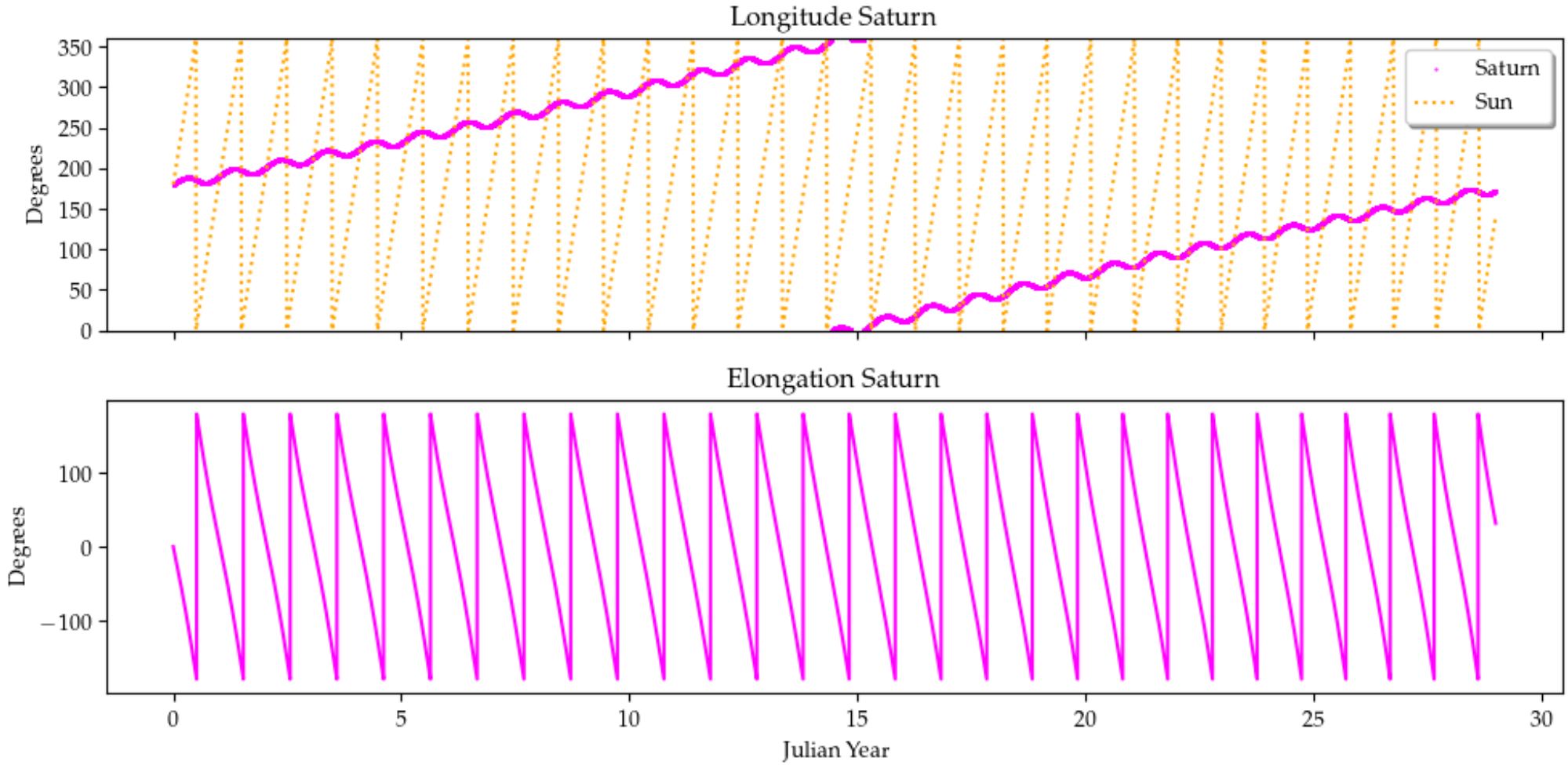



Longitude Mars
Degrees
350
300
250
200
150
100
50
0
Longitude Sun
Degrees
350
300
250
200
150
100
50
0
Elongation Mars
Degrees
100
0
−100
0
500
1000
1500
2000
2500
3000
Julian Day

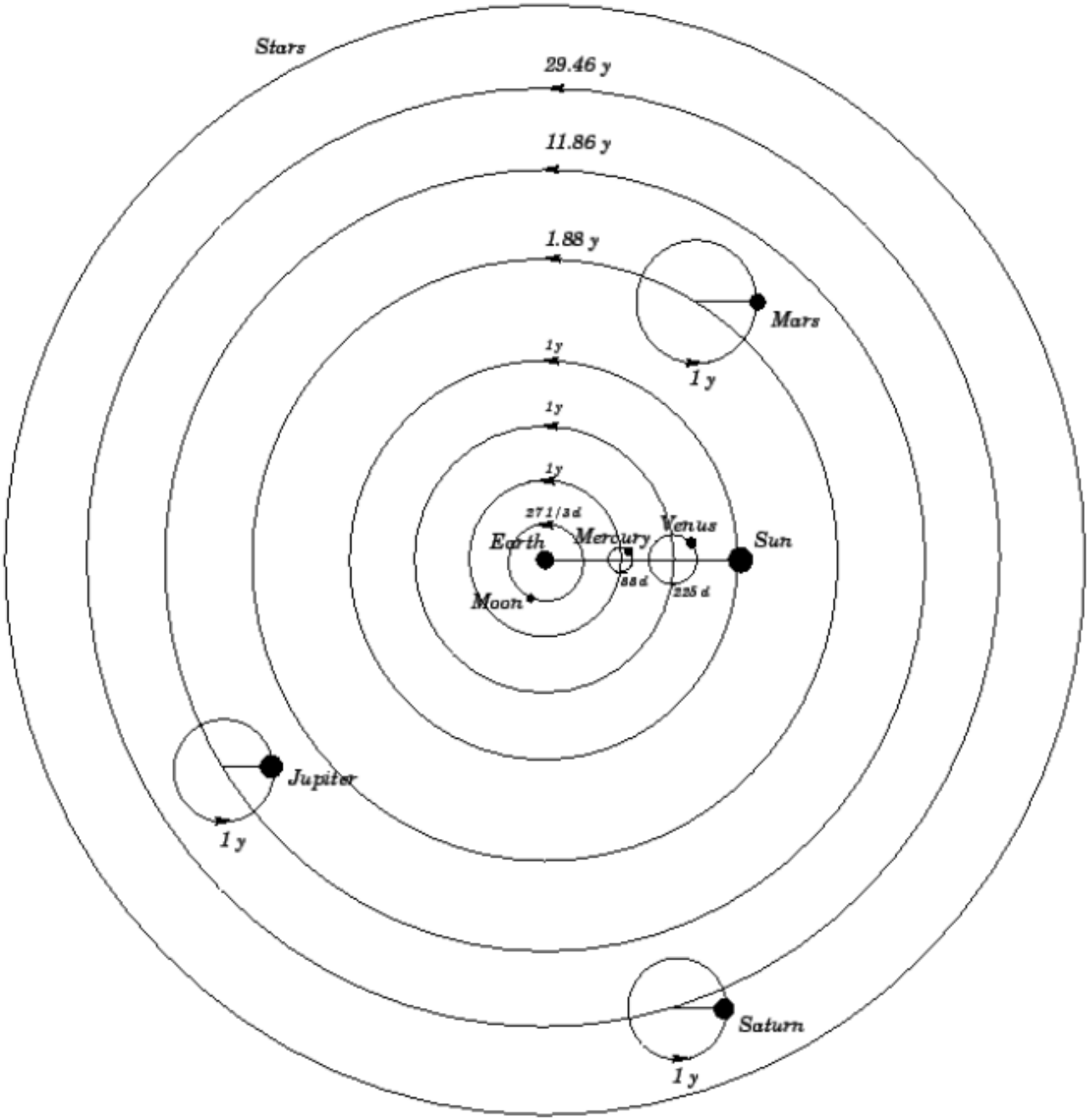

Longitude Venus
Degrees
350
300
250
200
150
100
50
0
Longitude Sun
Degrees
350
300
250
200
150
100
50
0
Elongation Venus
Degrees
50
0
−50
0
500
1000
1500
2000
2500
3000


Longitude Mercury
Degrees
350
300
250
200
150
100
50
0
Longitude Sun
Degrees
350
300
250
200
150
100
50
0
Elongation Mercury
Degrees
20
0
−20
0
500
1000
1500
2000
2500
3000

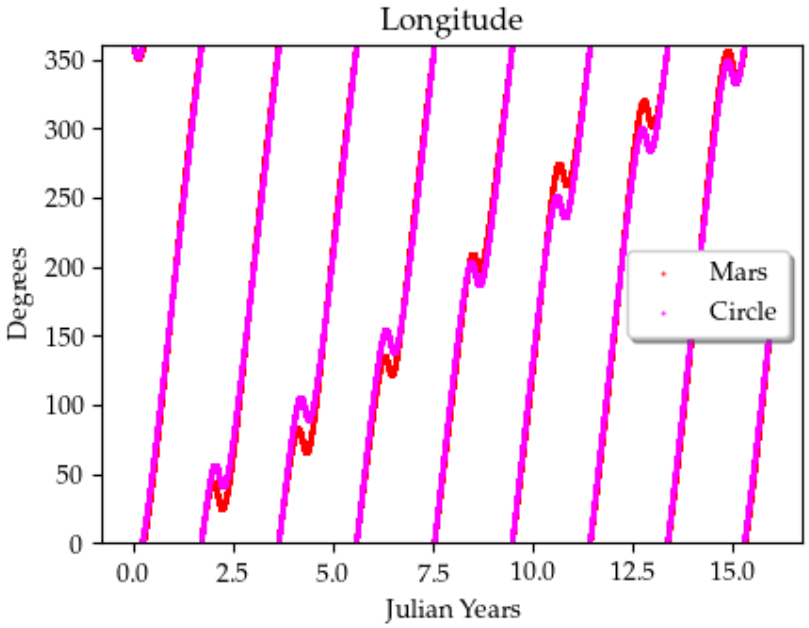

Longitude Jupiter
Degrees
350
300
250
200
150
100
50
0
Jupiter
Sun
Elongation Jupiter
Degrees
100
0
−100
0
5
10
15
20
25
30
Julian Year

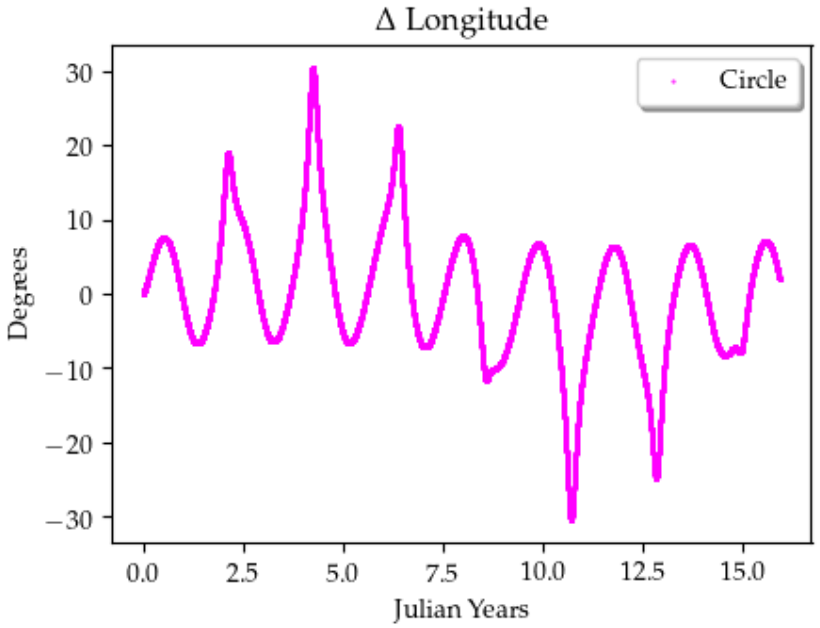

Longitude Saturn
Degrees
350
300
250
200
150
100
50
0
Saturn
Sun
Elongation Saturn
Degrees
100
0
−100
0
5
10
15
20
25
30
Julian Year

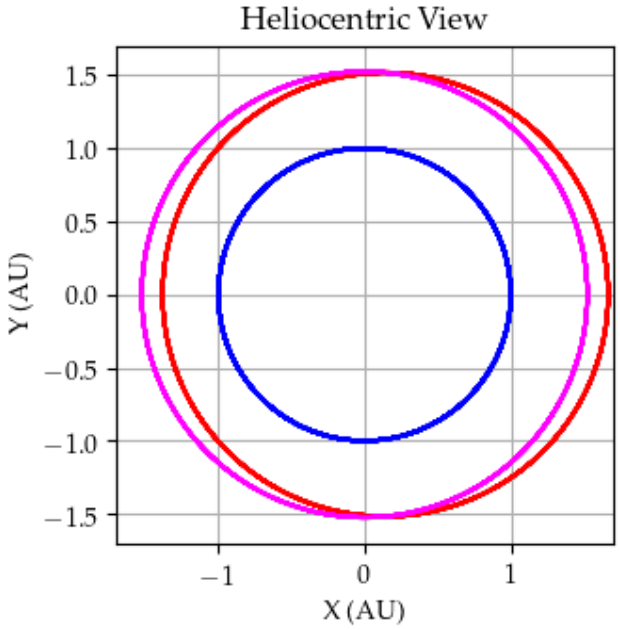
The Ptolemaic Model

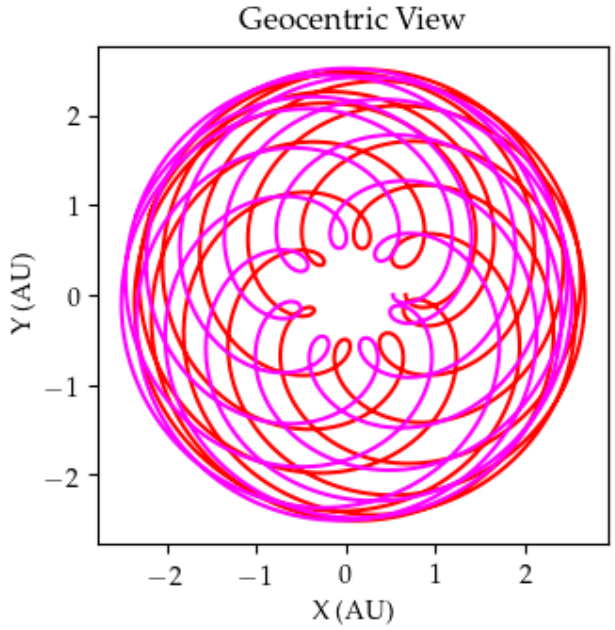


Circular and Centered Orbit

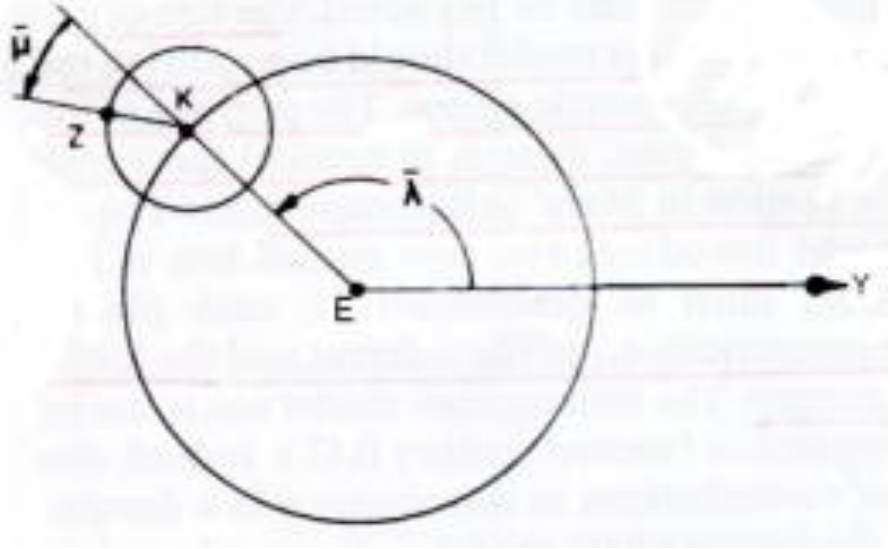






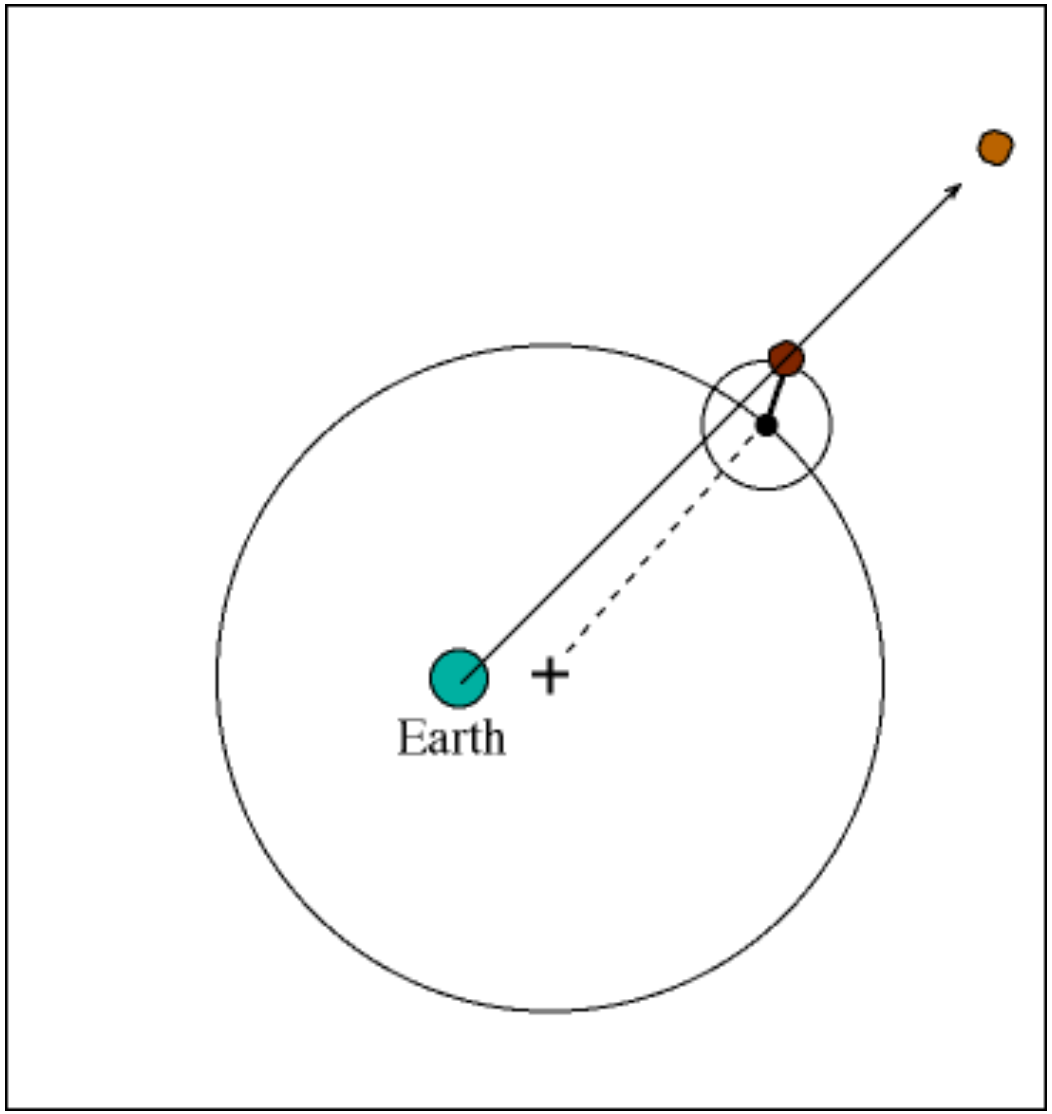


Regular Epicycles produce regular retrogradations

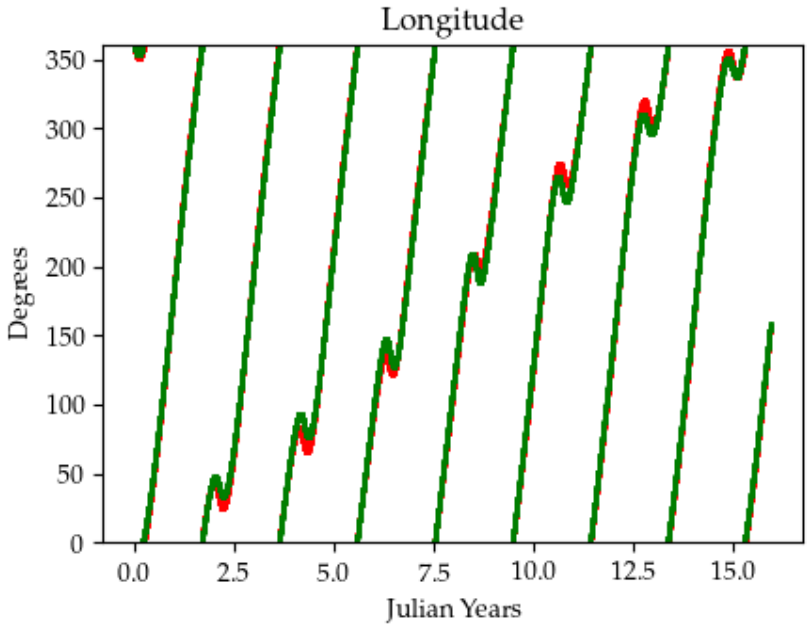


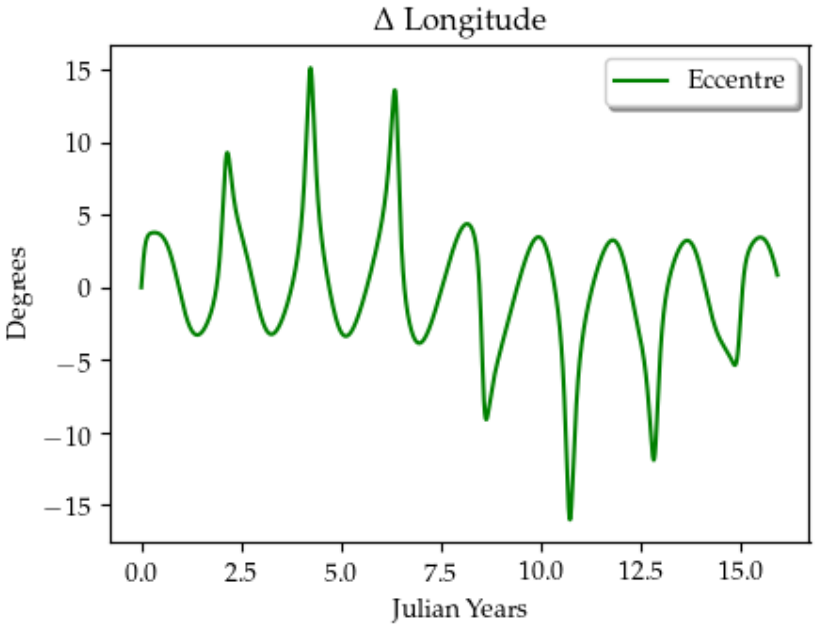
In reality the retrogradations vary both in position and duration.

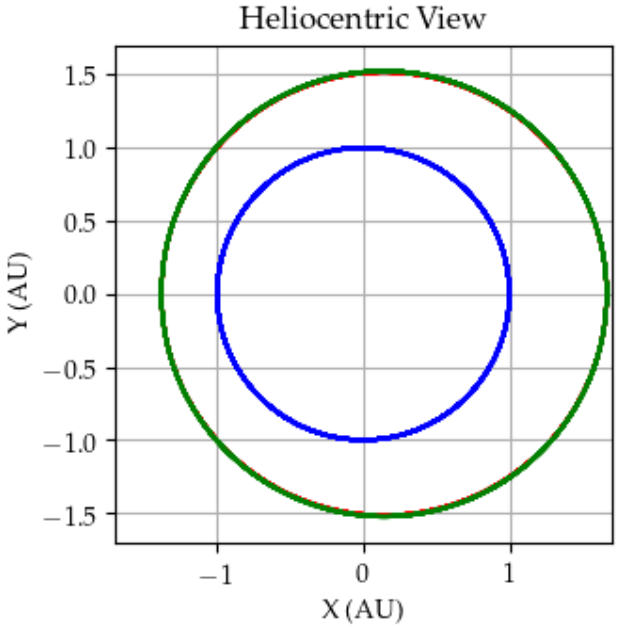
Move the Earth from the Center

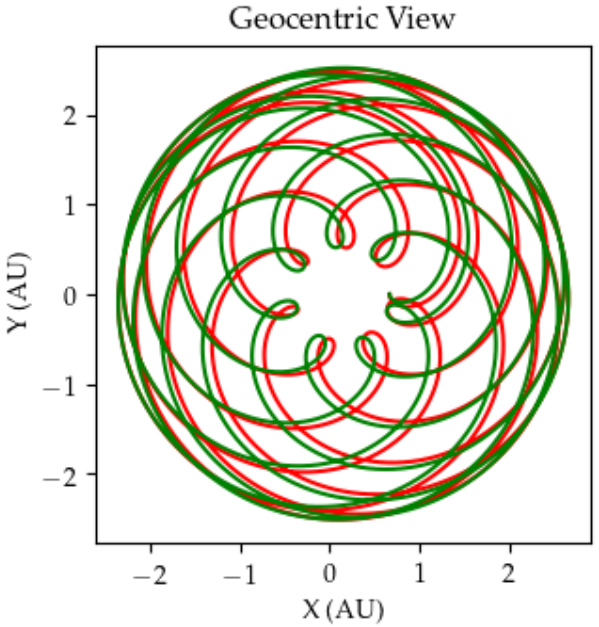


Circular Off-Centered Orbit (the Eccentre)

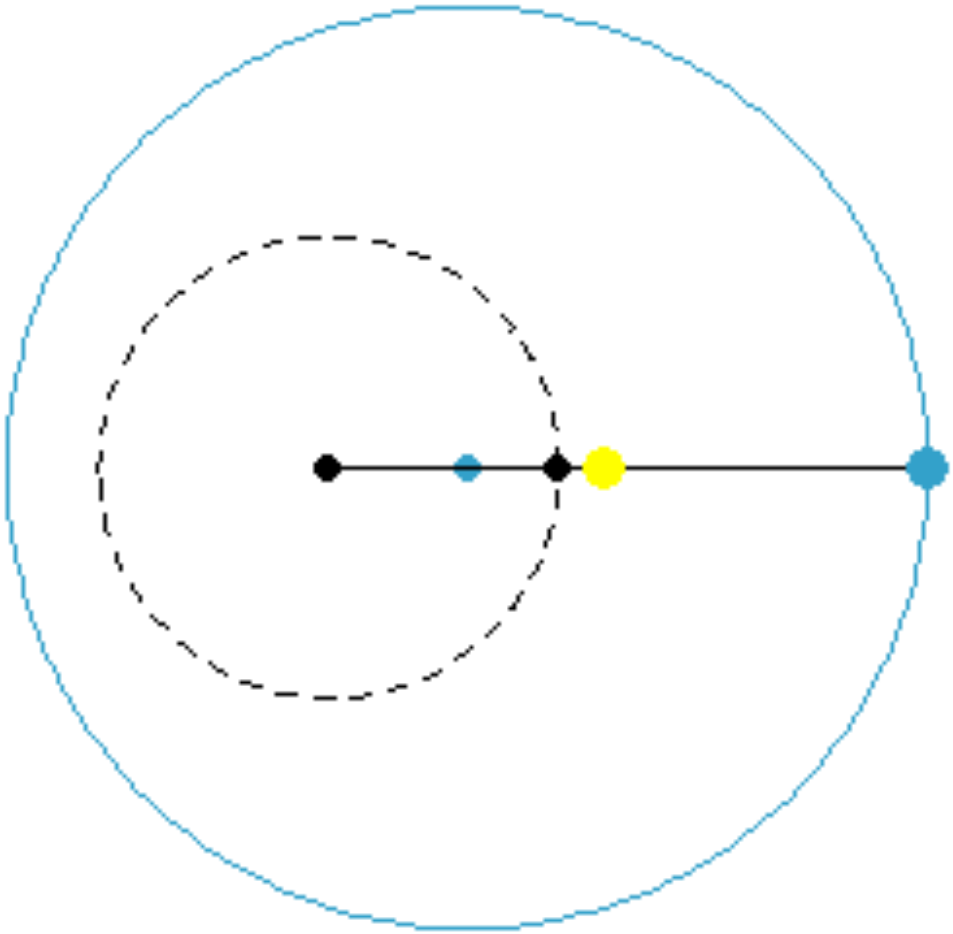






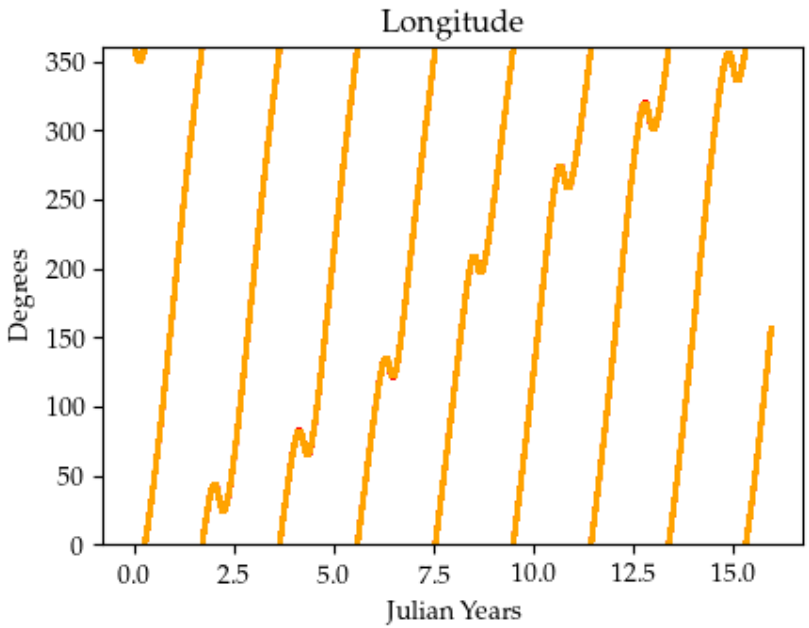


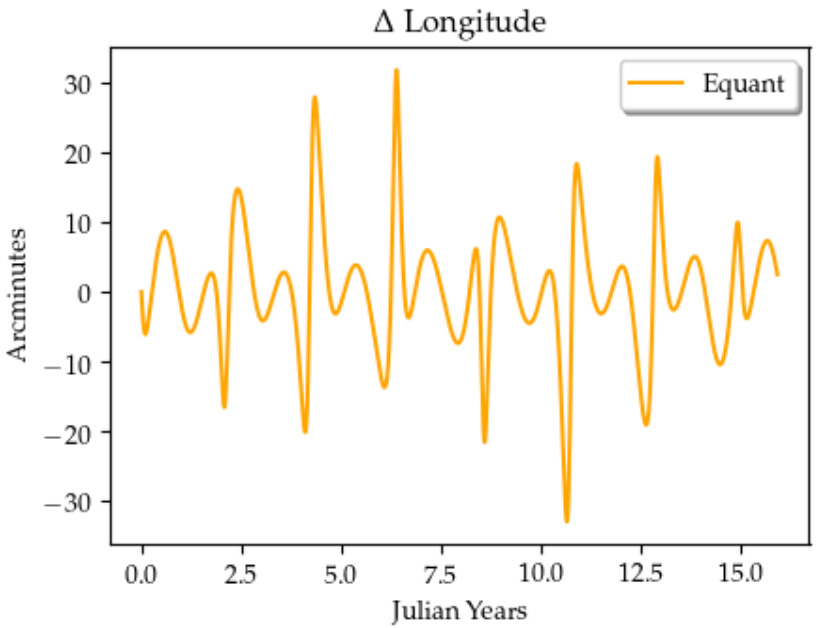
The Equant

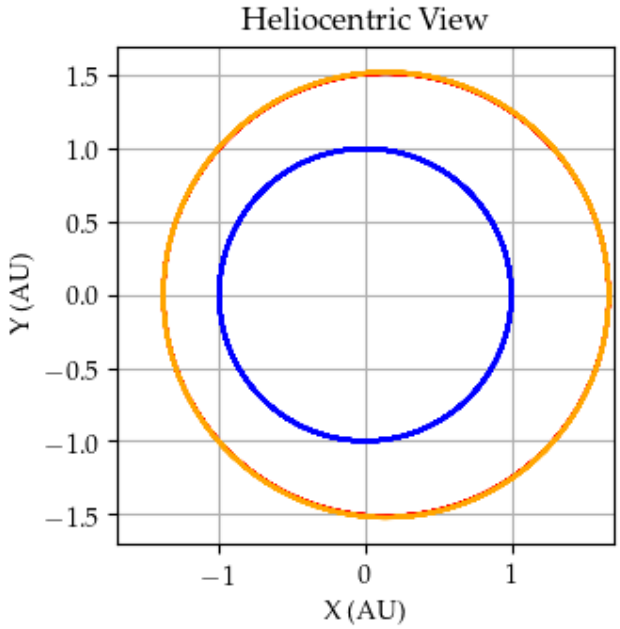


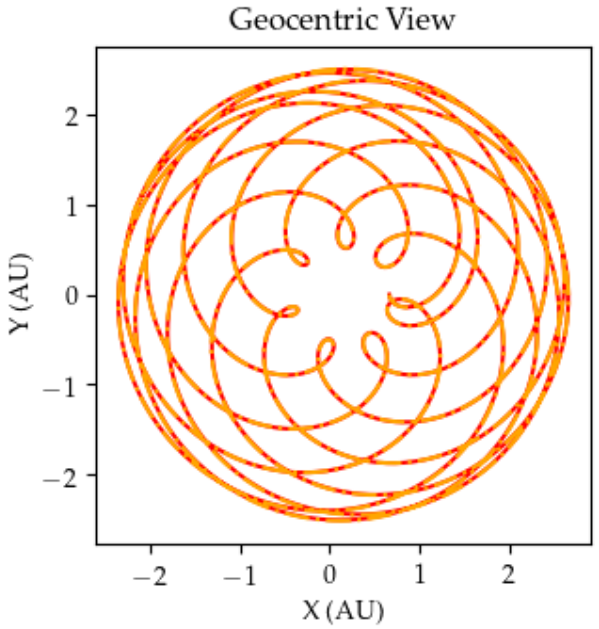
**Around a special point (the equant),
the motion is uniform**

The Equant

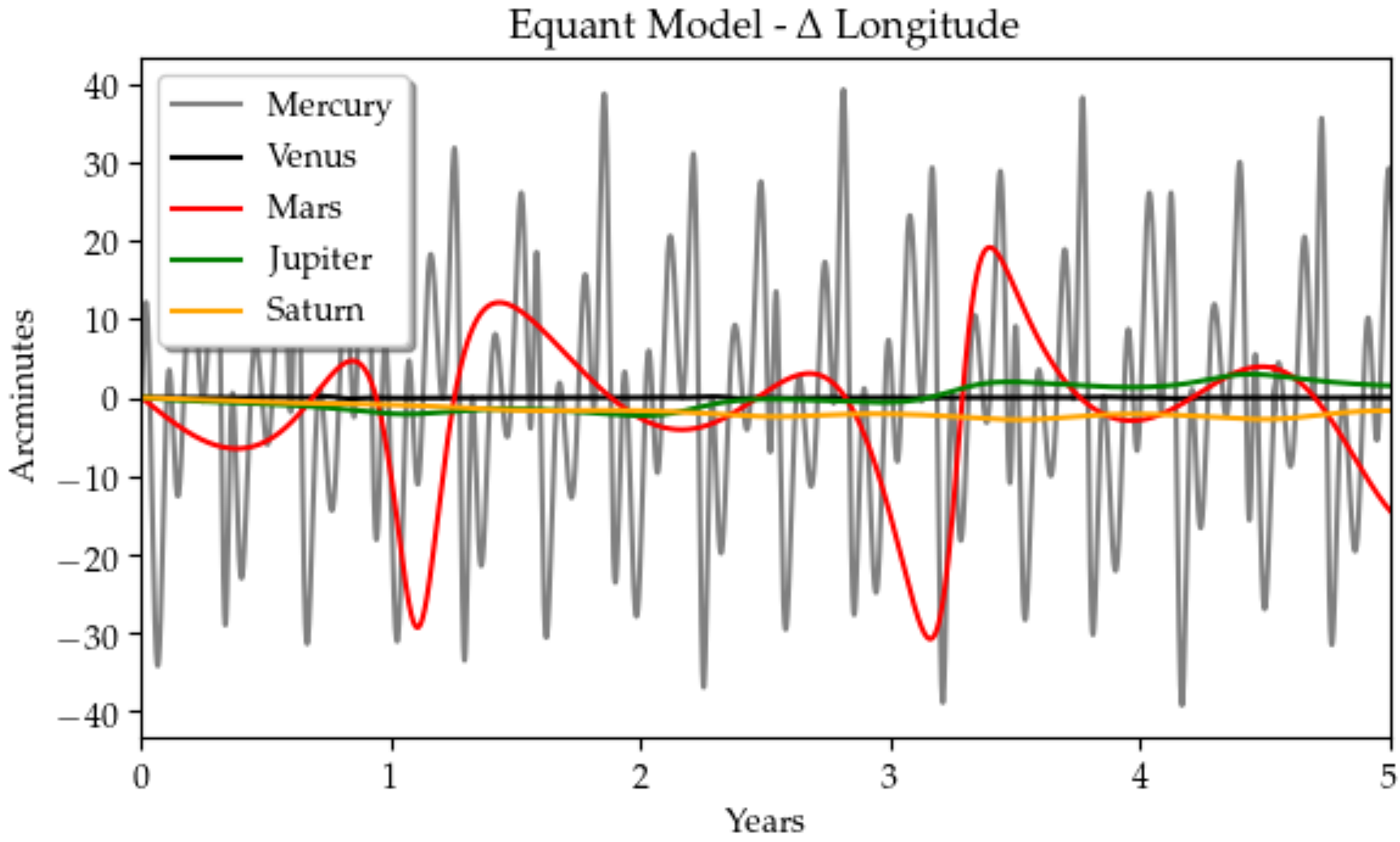

Longitude
Degrees
Julian Years
0
50
100
150
200
250
300
350
0.0
2.5
5.0
7.5
10.0
12.5
15.0


Δ Longitude
Arcminutes
Julian Years
Equant
30
20
10
0
−10
−20
−30
0.0
2.5
5.0
7.5
10.0
12.5
15.0

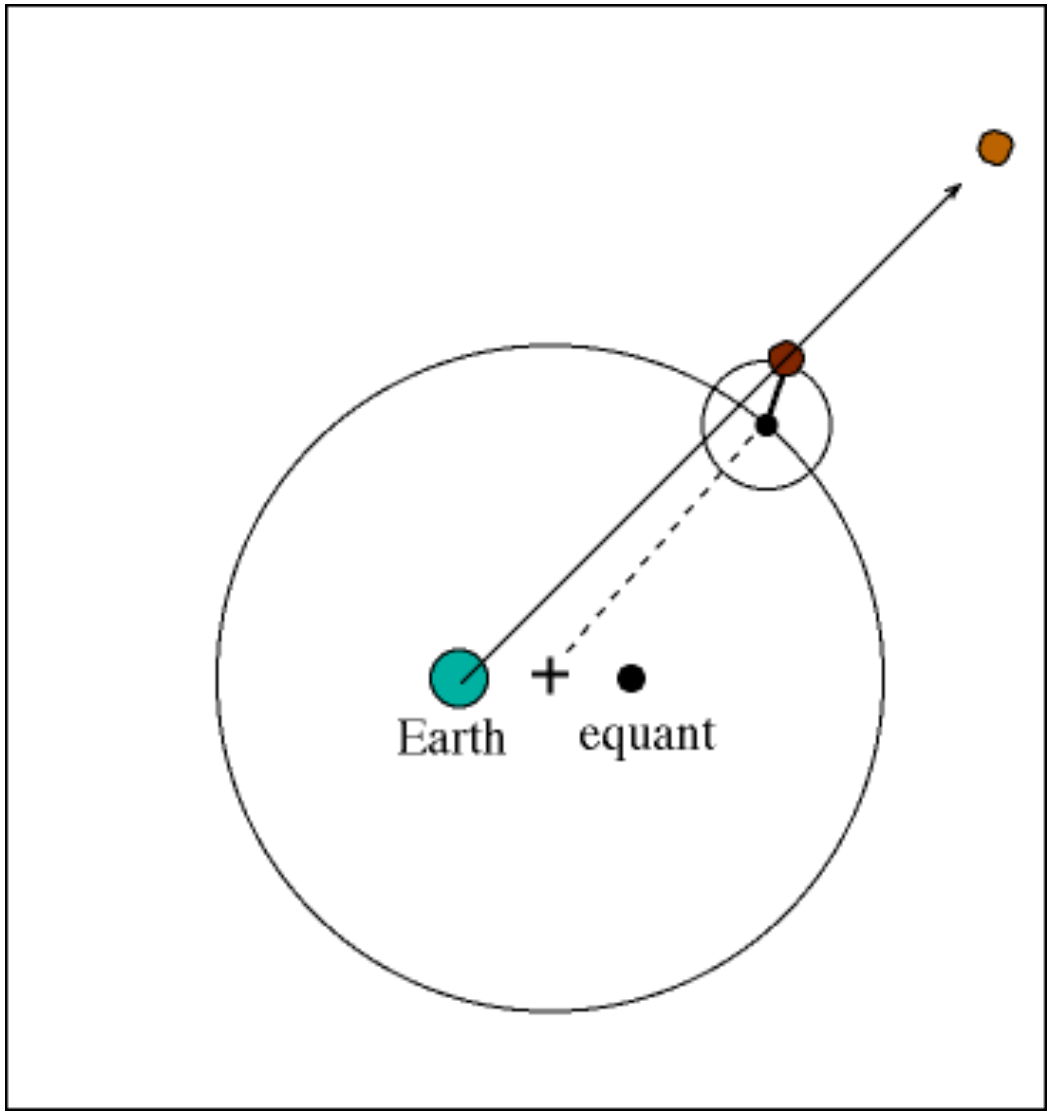

Heliocentric View
Y (AU)
X (AU)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1
0
1


Geocentric View
Y (AU)
X (AU)
2
1
0
−1
−2
−2
−1
0
1
2

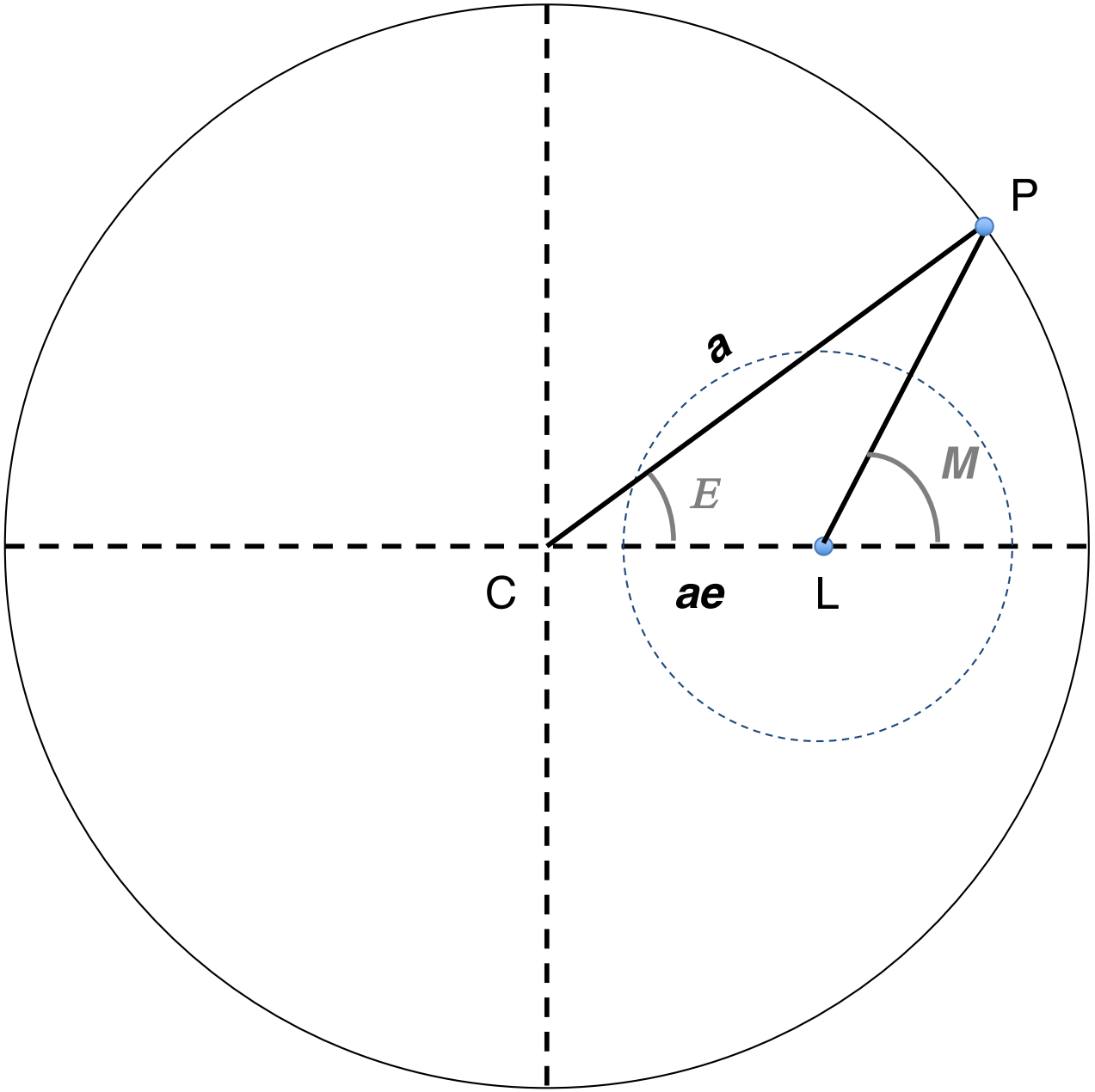
The Equant



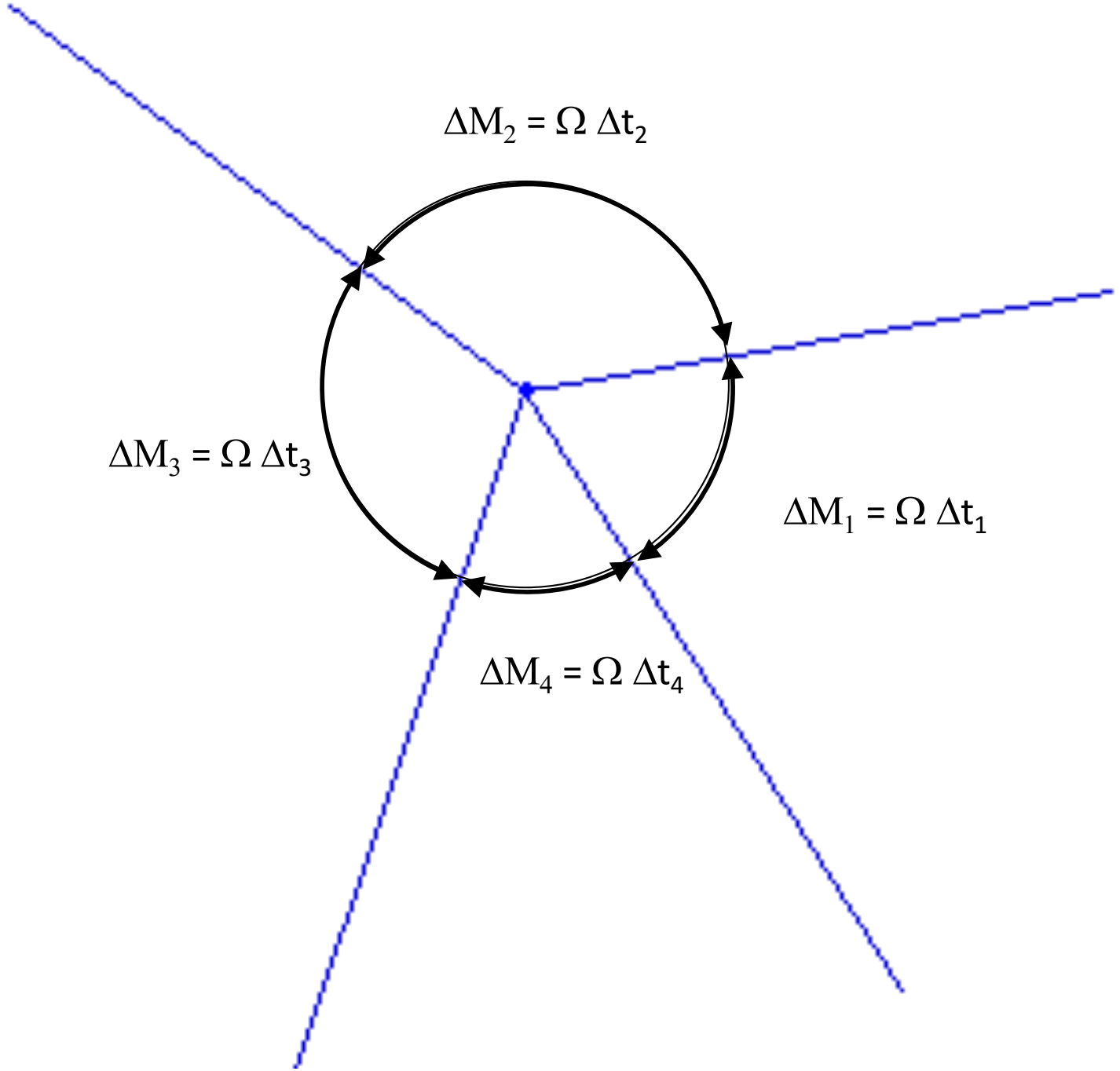
Even for the highest eccentricity (Mercury),
the equant reproduces the observations to less than a degree accuracy.


Earth
equant

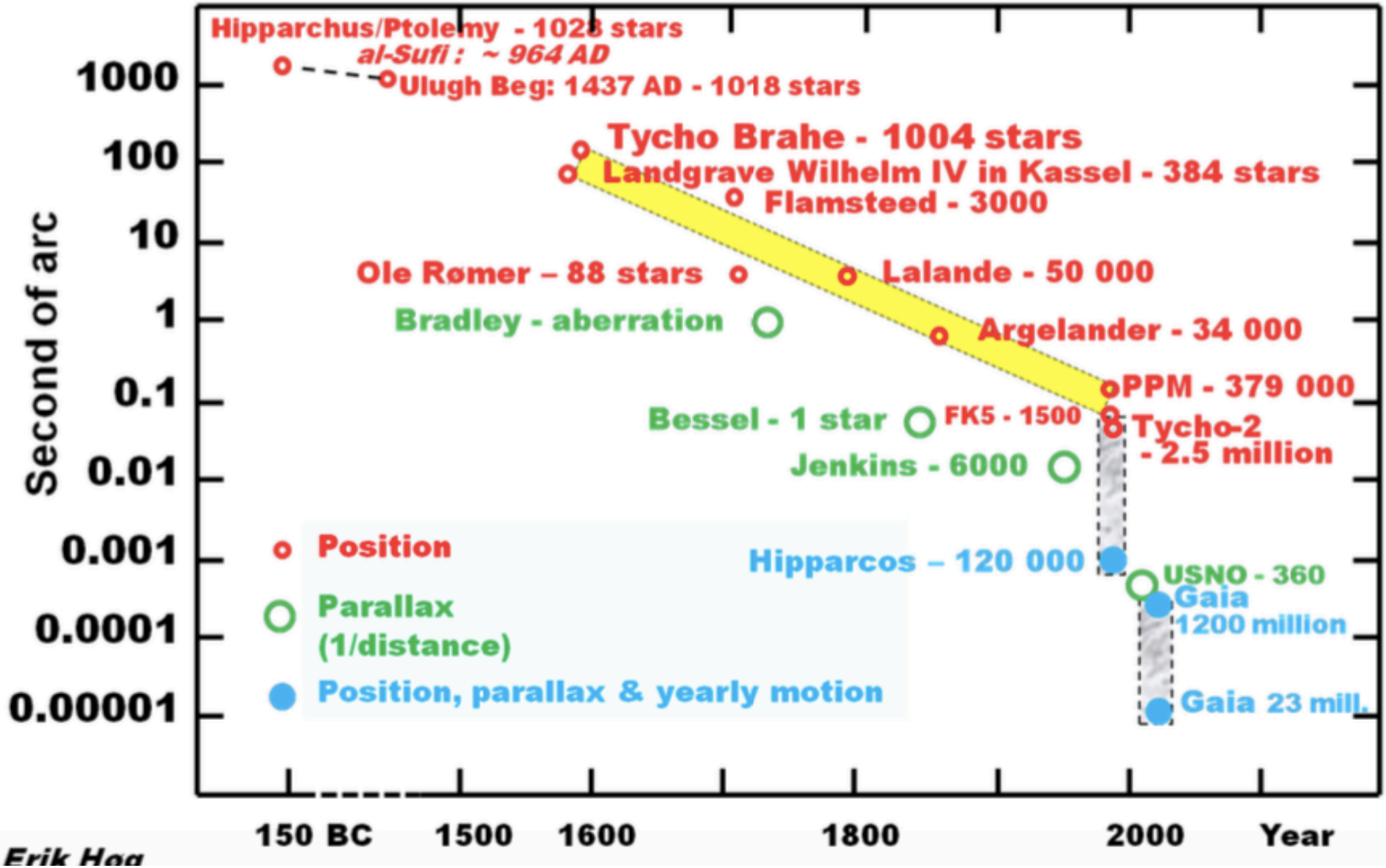
The Equant



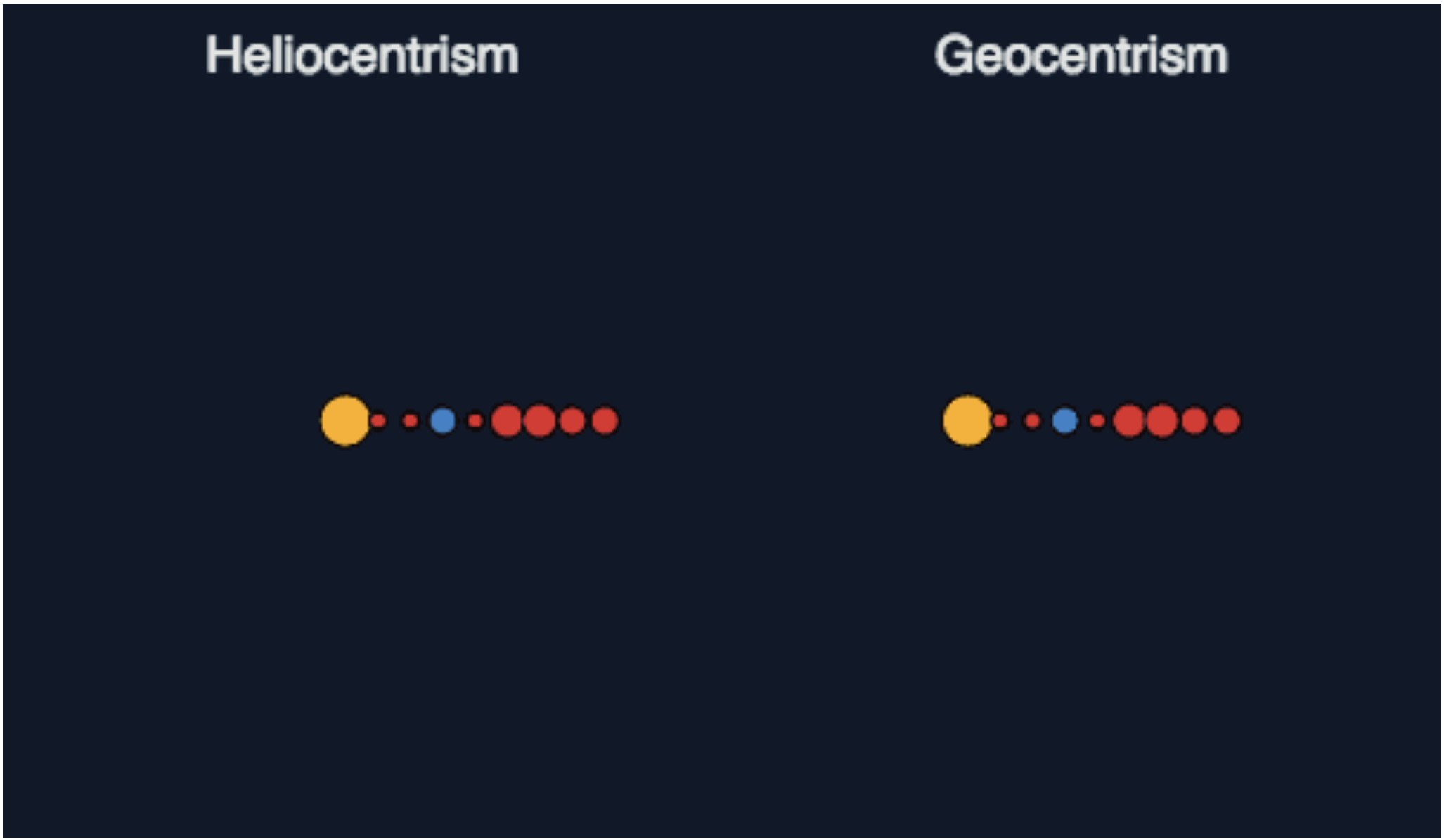
Equant Point of View



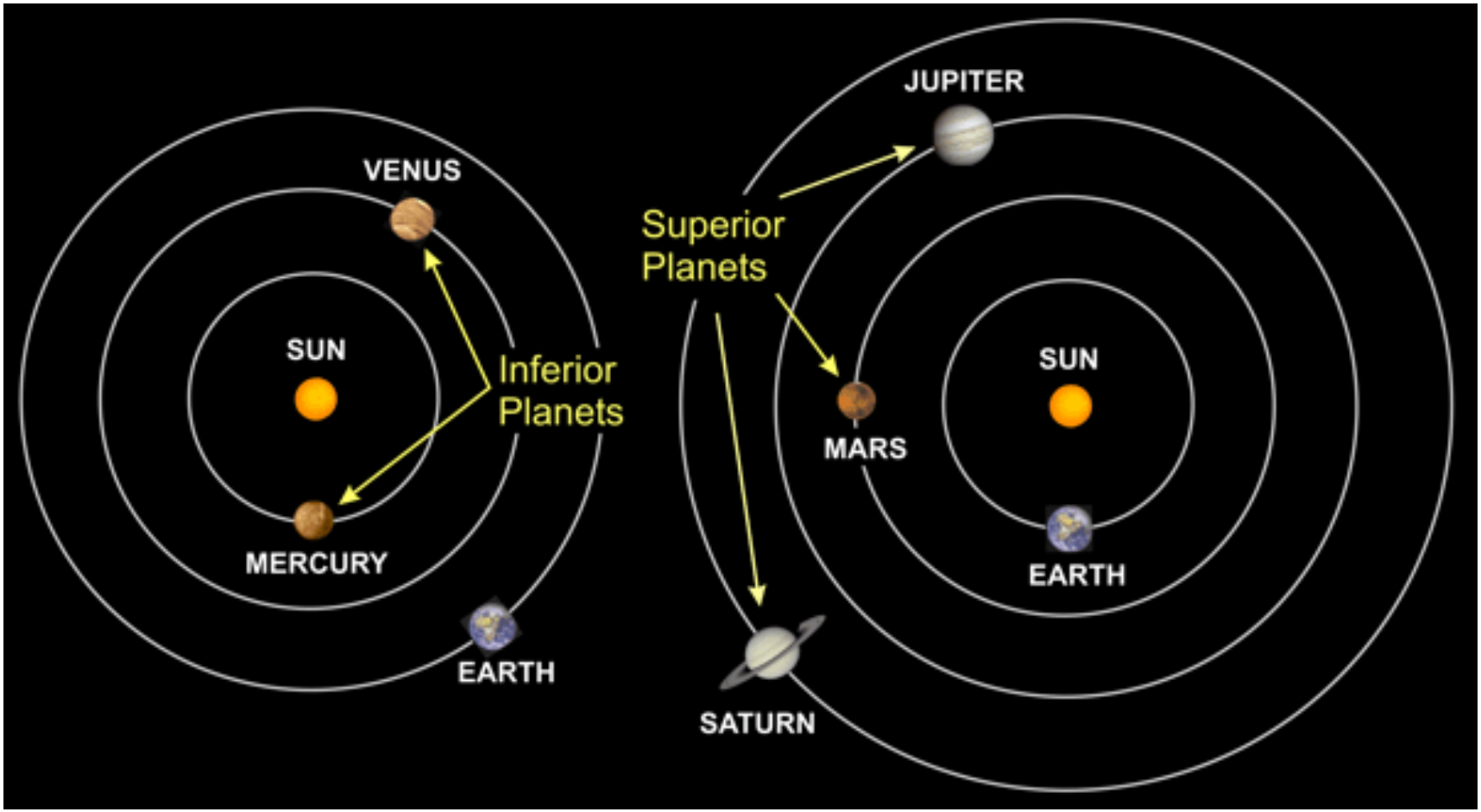
Astrometric Accuracy during 2000 Years



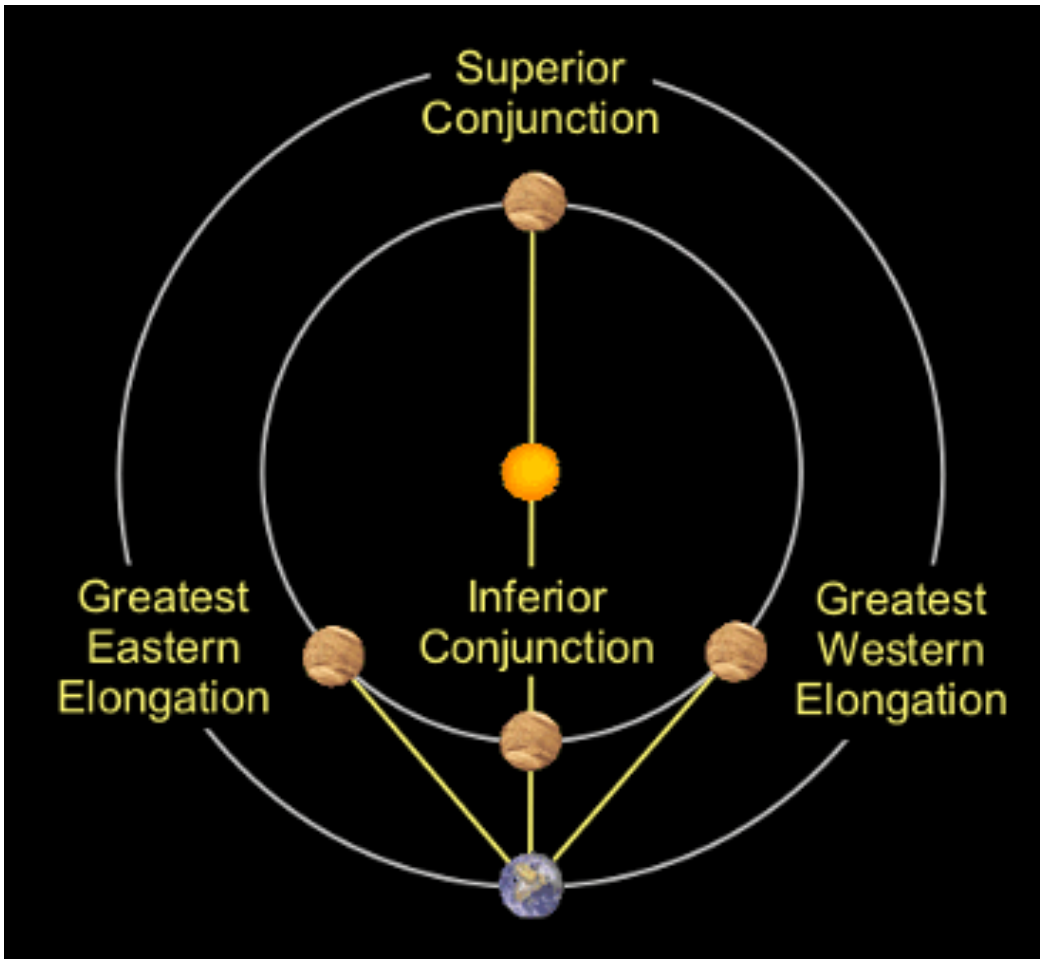
Erik Høg


Heliocentrism
Geocentrism

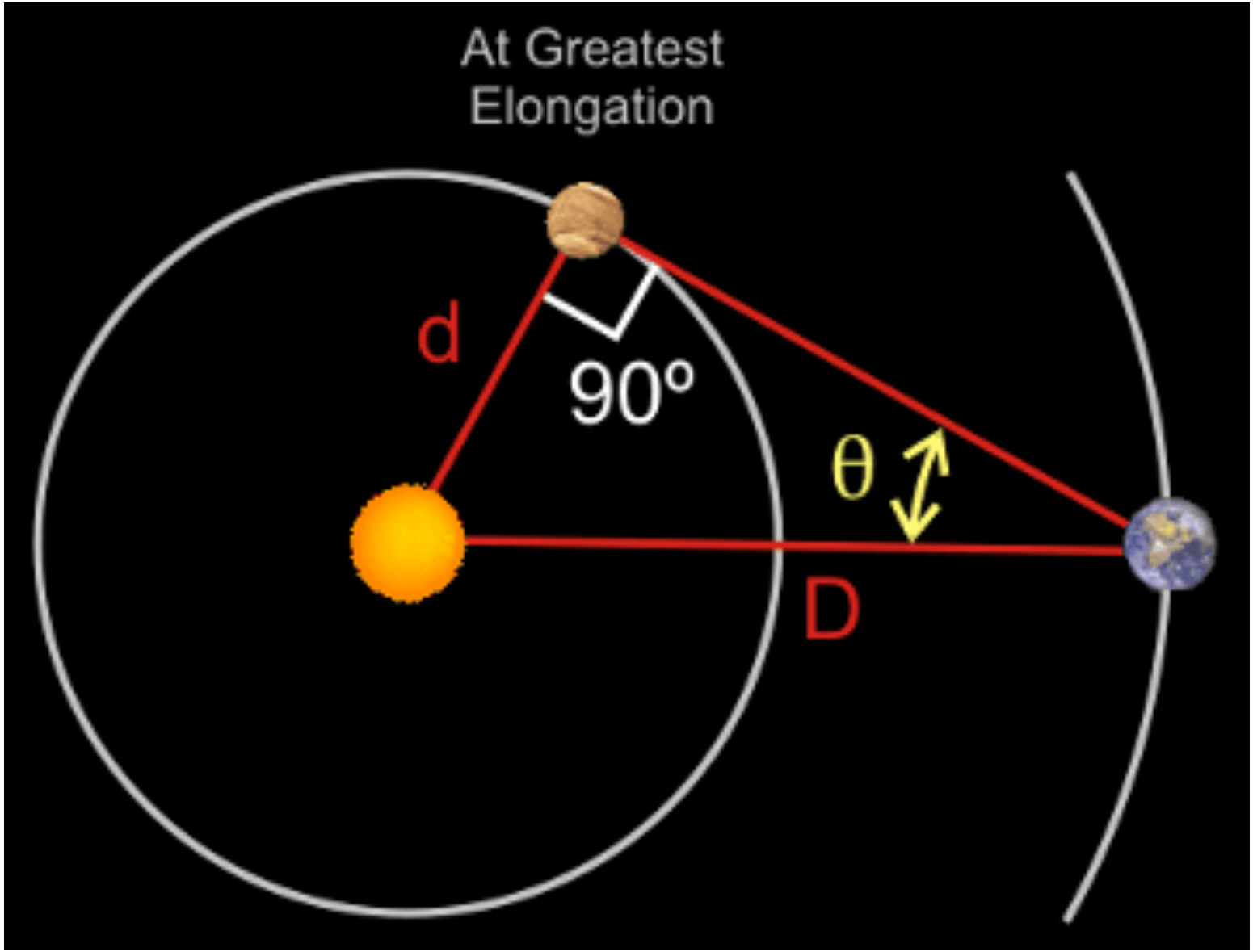
Inferior and Superior planets in the Copernican Model



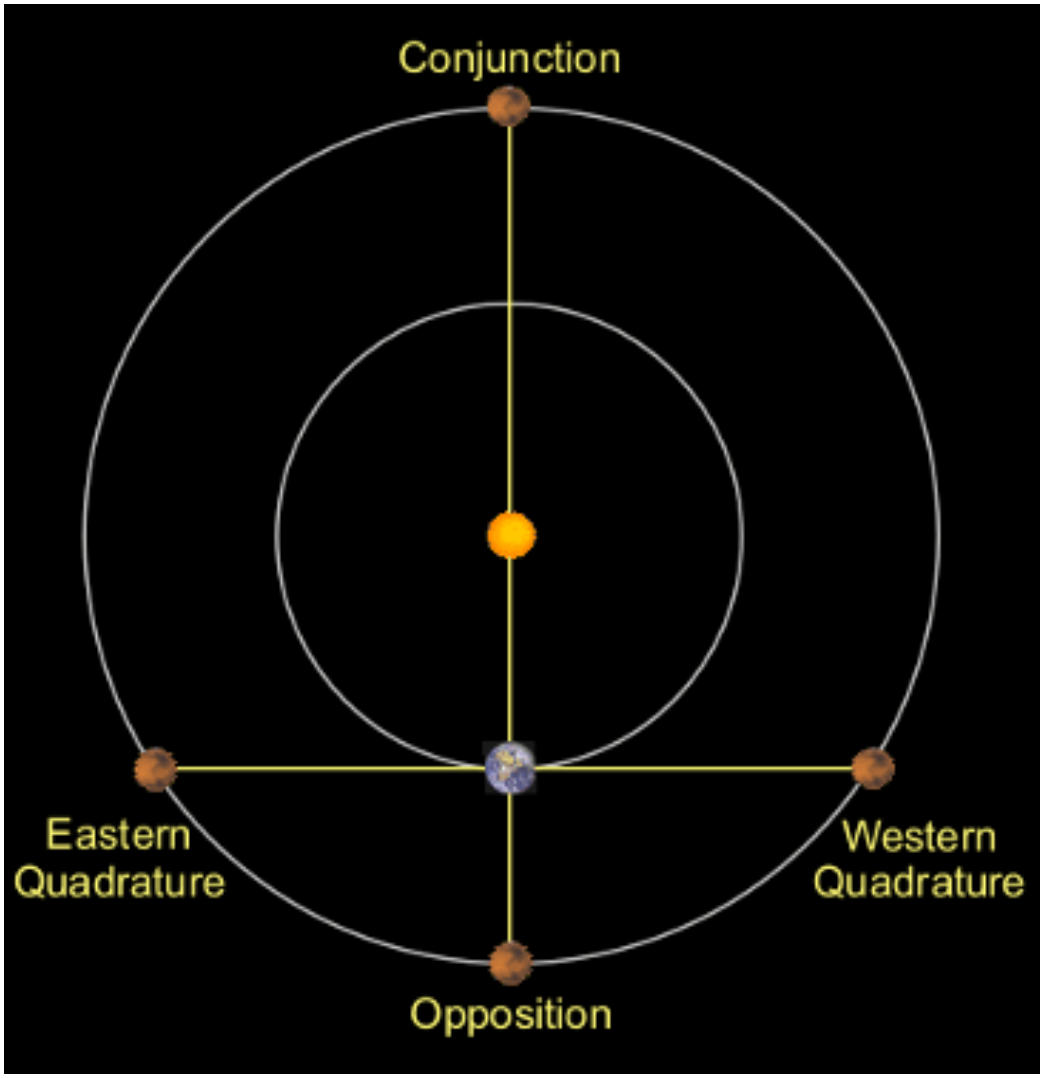
Inferior planets : special points

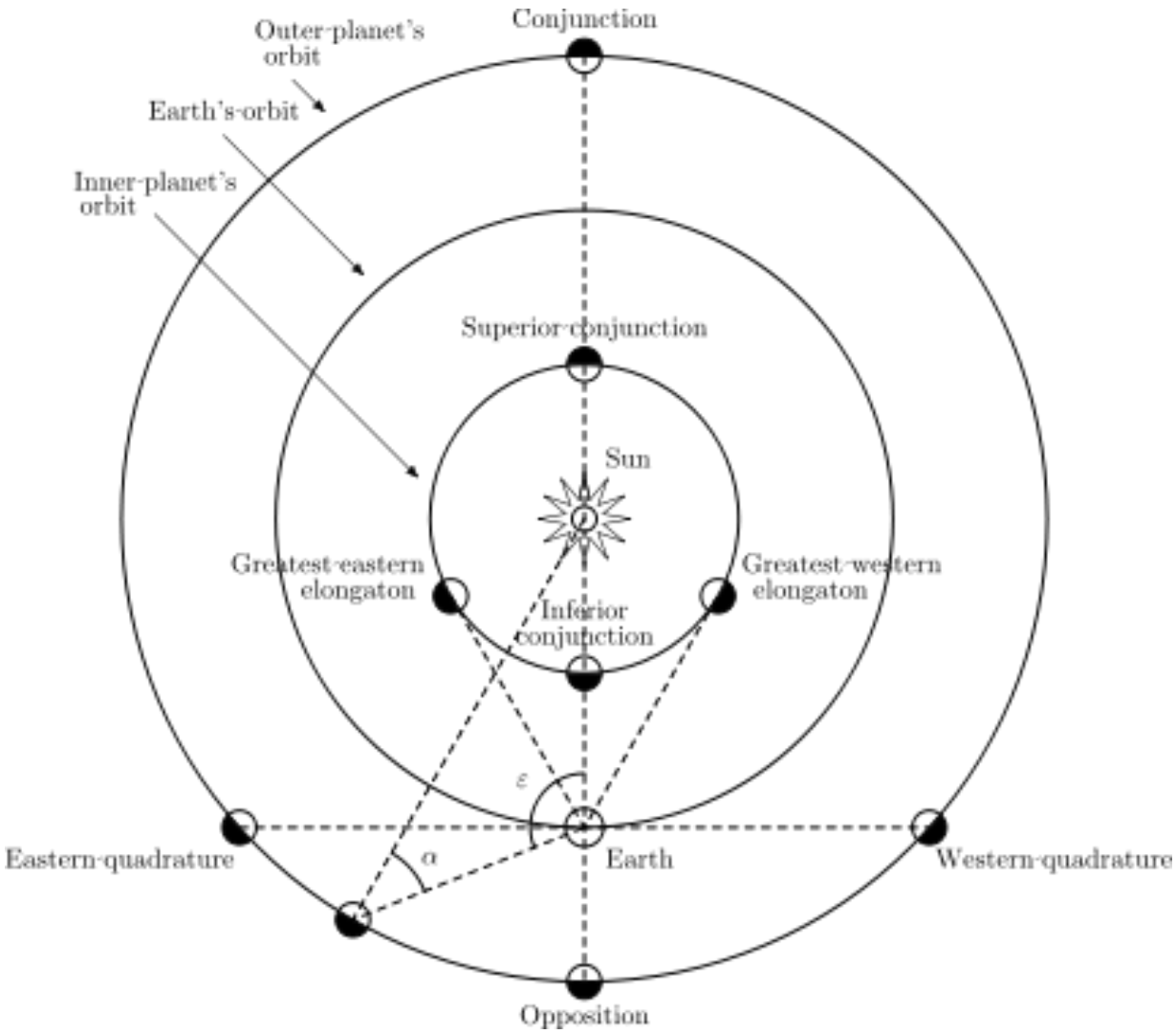


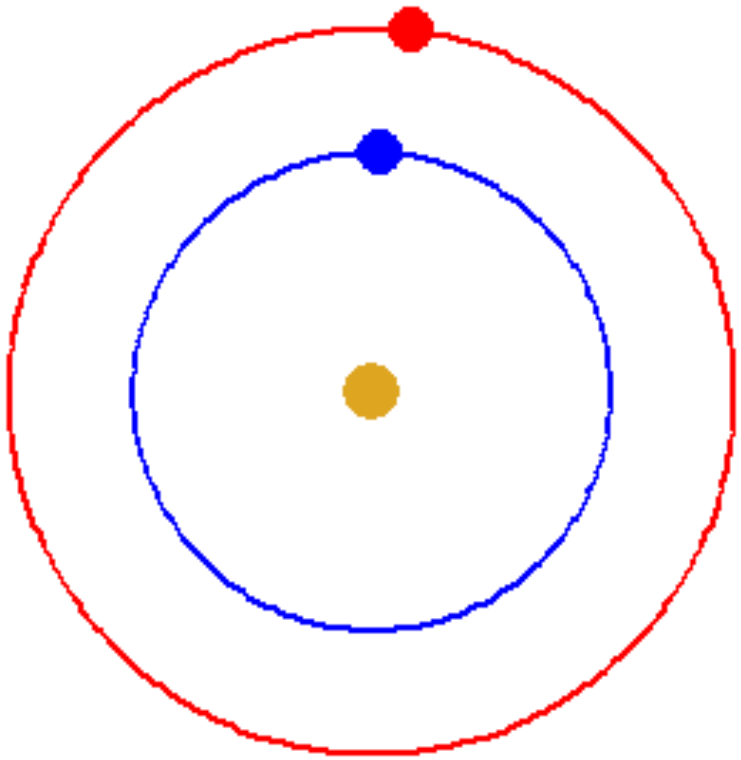
Distance to the inferior planets



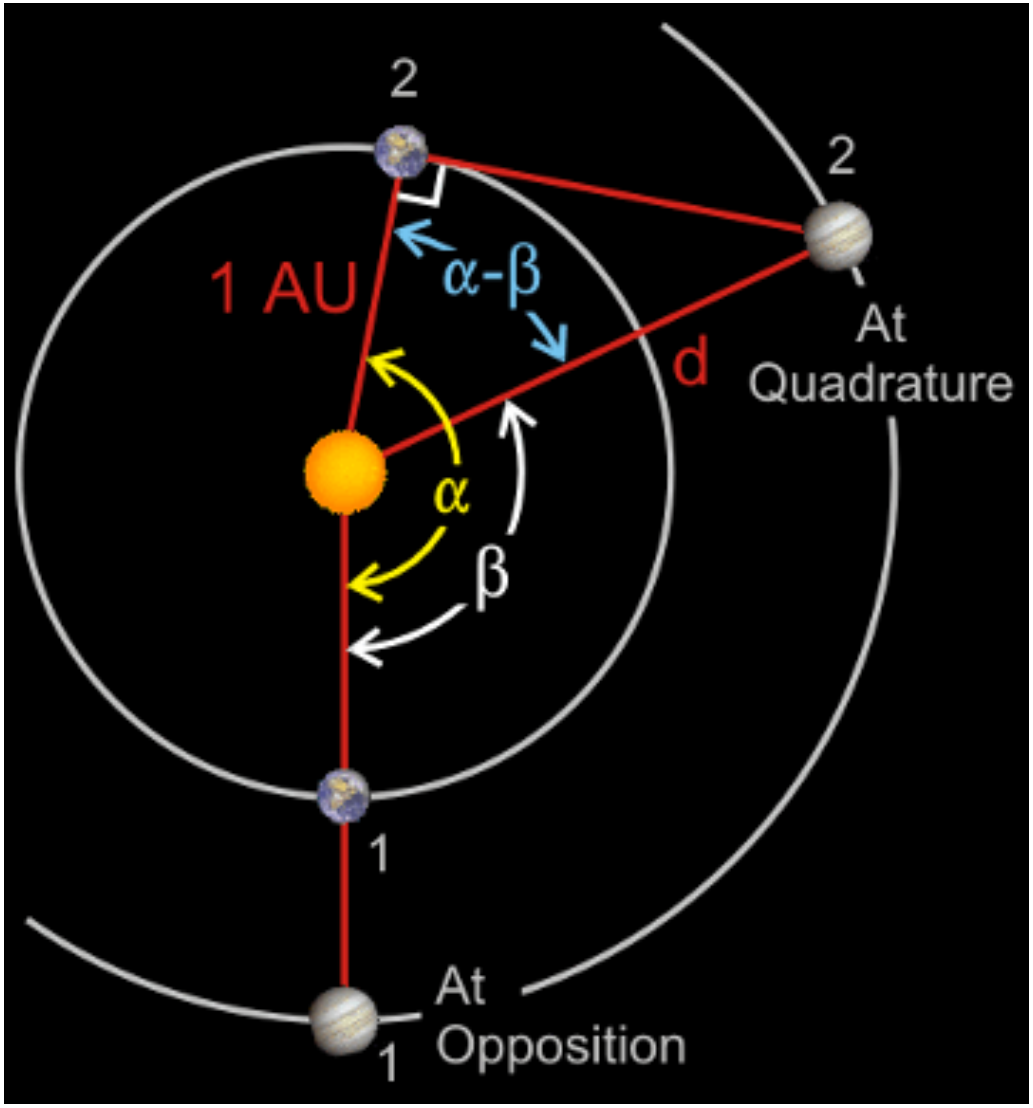
Superior planets : special points



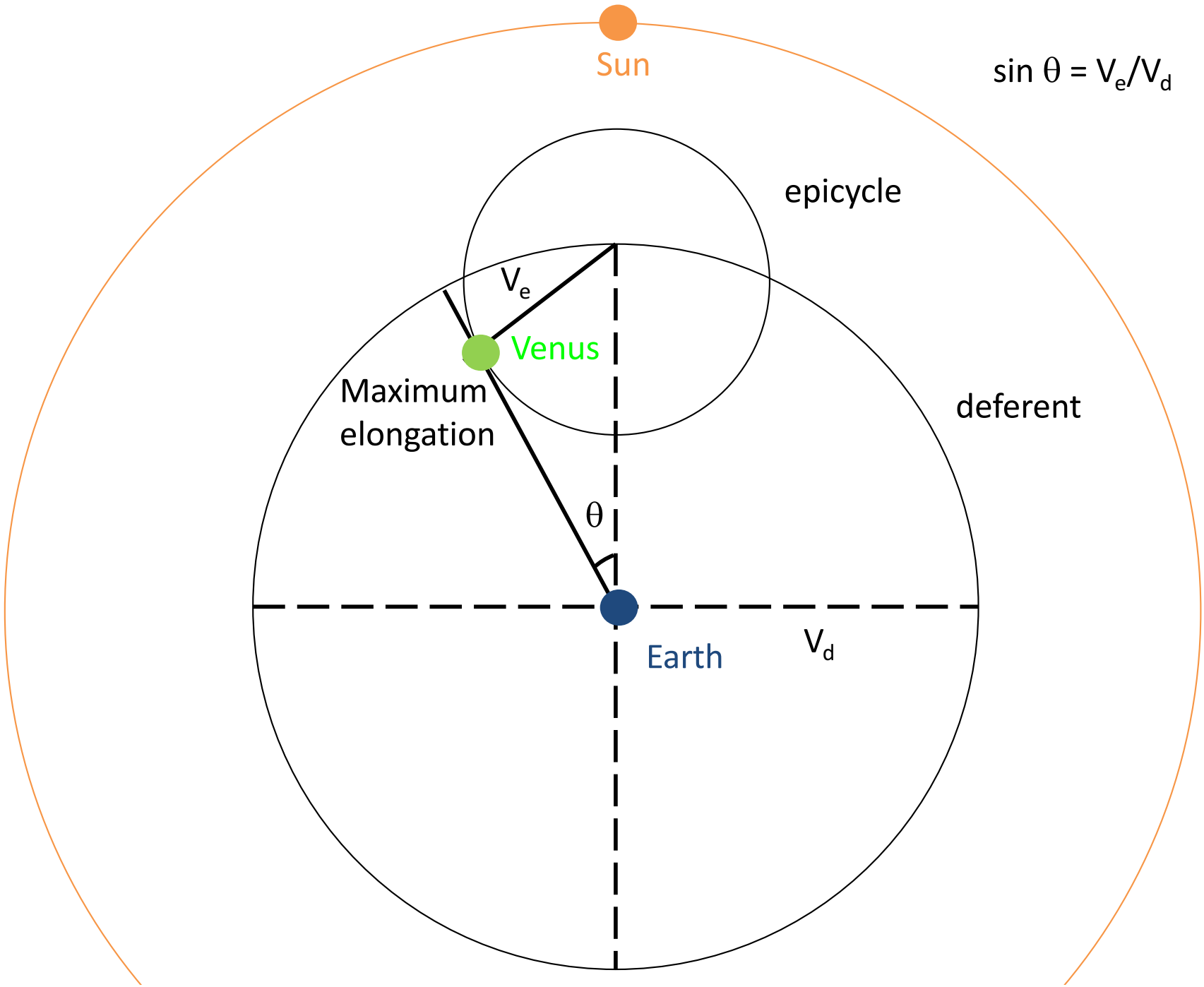

Outer-planet's orbit
Earth's-orbit
Inner-planet's orbit
Conjunction
Superior-conjunction
Sun
Greatest-eastern elongaton
Greatest-western elongaton
Inferior conjunction
ε
α
Eastern-quadrature
Earth
Western-quadrature
Opposition

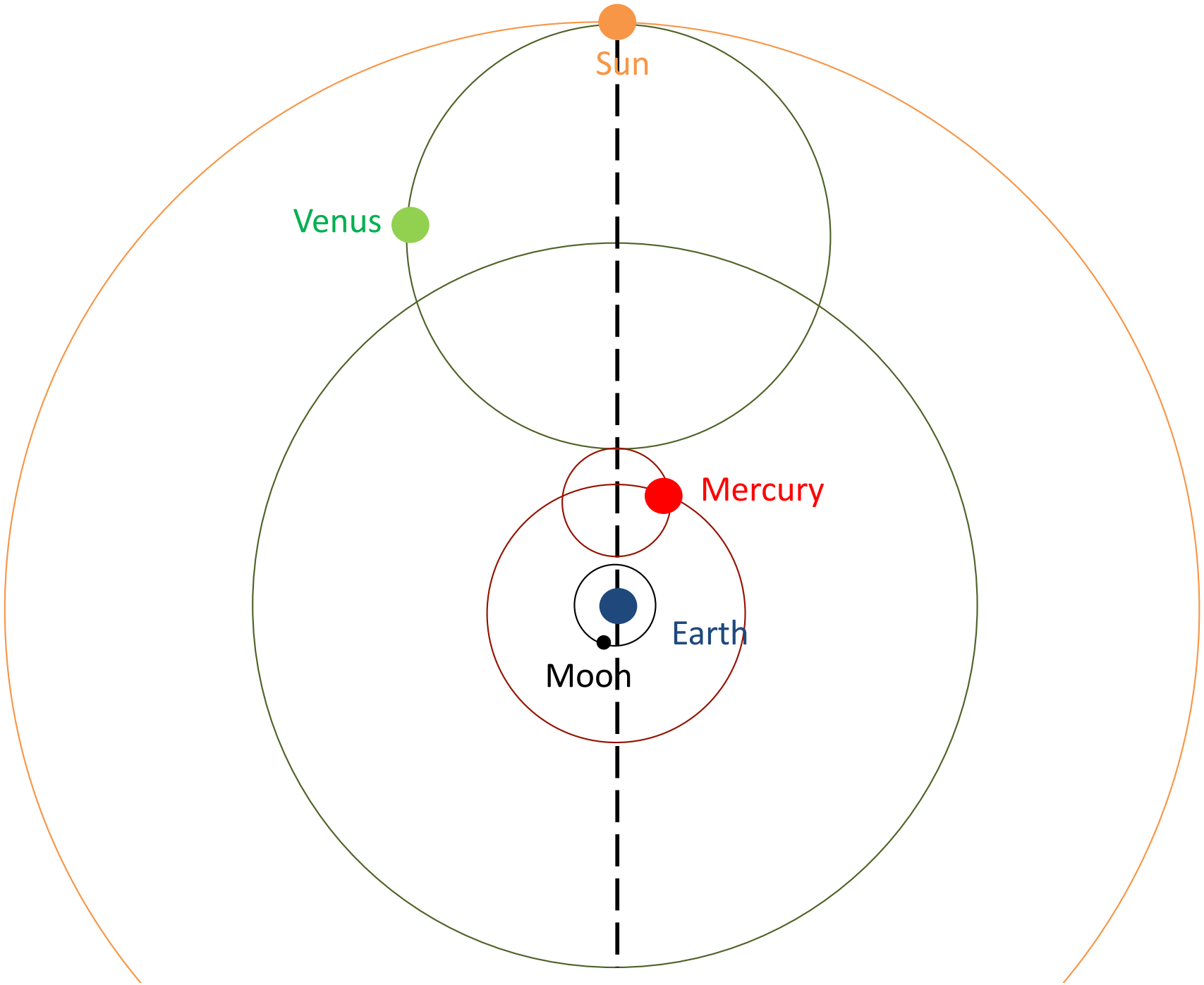


Distances to the superior planets

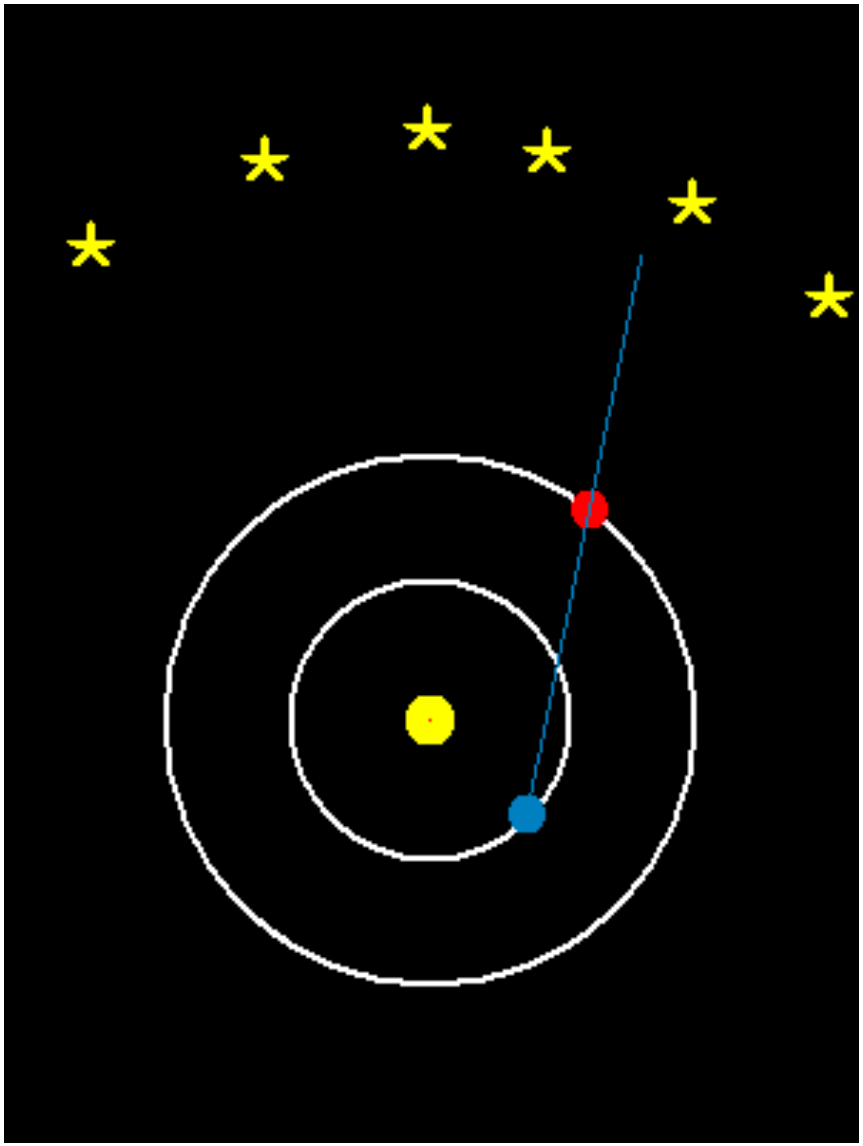


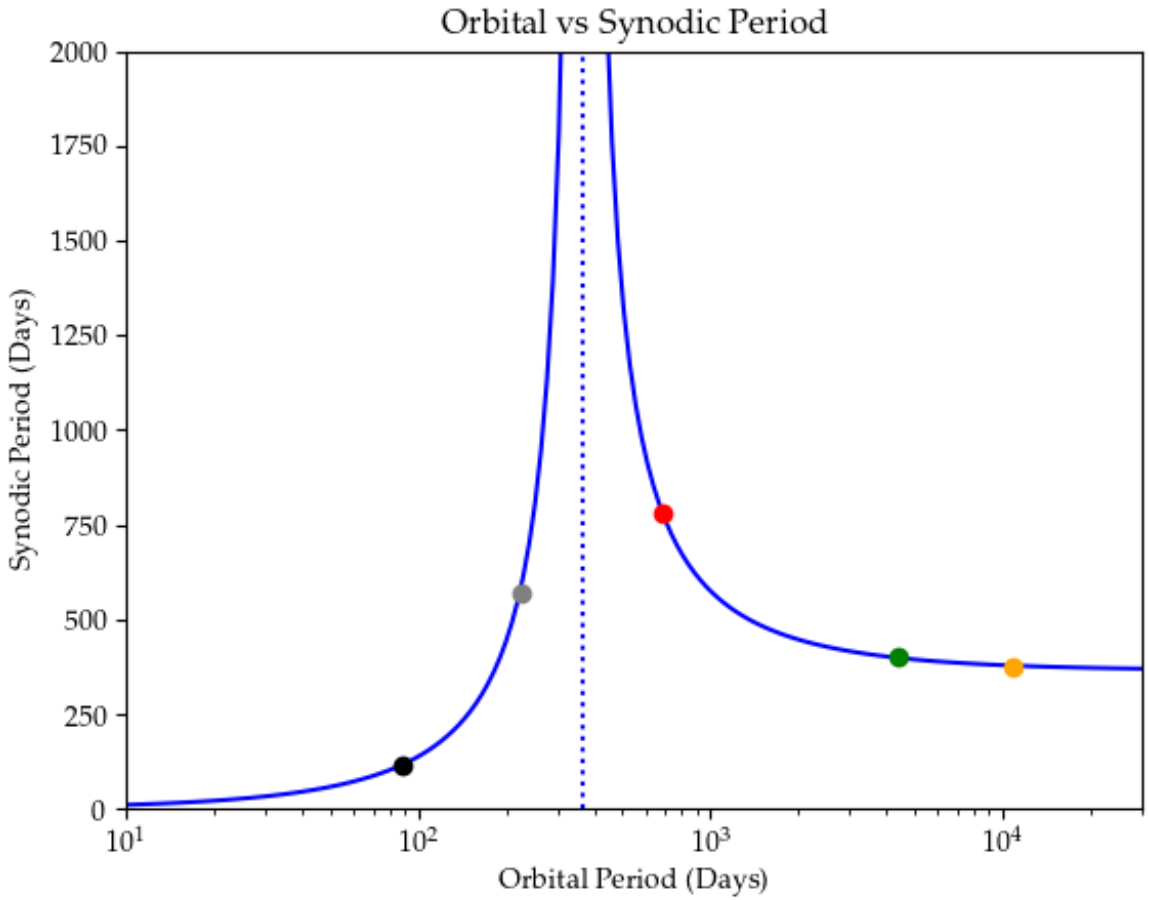
Distance to the inferior planets: Ptolemaic System



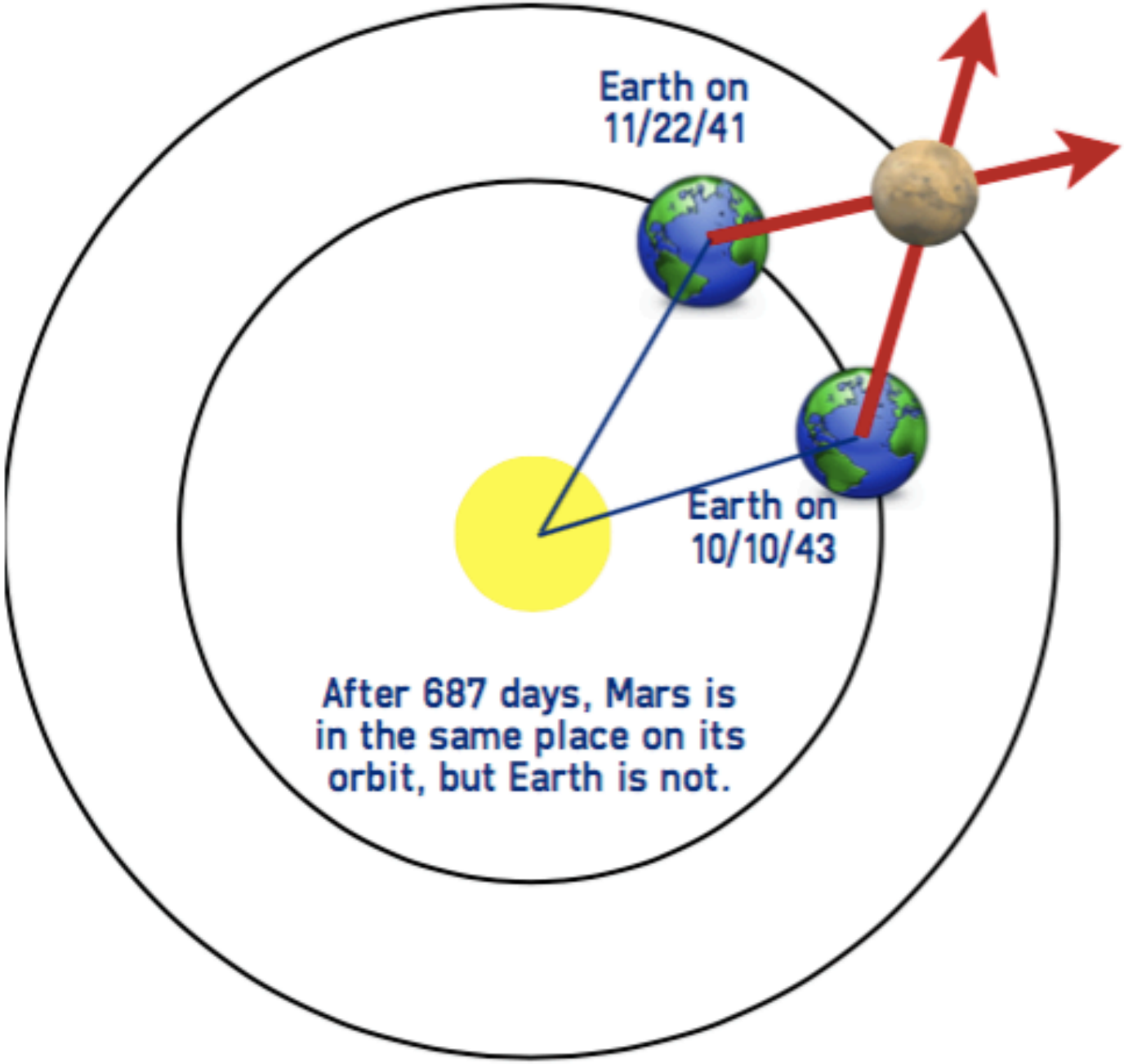

Sun
Venus
Mercury
Earth
Mooh

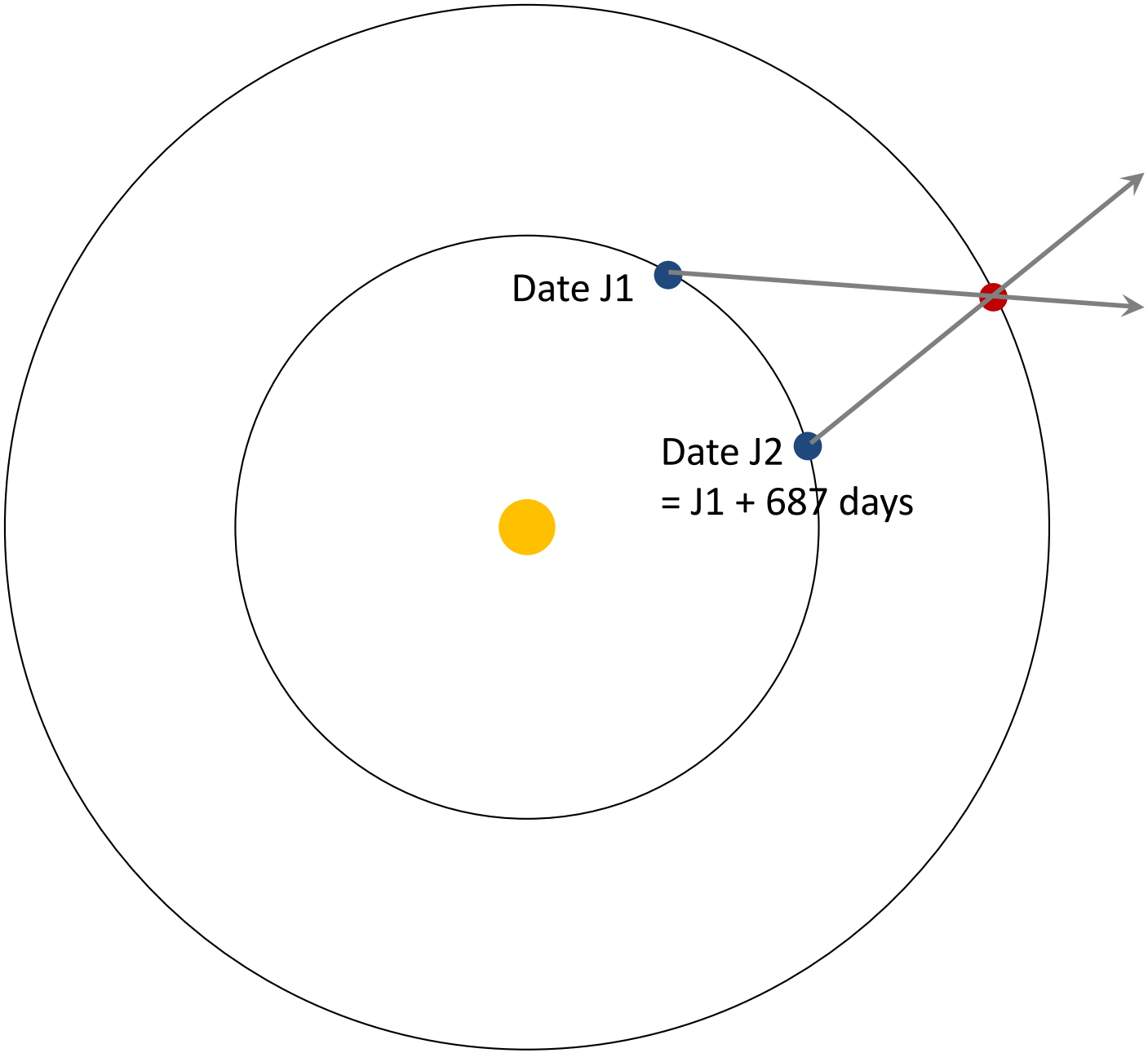
Retrograde Motion

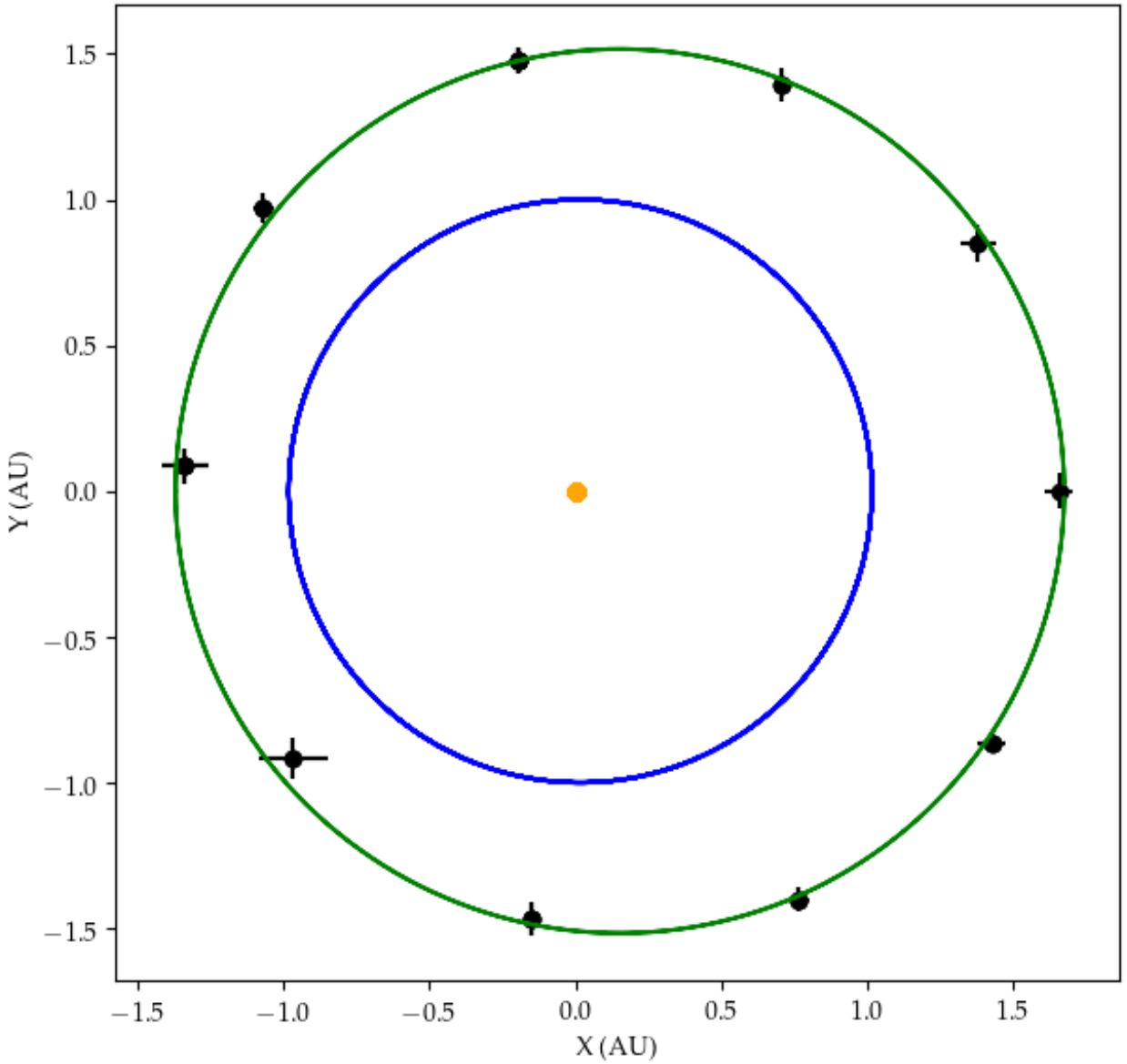


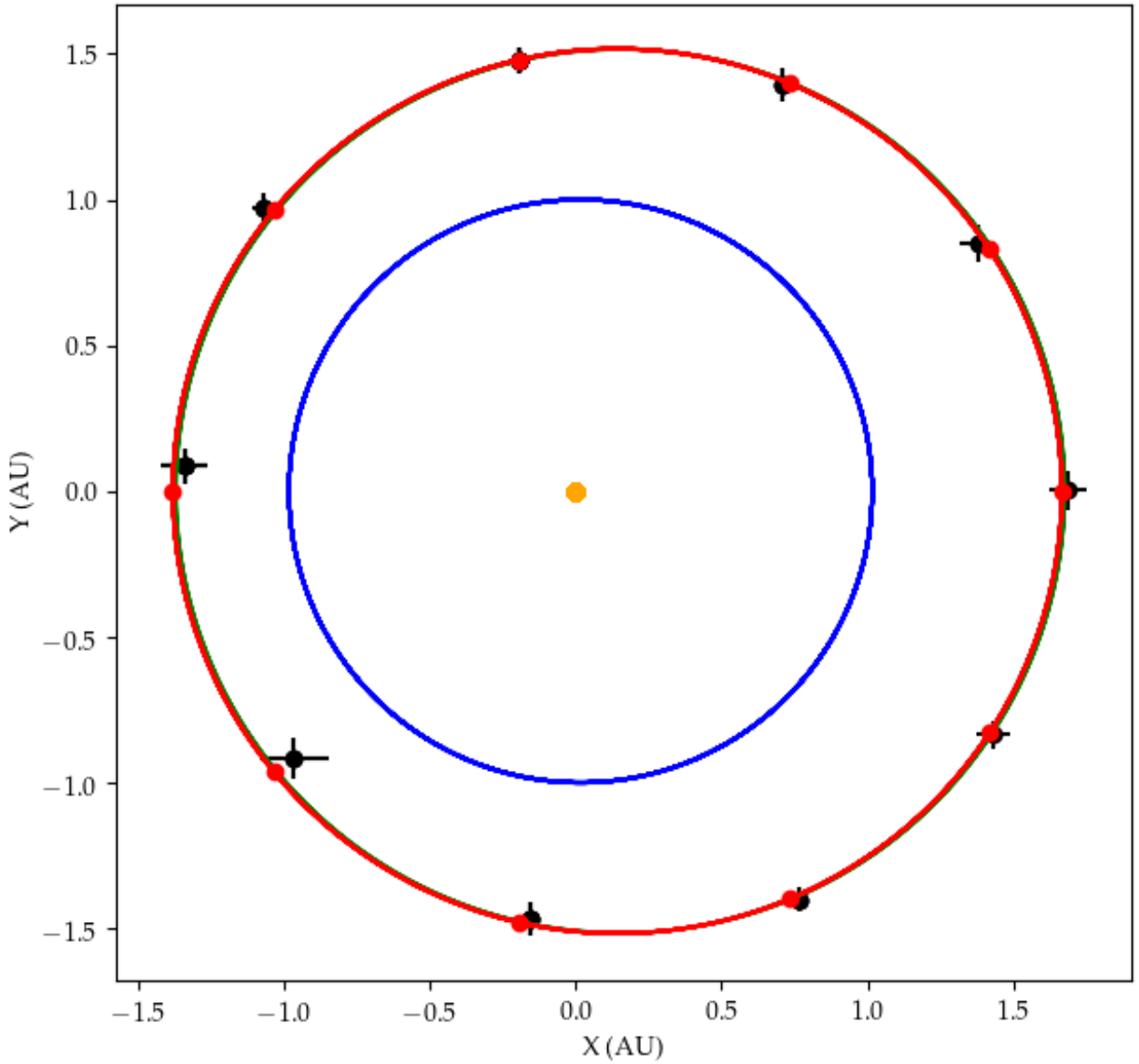

Orbital vs Synodic Period
Synodic Period (Days)
Orbital Period (Days)
2000
1750
1500
1250
1000
750
500
250
0
10^1
10^2
10^3
10^4

Constructing an Orbit




Date J1
Date J2
= J1 + 687 days


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Y (AU)
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X (AU)


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Y (AU)
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X (AU)

① inclination of planet
② helical/synodic anomaly
→ epicycle
③ zodiacal/ecliptic
→ eccentric
④ precession
- the movement of the fixed stars, planets
(A D K) 1°/100 years
⑤ equant

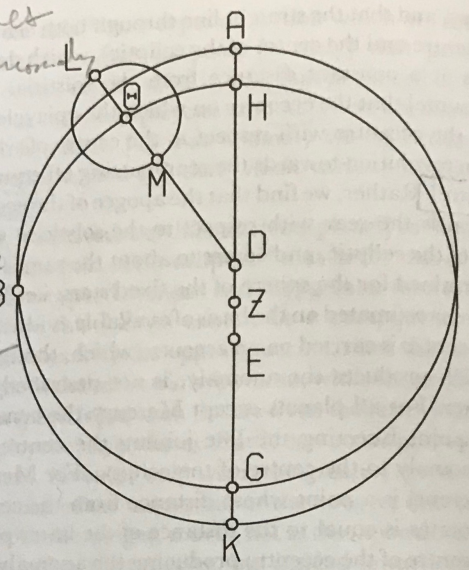


Fig. 9.1

planets. Next, we say that the whole plane [of the eccentre] moves uniformly about centre E towards the rear [i.e. in the order] of the signs, shifting the position of apogee and perigee 1° in 100 years, and that diameter LΘM of the epicycle rotates uniformly about centre D, again towards the rear [i.e. in the
H255 order] of the signs, with a speed corresponding to the planet's return in longitude, and that it carries with it points L and M of the epicycle, and centre Θ of the epicycle (which always moves on the eccentre HΘK), and also carries with it the planet; the planet, for its part, moves with uniform motion on the epicycle LM and performs its return always with respect to that diameter [of the epicycle] which points towards centre D, with a speed corresponding to the mean period of the synodic anomaly, and [a sense of rotation] such that its motion at the apogee L takes place towards the rear.

We can visualise the peculiar features of the hypothesis for Mercury as follows. Let [Fig. 9.2] the eccentre producing the anomaly be ABG about centre D, and let the diameter through D and centre E of the ecliptic be ADE [passing] through the apogee at A. On AG take DZ towards the apogee A, equal to DE. Then everything else remains the same, namely the whole plane [revolving] about centre E, shifts the apogee towards the rear by the same amount as for the other planets, the epicycle is revolved uniformly about centre D towards the rear, as [here] by the line DB, and furthermore the planet moves on the epicycle in the same way as the others. But in this case the centre of the other eccentre, which is, again, equal in size to the first eccentre, and on which
H256 the epicycle centre is always located, is carried around point Z in the opposite sense to the motion of the epicycle, namely in advance [i.e. in the reverse order of the signs, but uniformly and with the same speed as the epicycle, as [here] the line ZHΘ. Thus in one year each of the lines DB and ZHΘ performs one

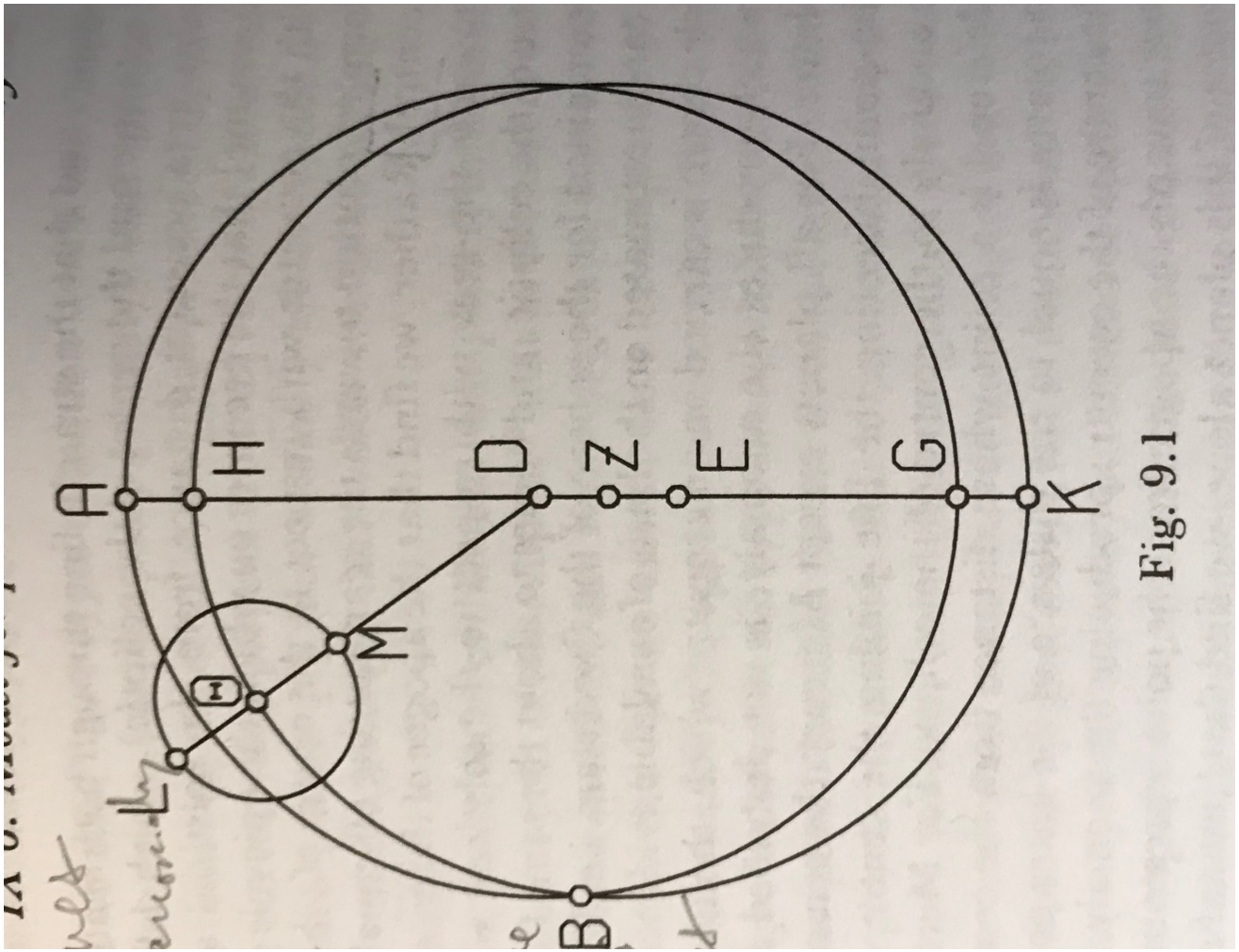
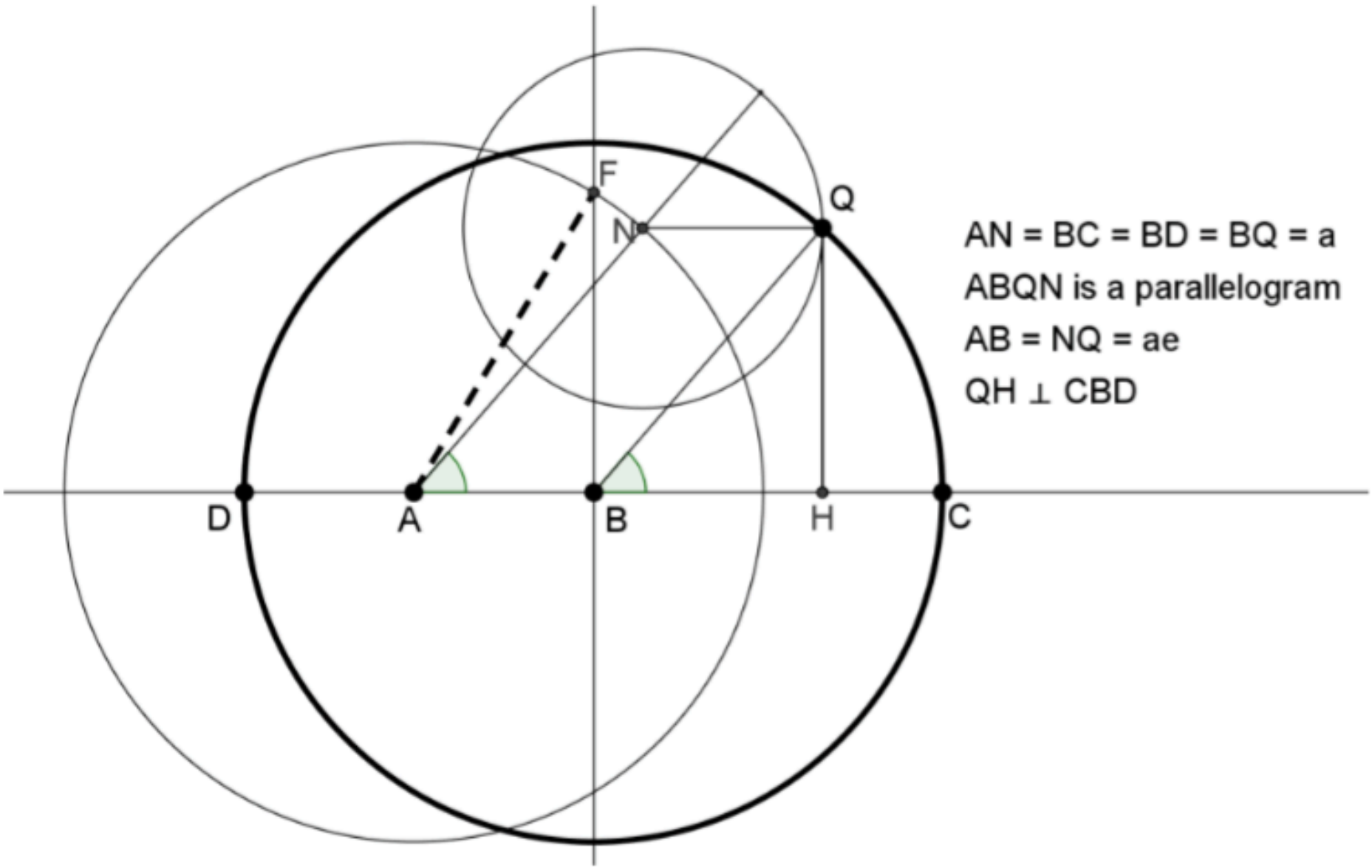


Fig. 9.1

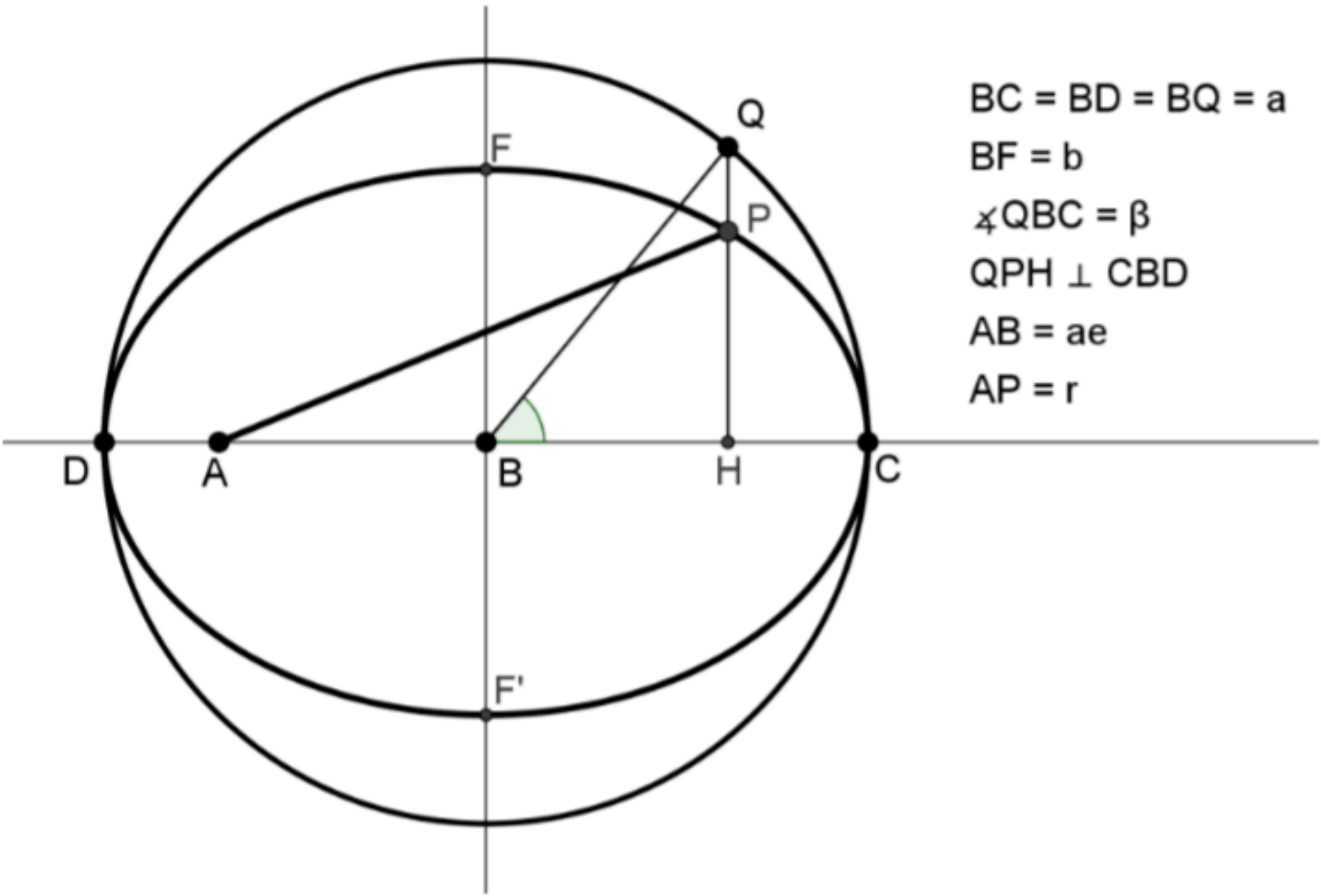
D – Equant
E – Centre of ecliptic (Earth)

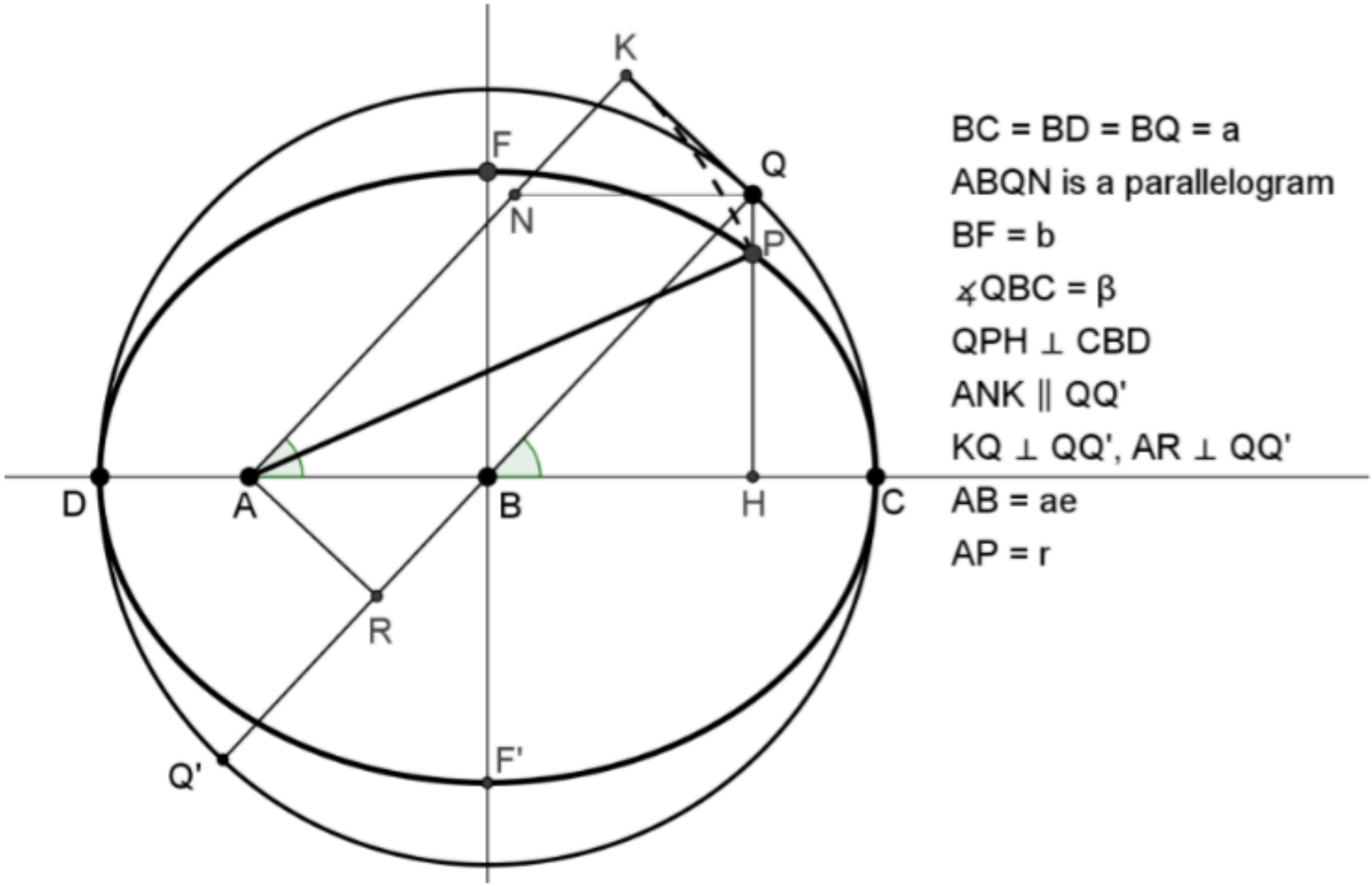
A – apogee
G – perigee

HΘK – deferent
ABG – eccentre


F
Q
N
D
A
B
H
C
AN = BC = BD = BQ = a
ABQN is a parallelogram
AB = NQ = ae
QH ⊥ CBD

$$AB = ae = \tfrac{1}{2}(CA - DA).$$





BC = BD = BQ = a

ABQN is a parallelogram

BF = b

$\measuredangle$QBC = β

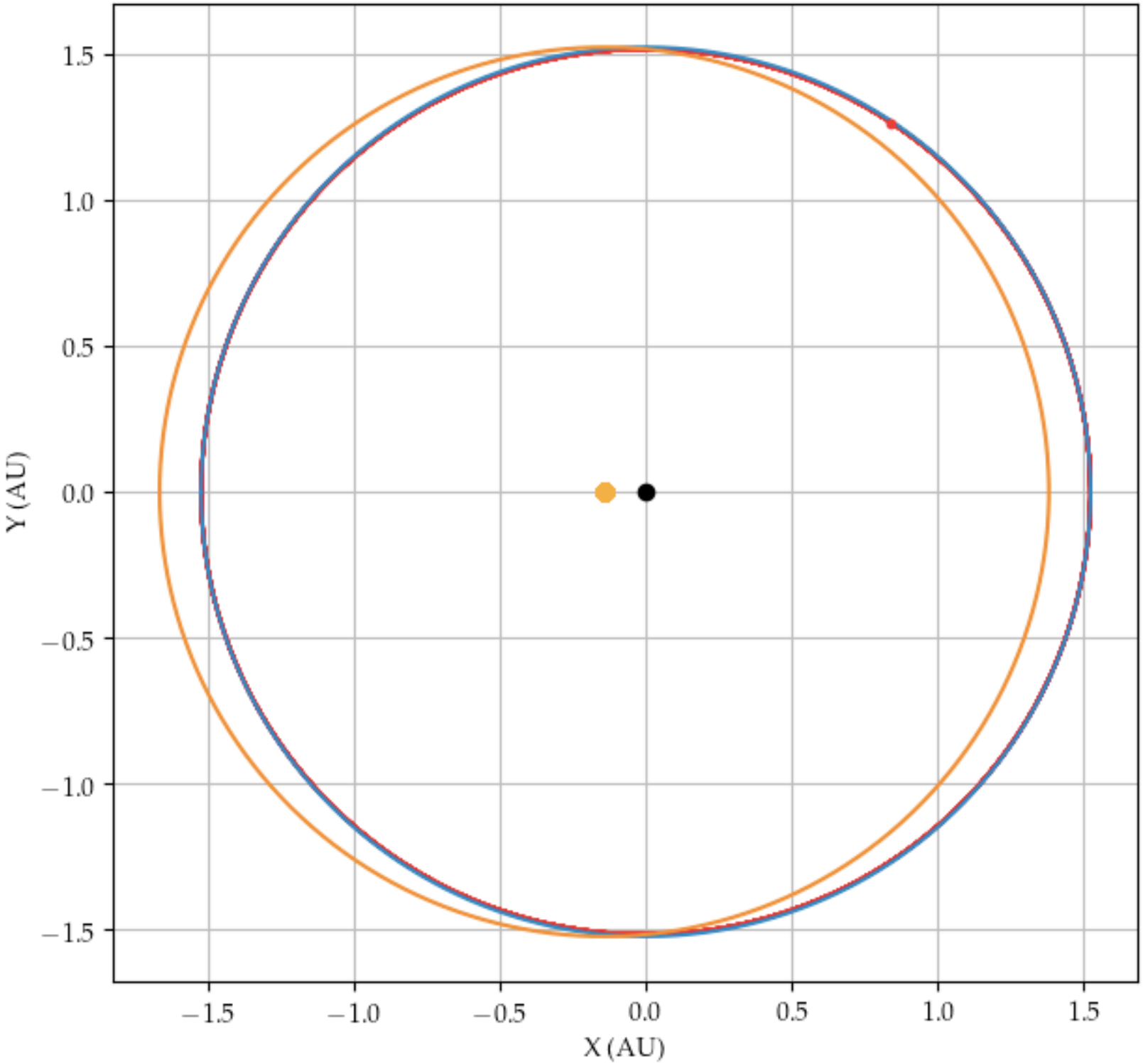
QPH ⊥ CBD

ANK ∥ QQ'

KQ ⊥ QQ', AR ⊥ QQ'

AB = ae

AP = r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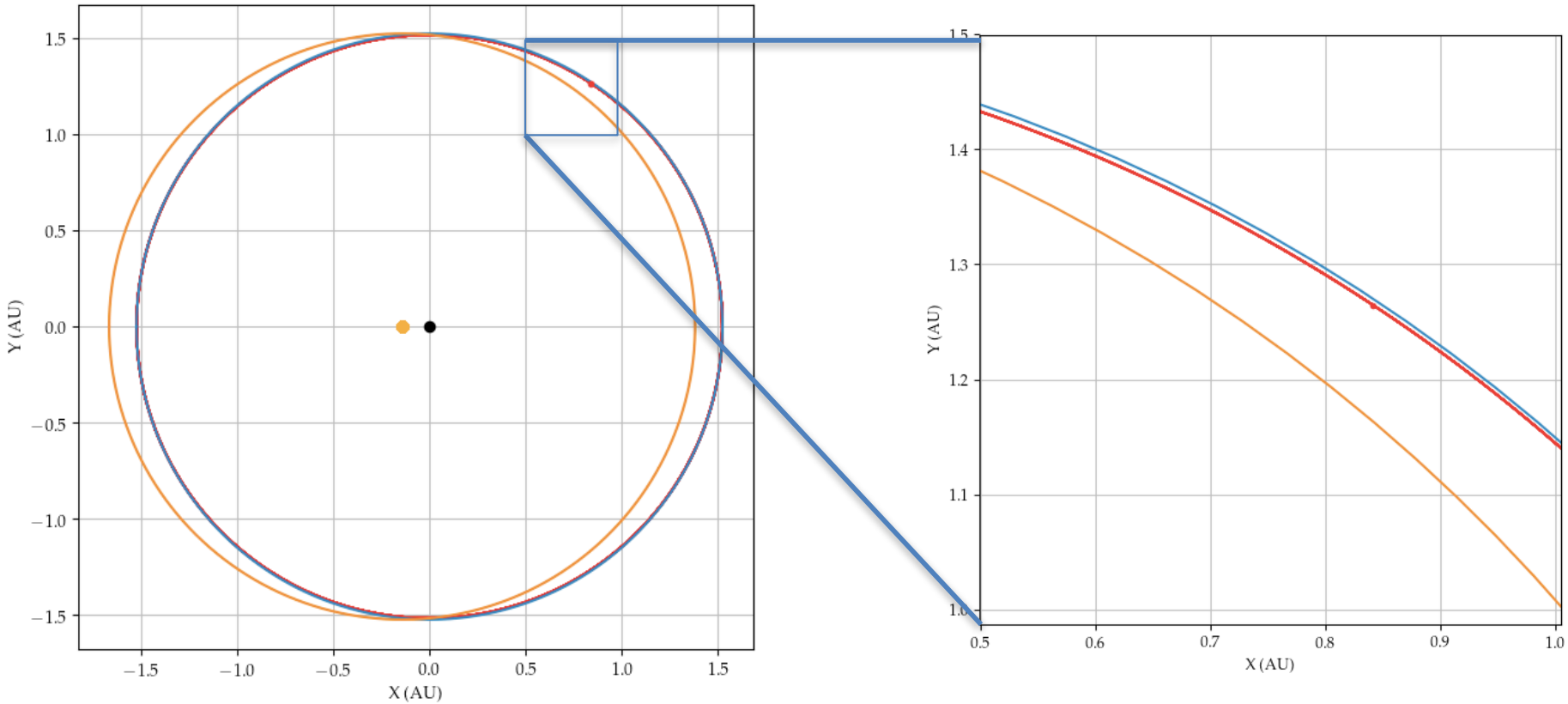
1.0
0.5
0.0
−0.5
−1.0
−1.5
Y (AU)
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X (AU)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Y (AU)
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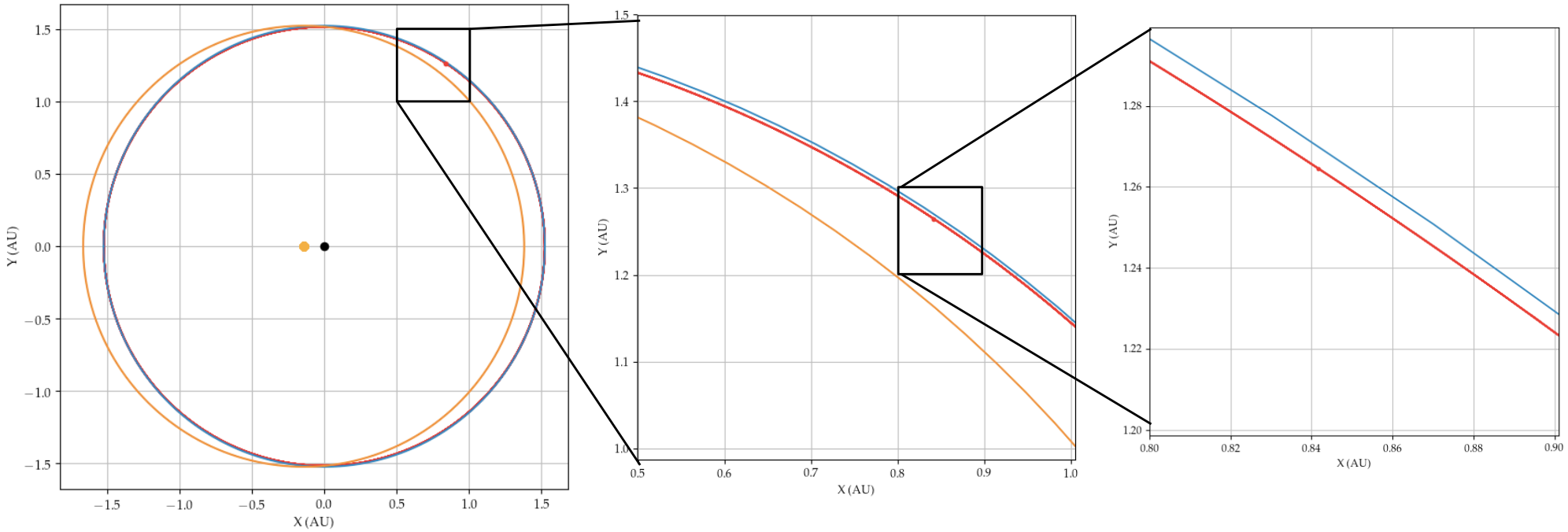
0.0
0.5
1.0
1.5
X (AU)
1.4
1.3
1.2
1.1
1.0
Y (AU)
0.5
0.6
0.7
0.8
0.9
1.0
X (AU)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Y (AU)
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1.5
X (AU)
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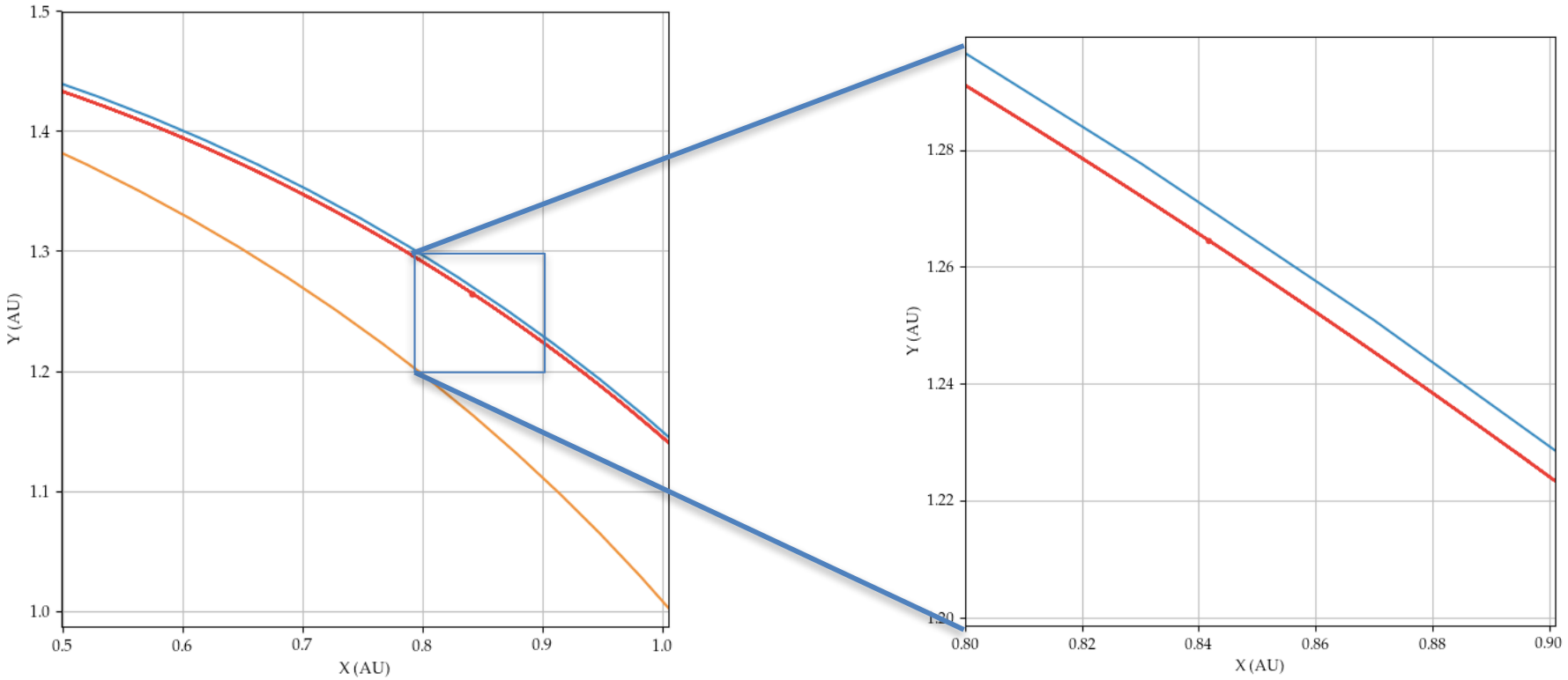
1.4
1.3
1.2
1.1
1.0
Y (AU)
0.5
0.6
0.7
0.8
0.9
1.0
X (AU)
1.28
1.26
1.24
1.22
1.20
Y (AU)
0.80
0.82
0.84
0.86
0.88
0.90
X (AU)


1.5
1.4
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1.1
1.0
Y (AU)
0.5
0.6
0.7
0.8
0.9
1.0
X (AU)
1.28
1.26
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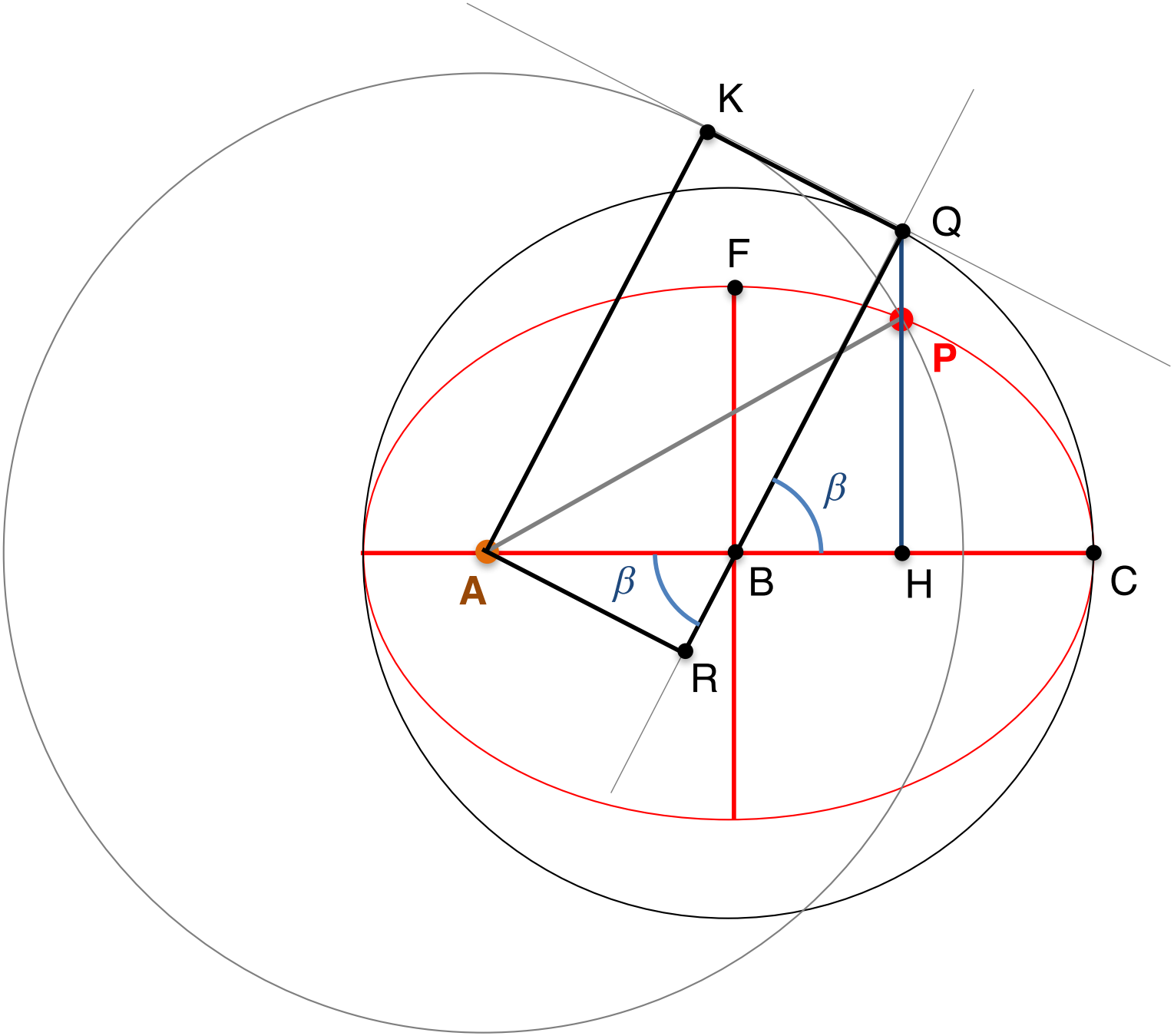
Y (AU)
0.80
0.82
0.84
0.86
0.88
0.90
X (AU)


K
Q
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β
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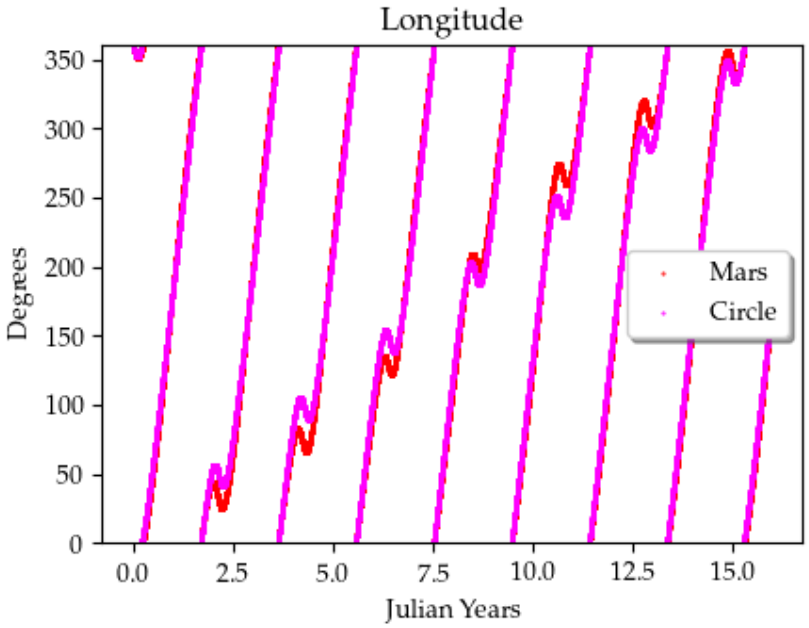
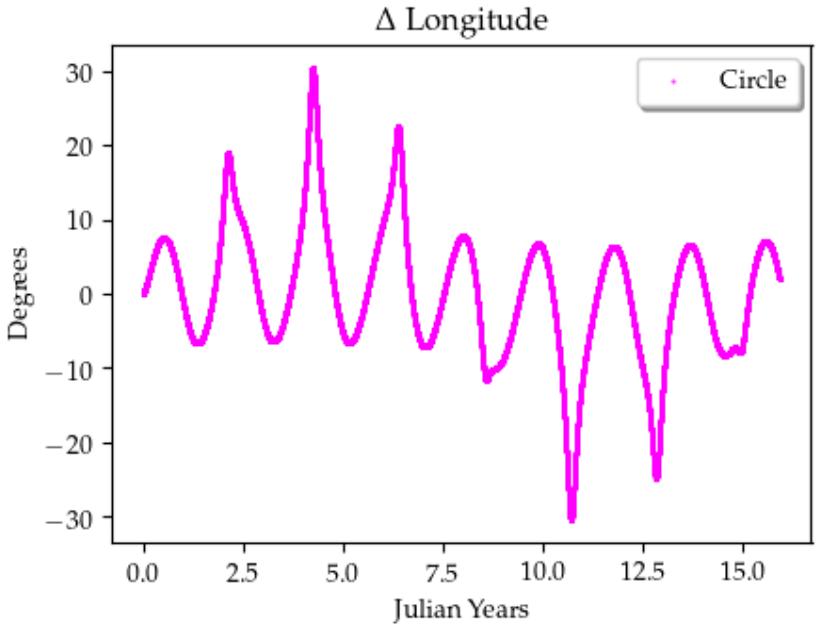
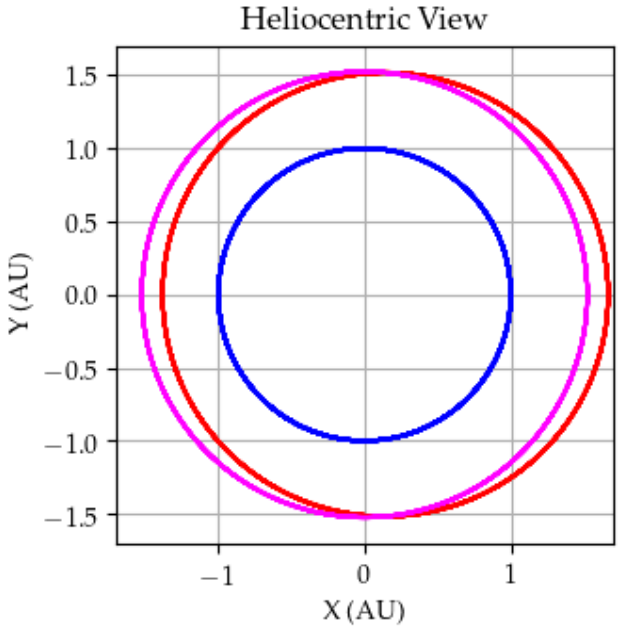
C
R

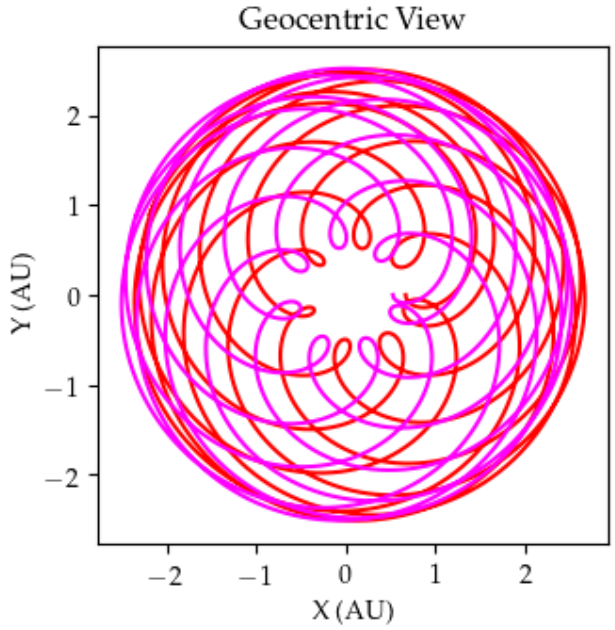
2nd law

Circular and Centered Orbit

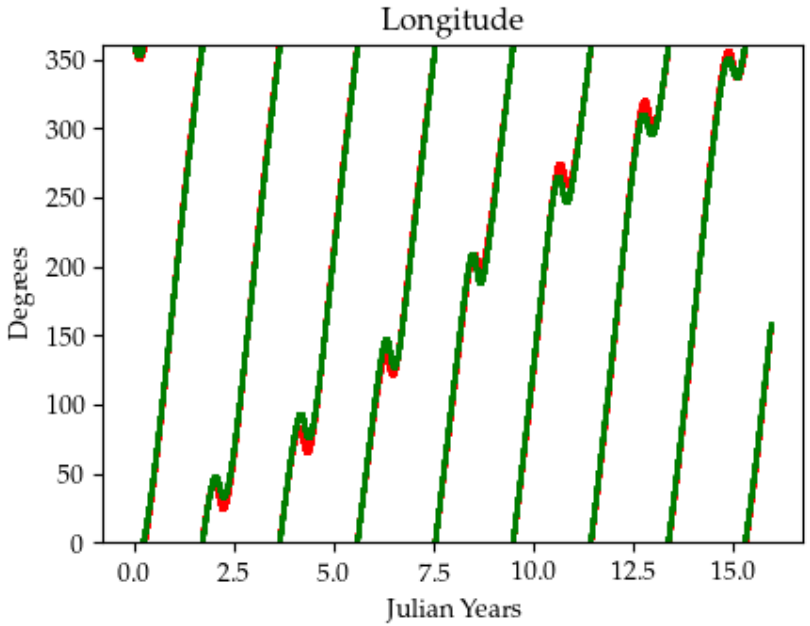


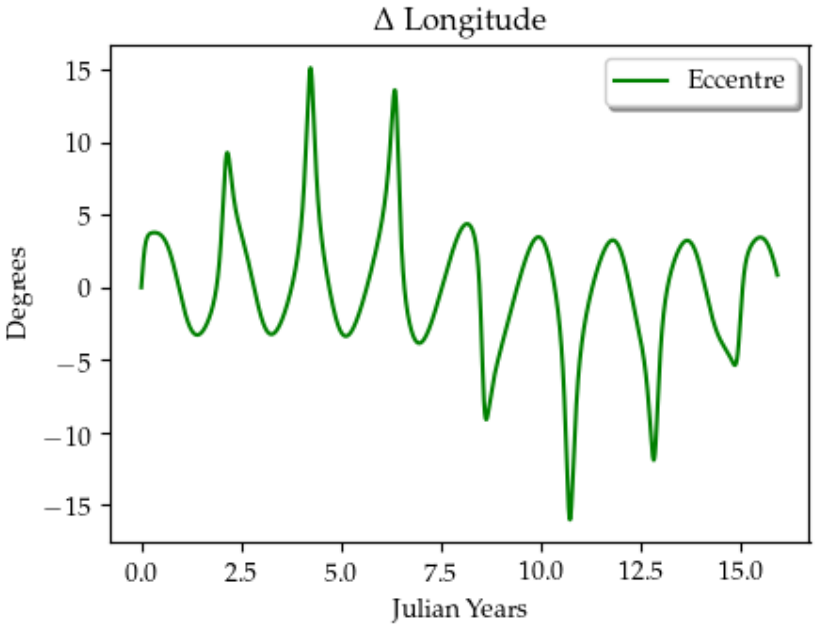


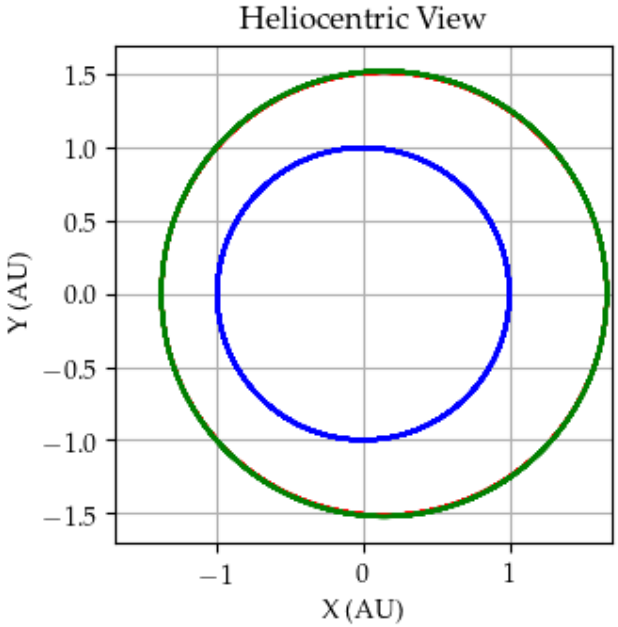


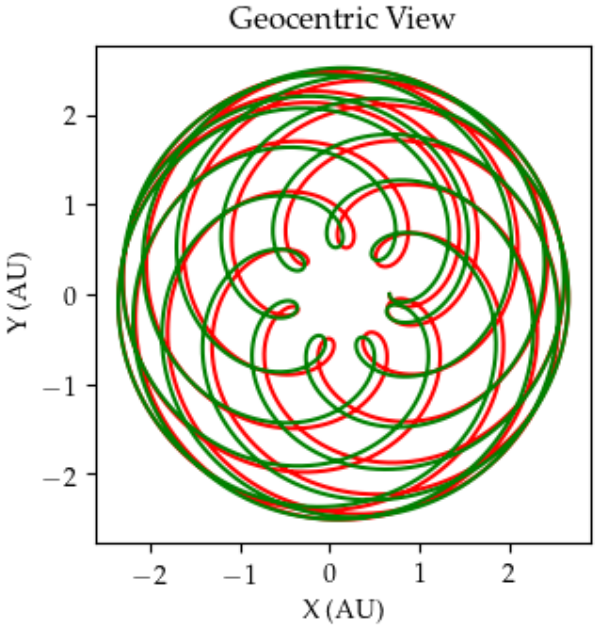


Circular Off-Centered Orbit (the Eccentre)

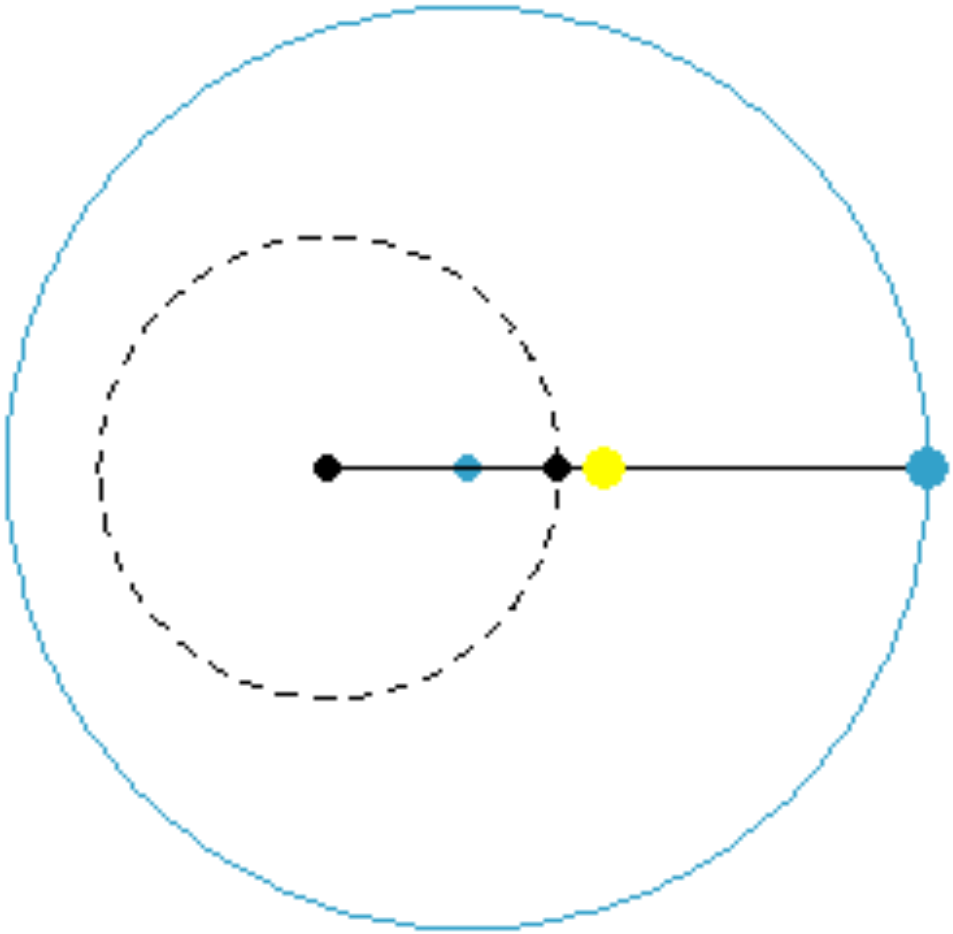






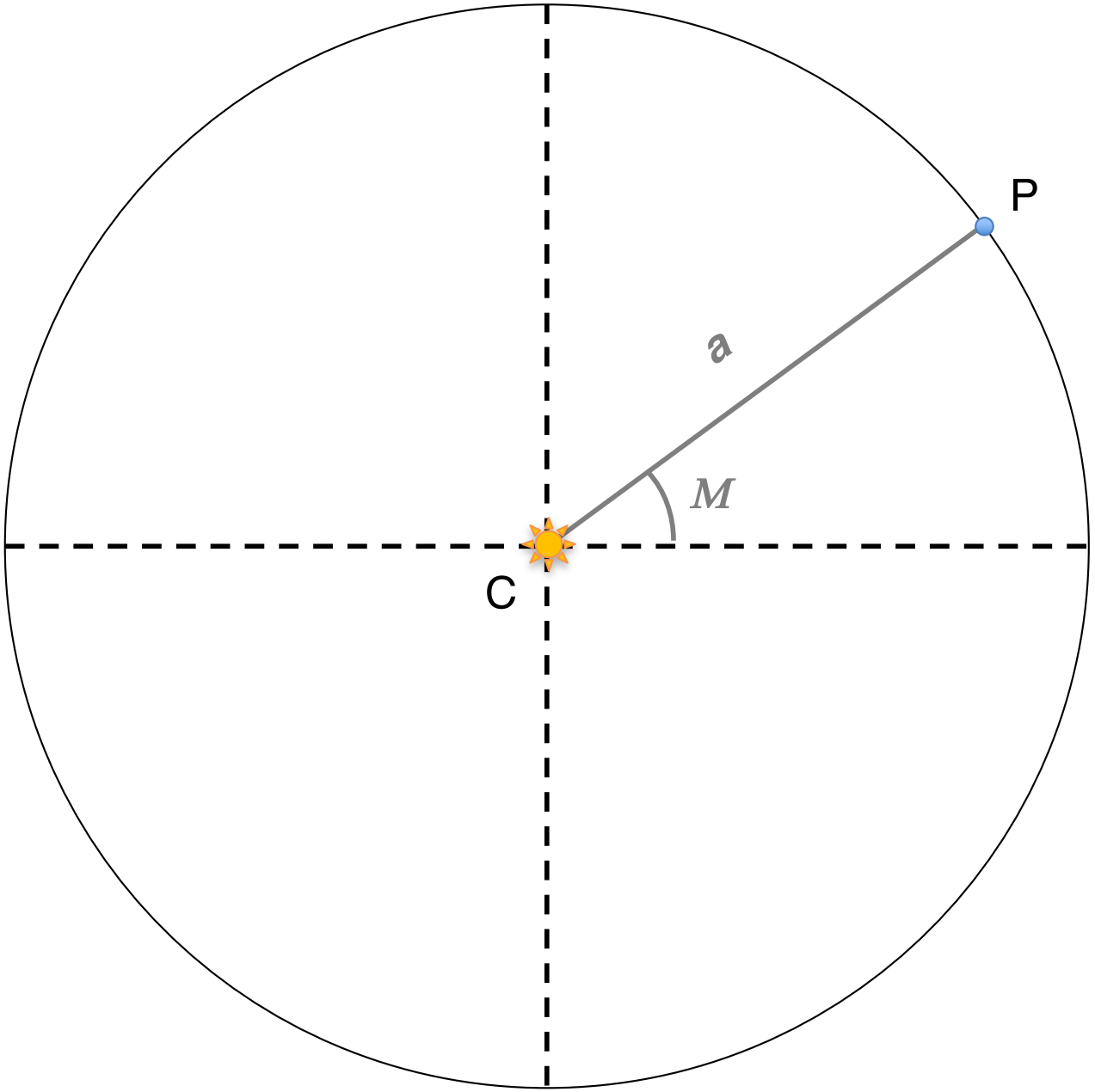


The Equant

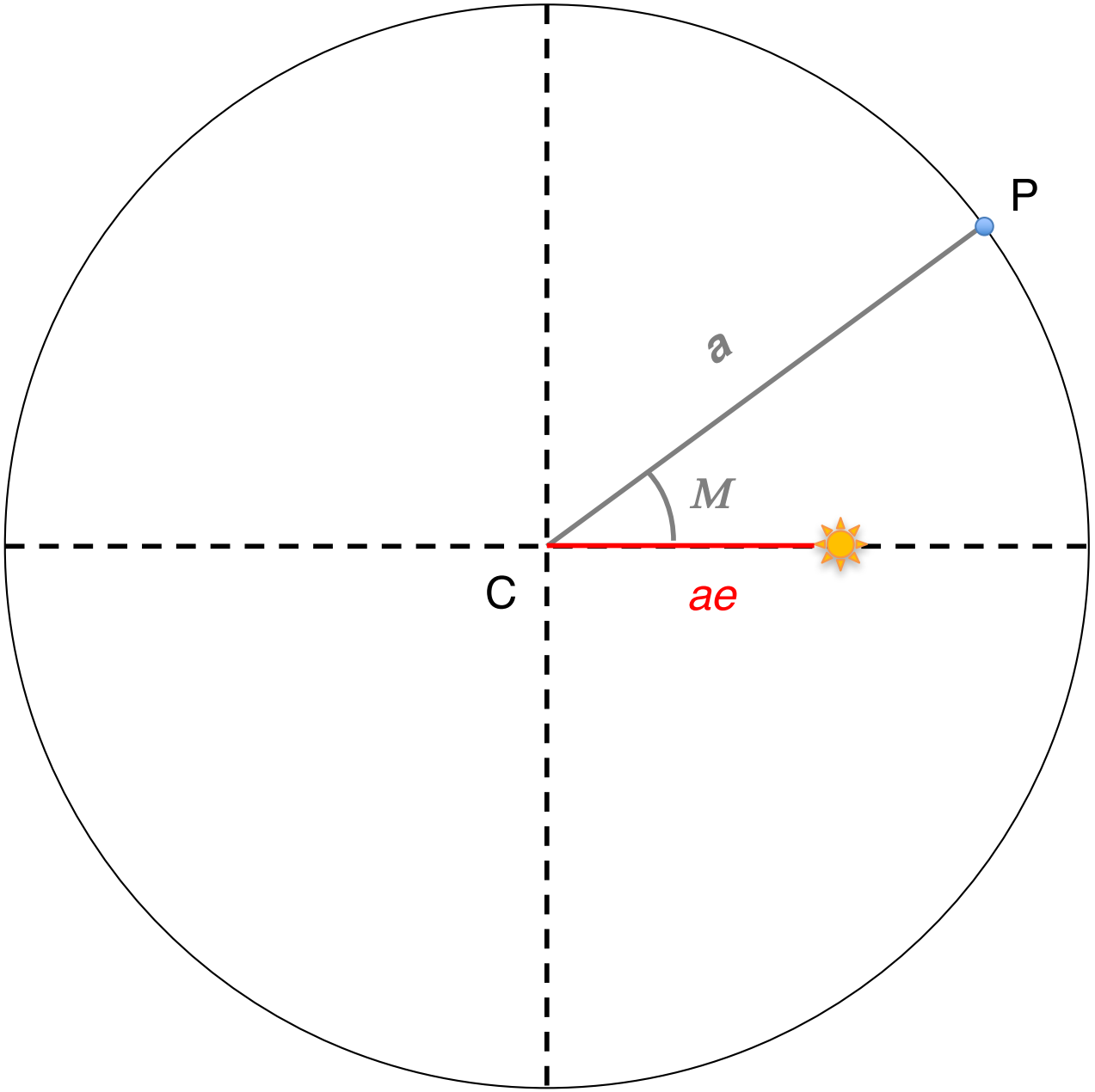


**Around a special point (the equant),
the motion is uniform**

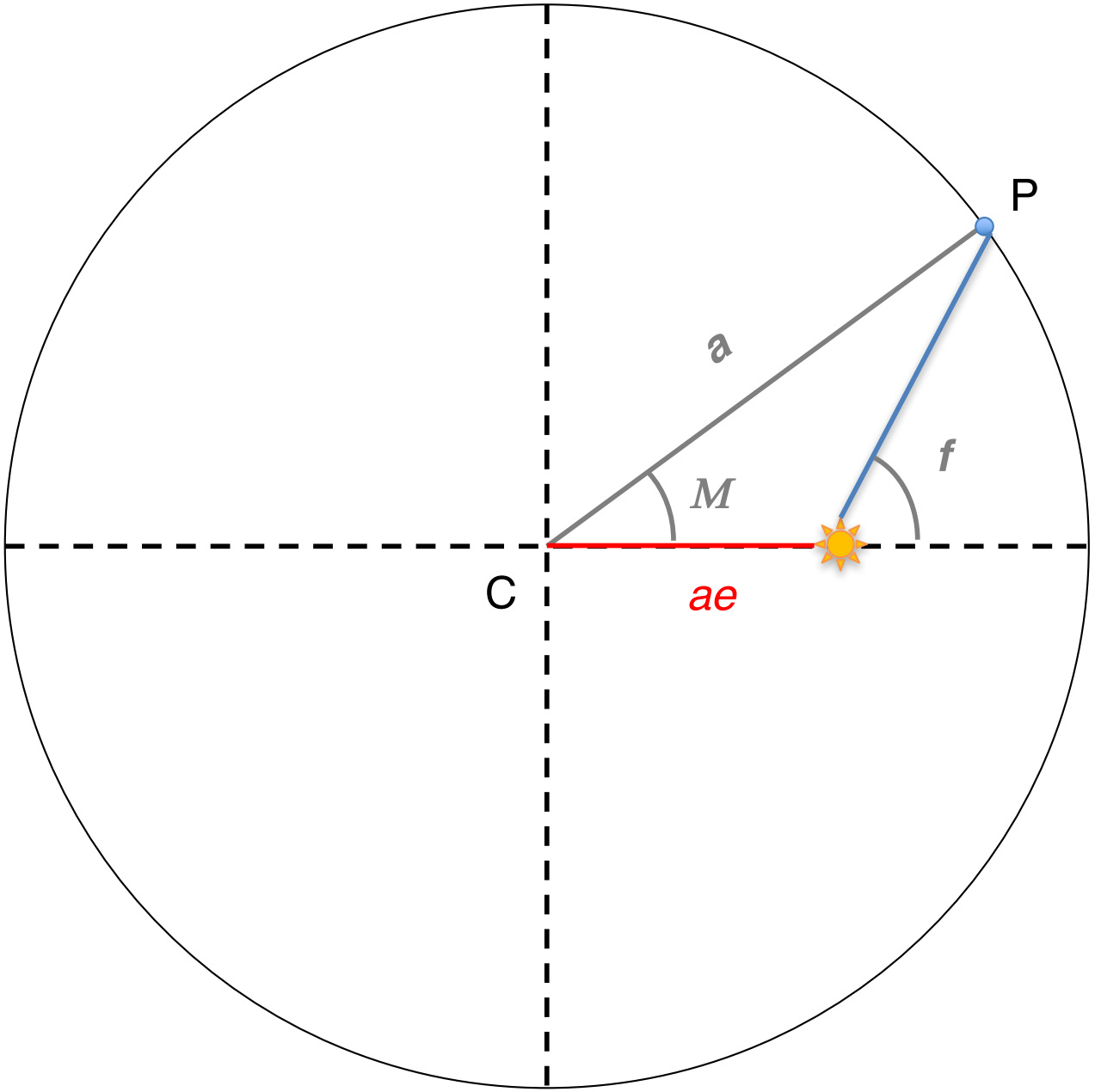
The Circle



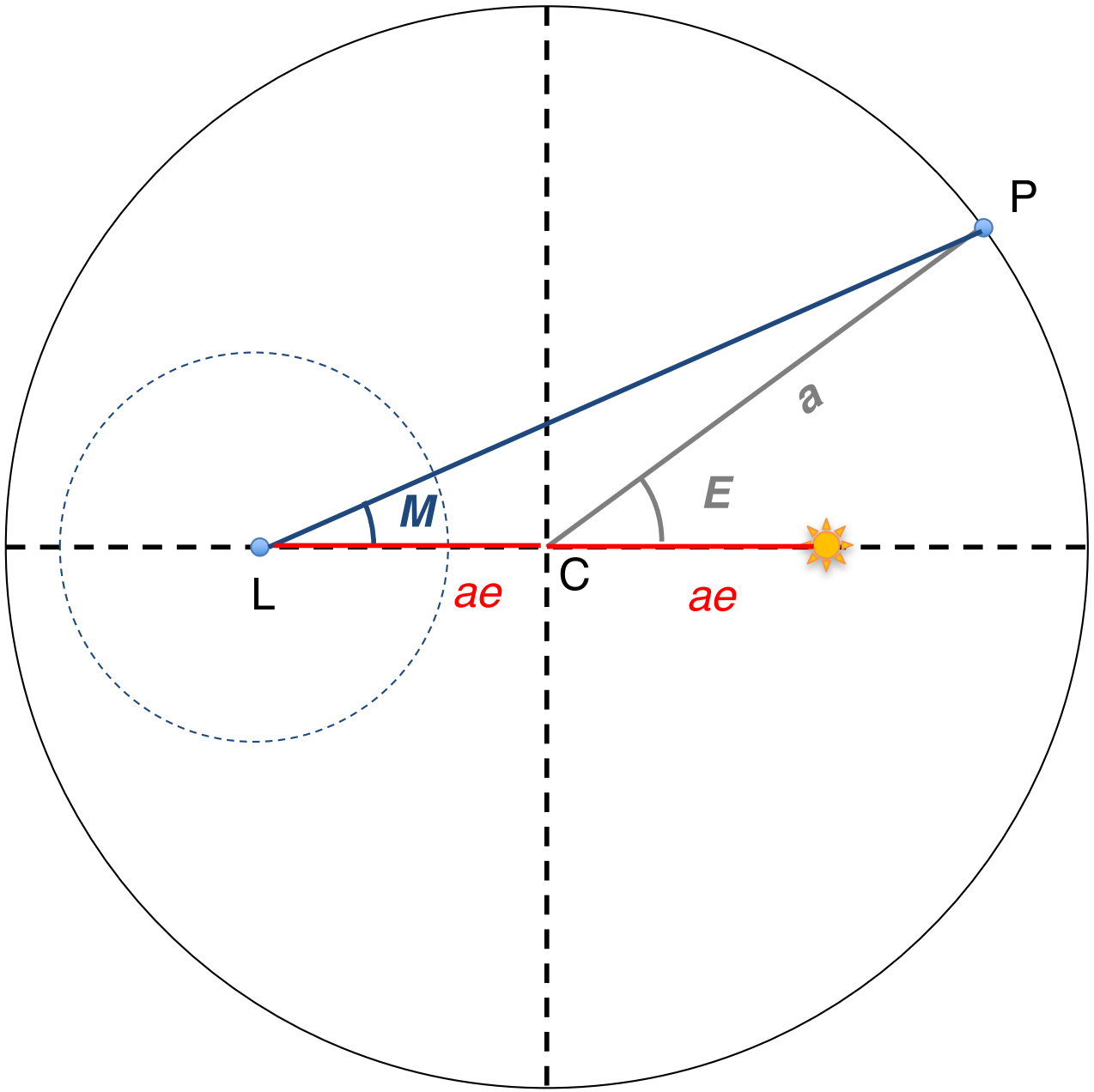
The Eccentre with uniform motion



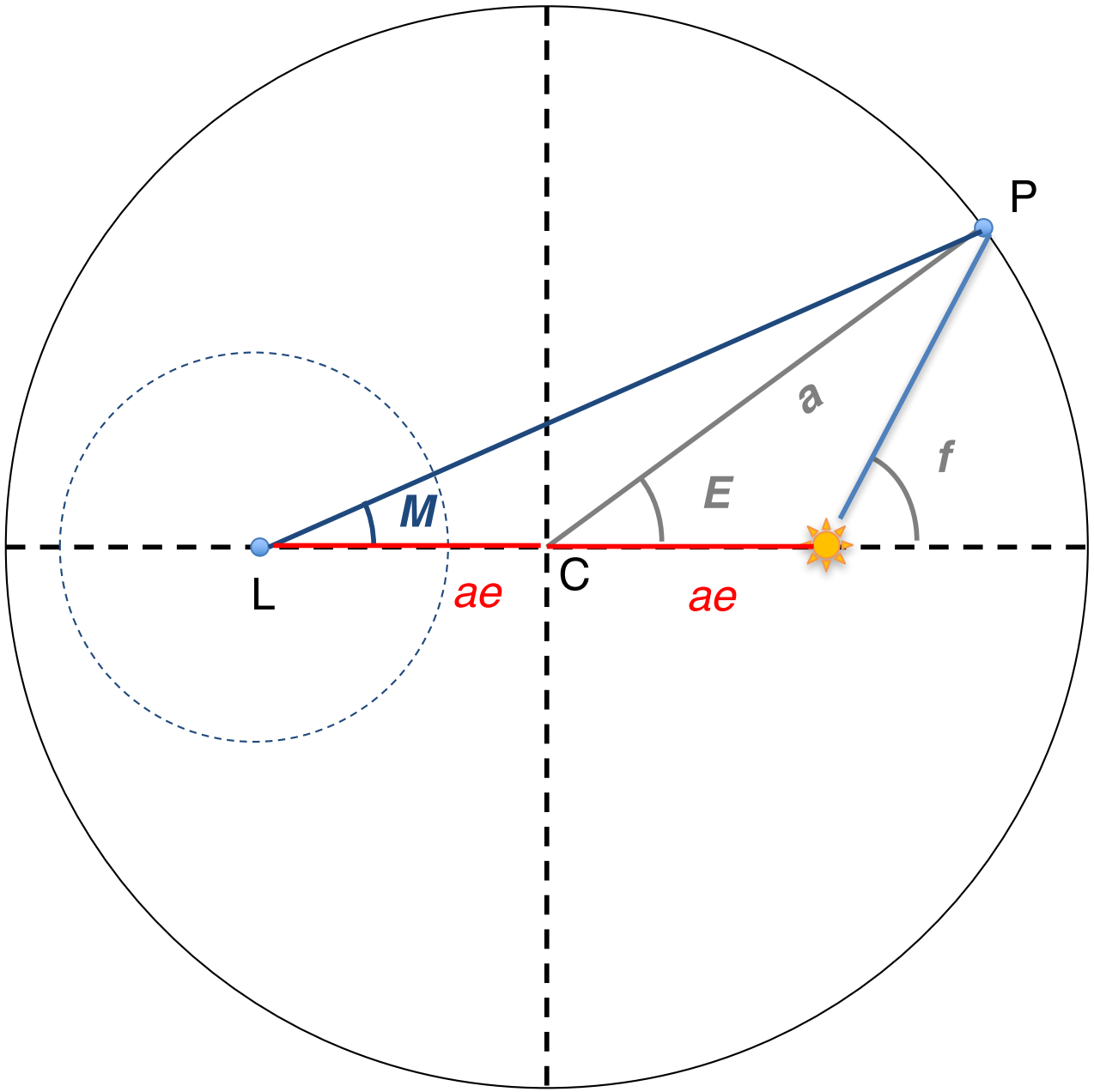
The Eccentre with uniform motion



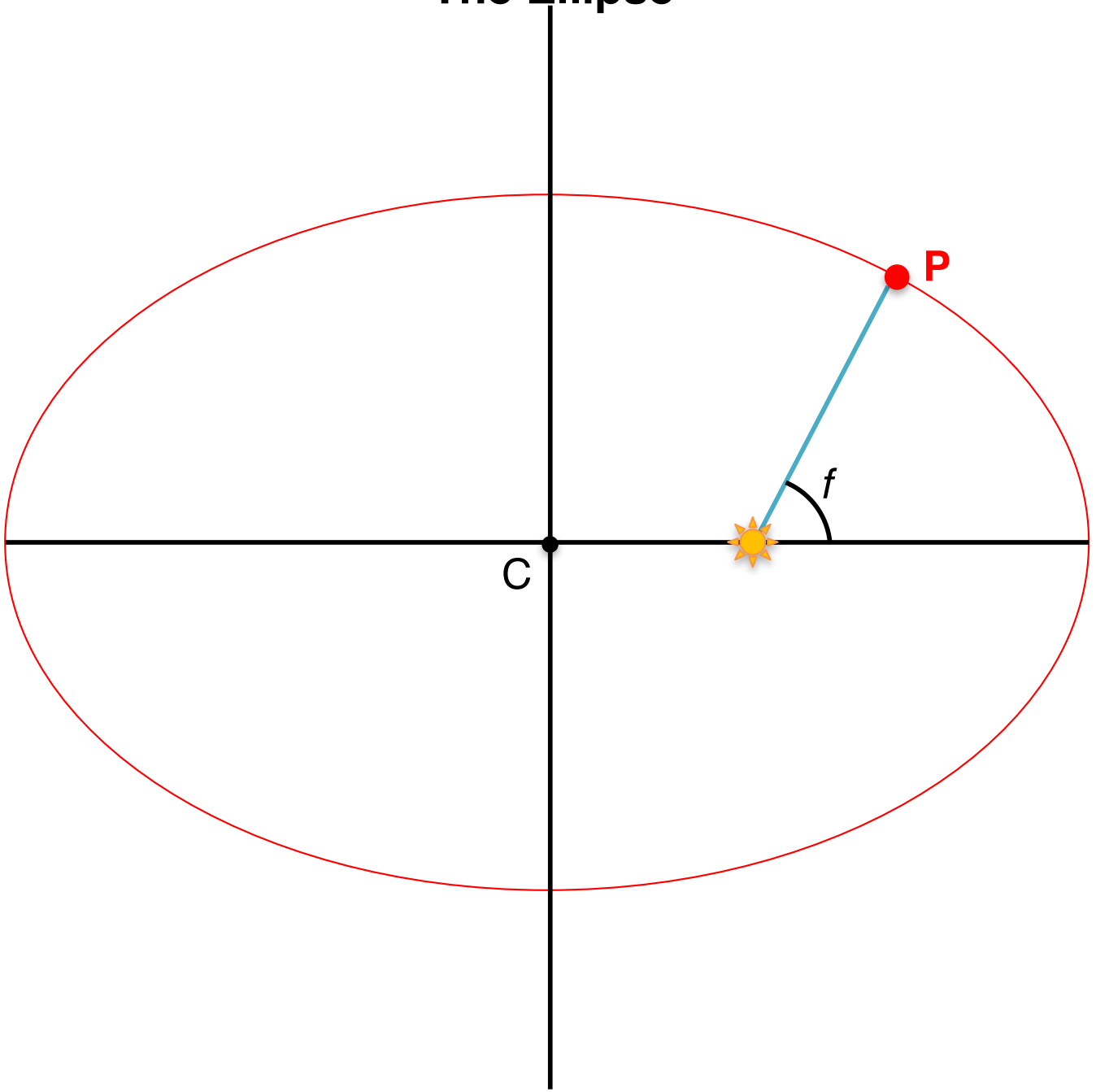
The Eccentre with equant

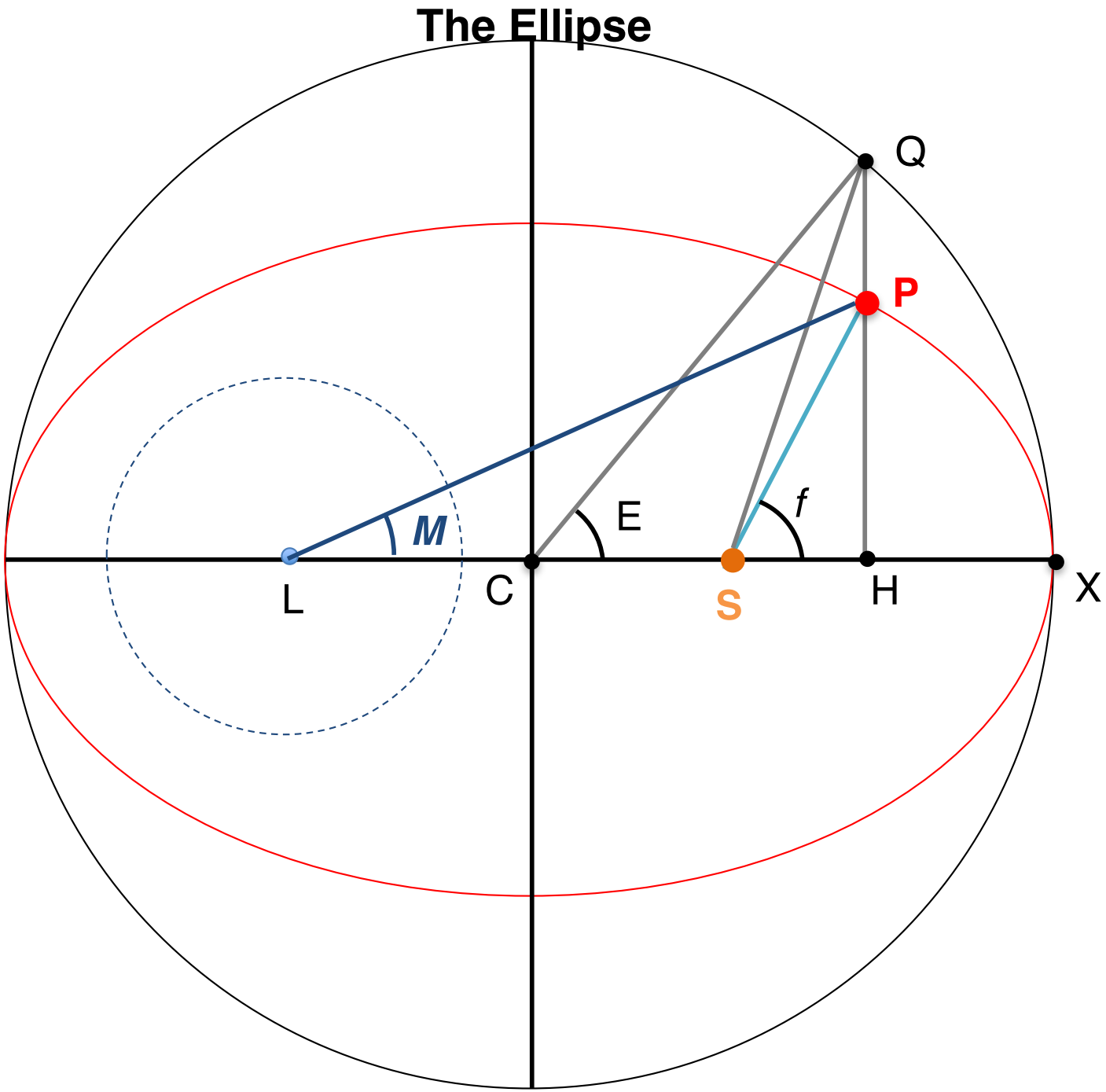


The Eccentre with equant



The Ellipse




The Ellipse
Q
P
M
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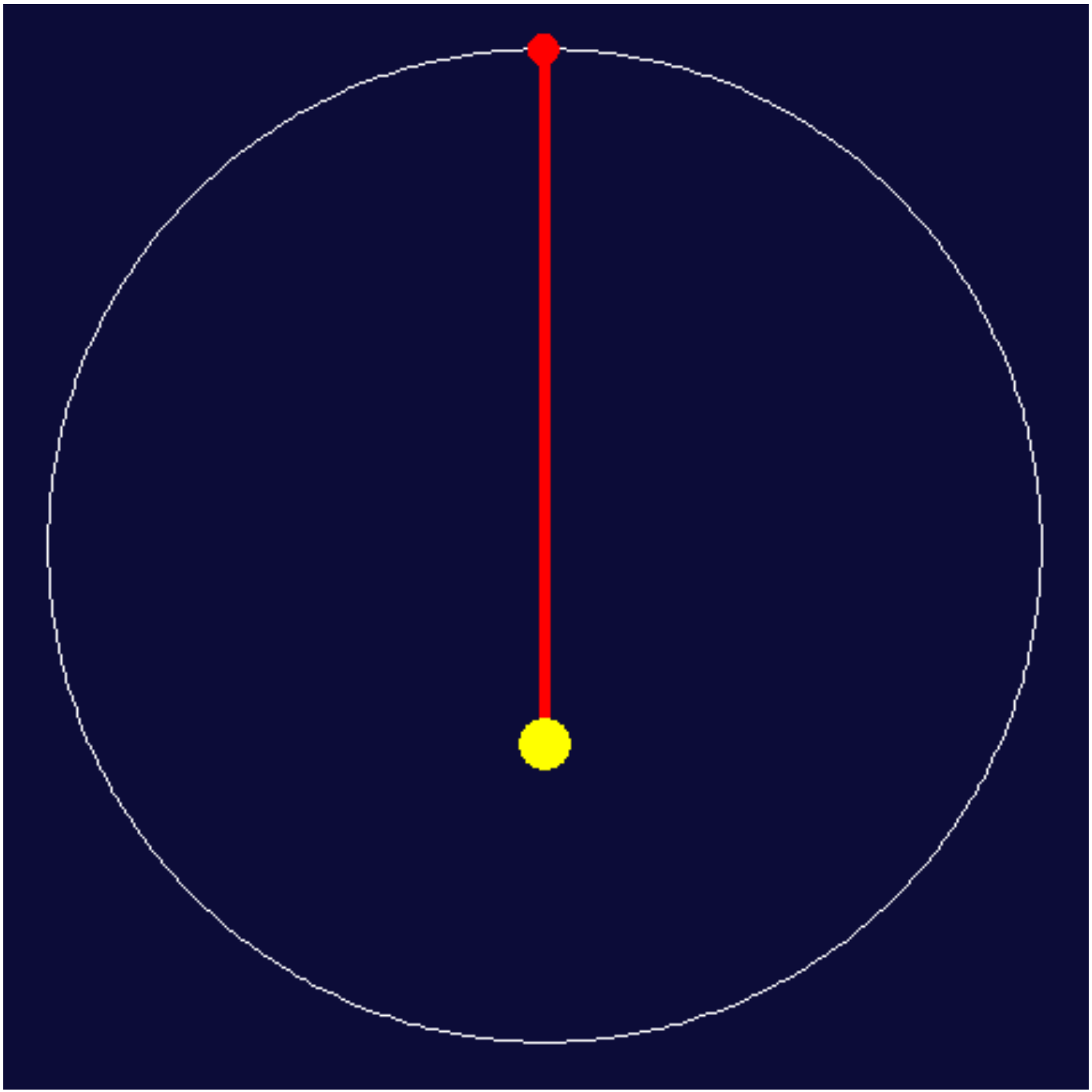
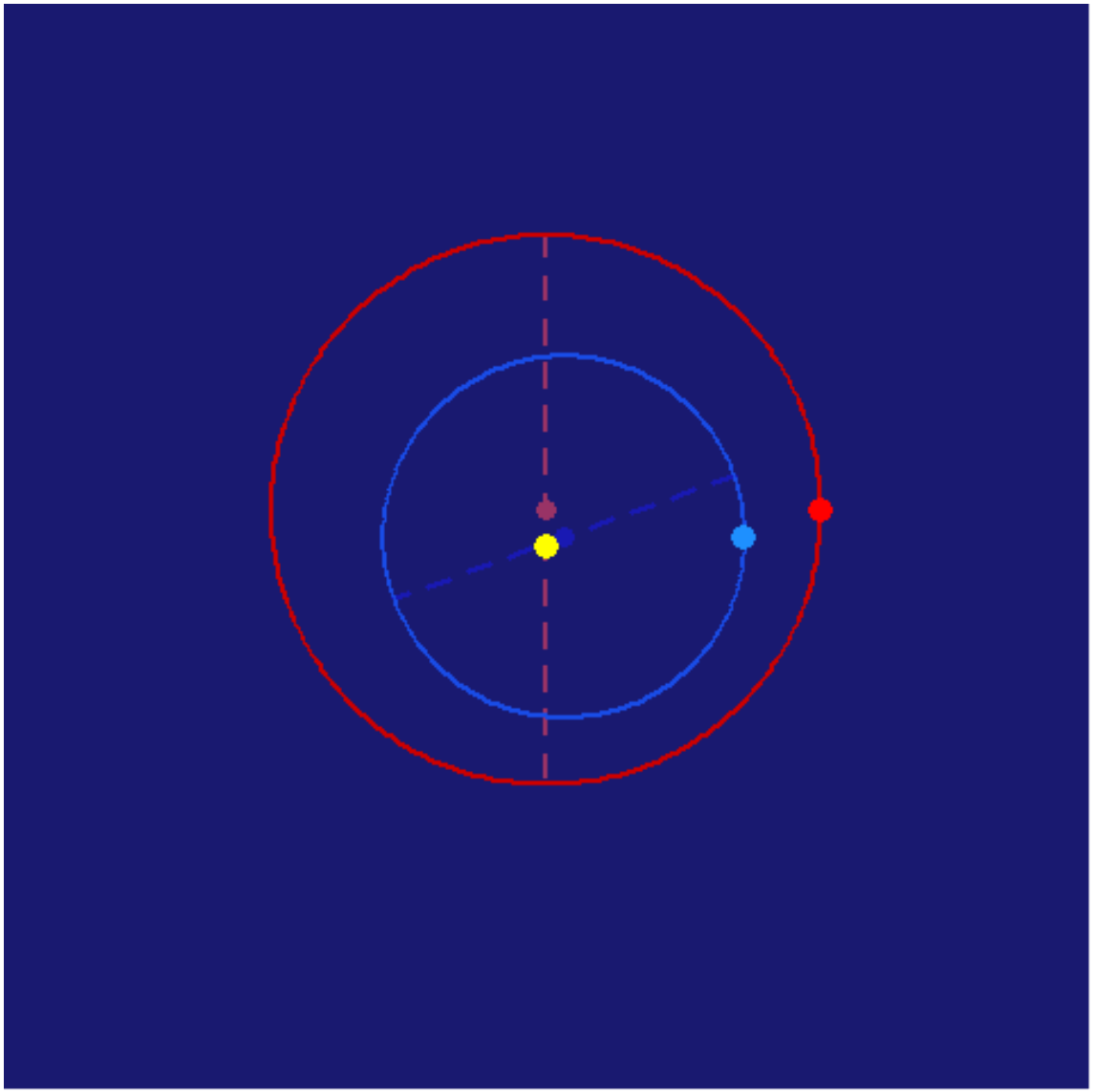


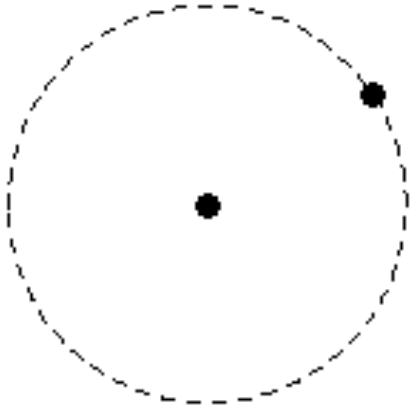
Changing to the Sun's Reference Frame: Observations at Opposition



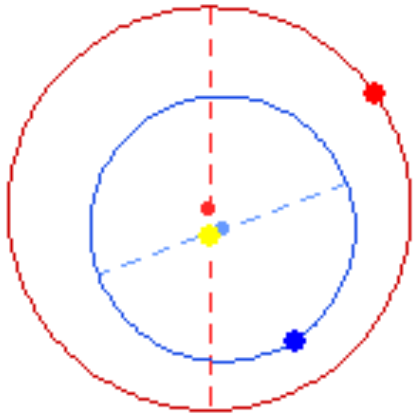
Locating the Equant

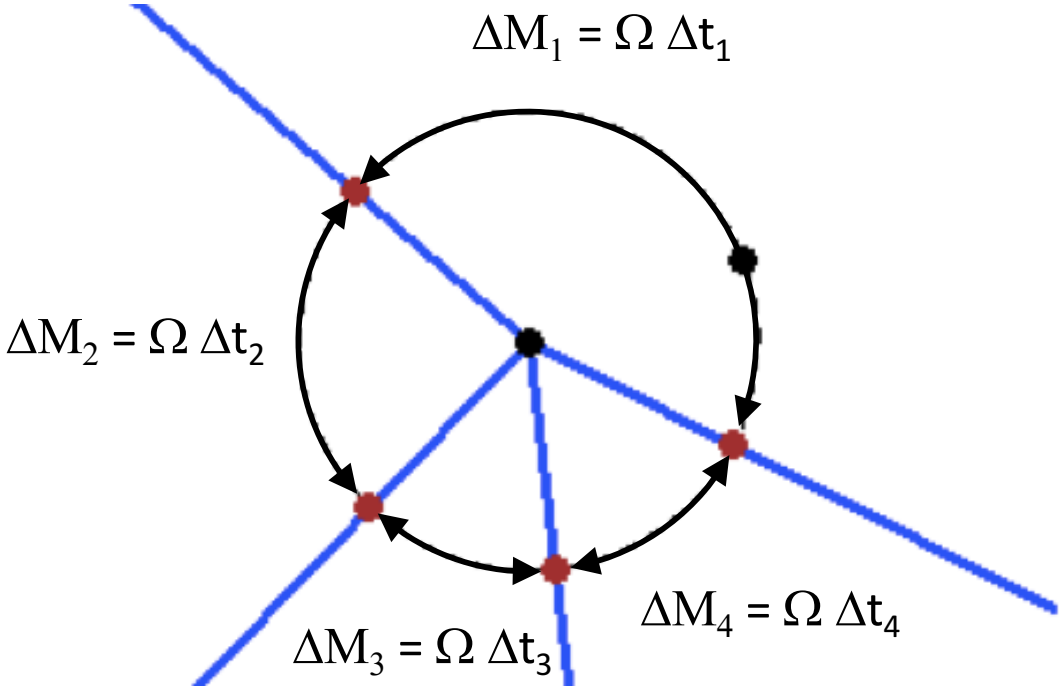
Equant (Mean)

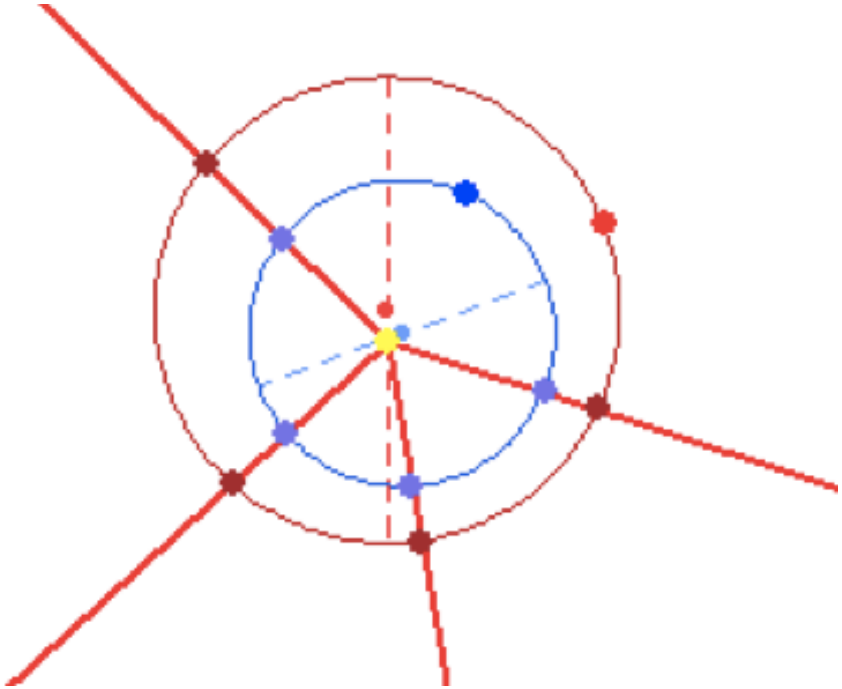
Time between Oppositions



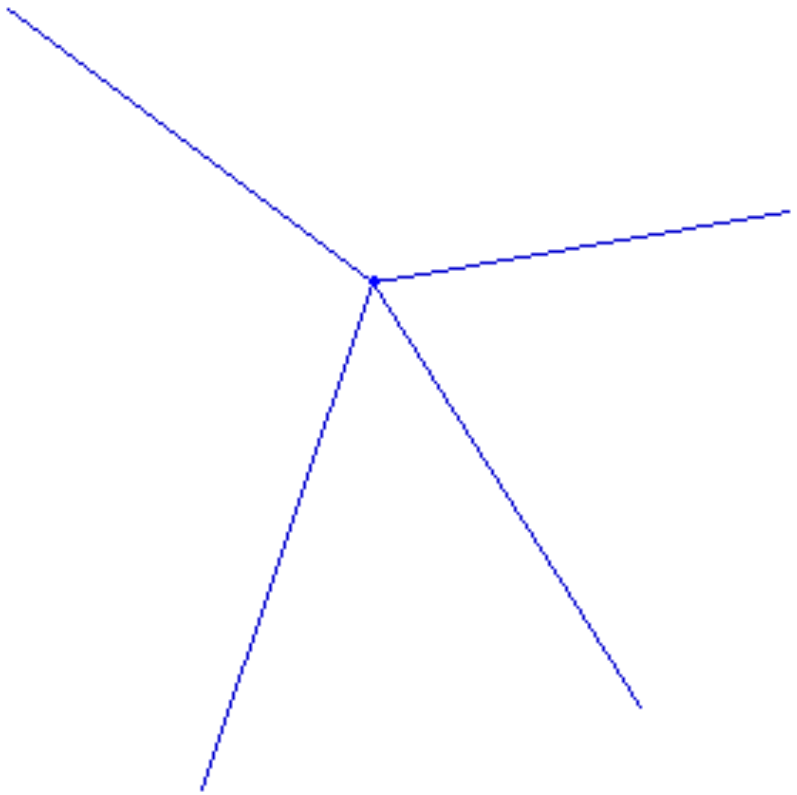
Observed (Apparent)

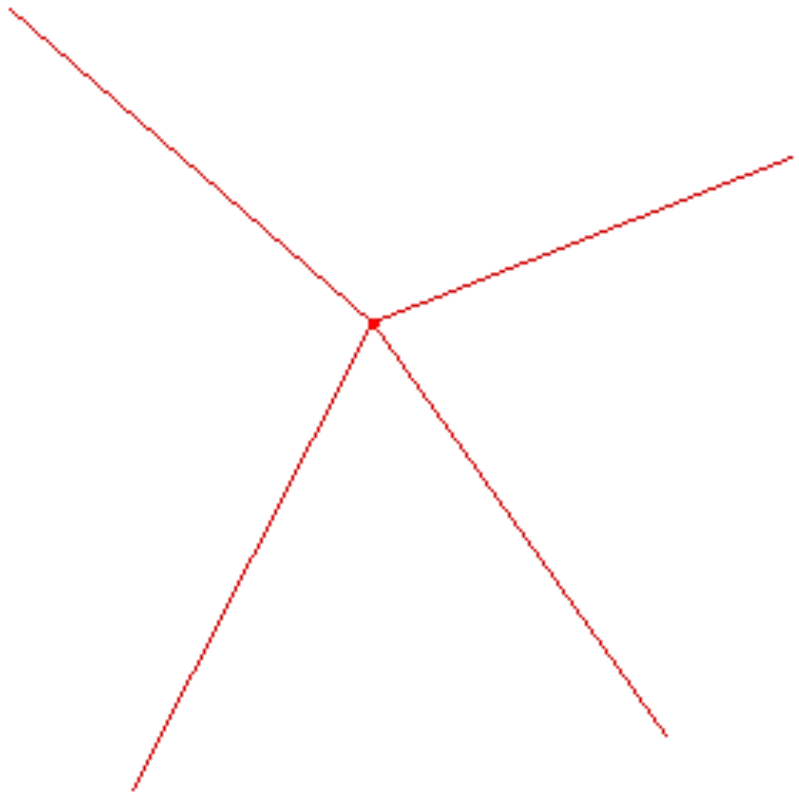



$\Delta M_1 = \Omega\ \Delta t_1$
$\Delta M_2 = \Omega\ \Delta t_2$
$\Delta M_3 = \Omega\ \Delta t_3$
$\Delta M_4 = \Omega\ \Delta t_4$

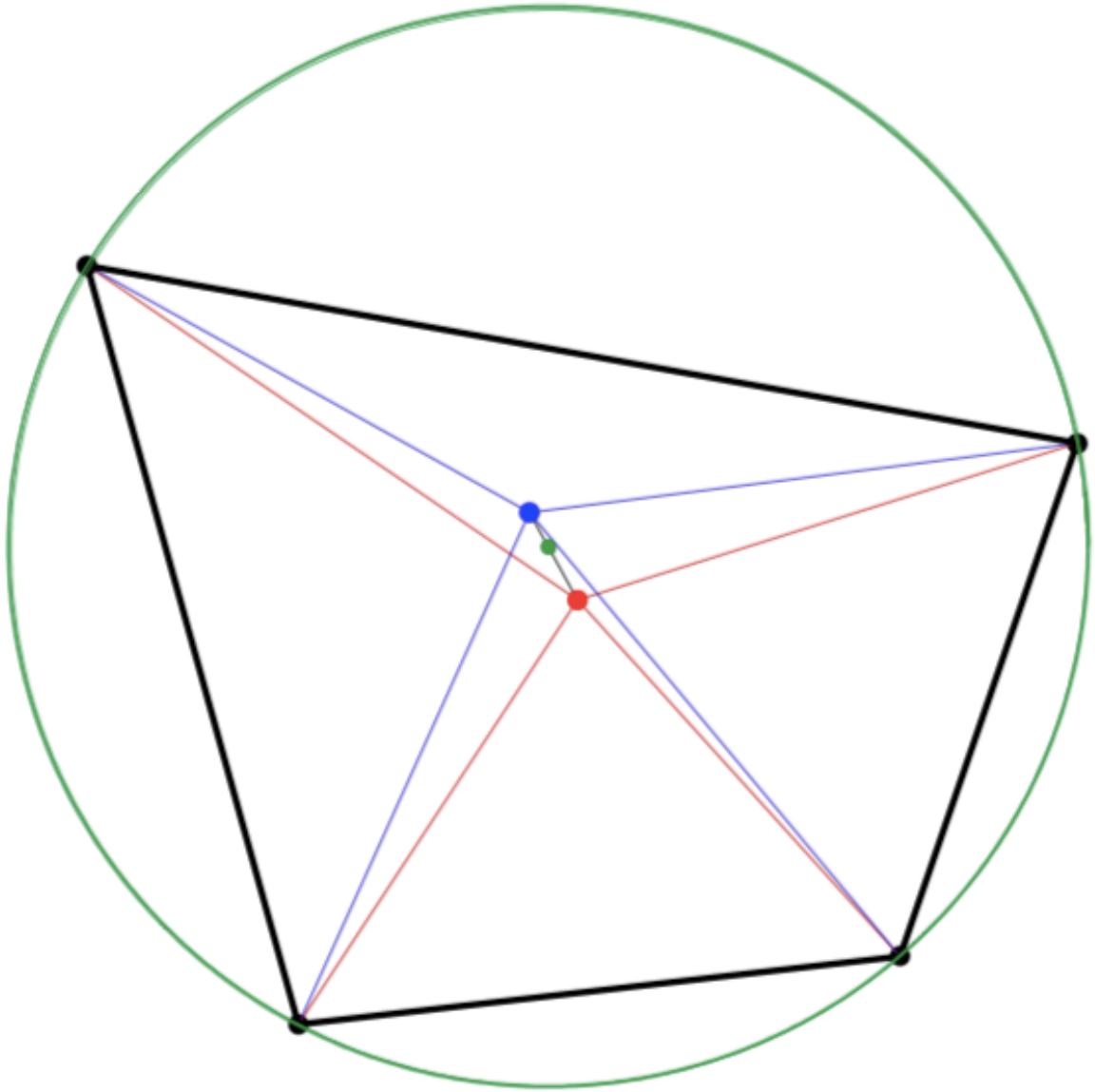


Locating the Equant

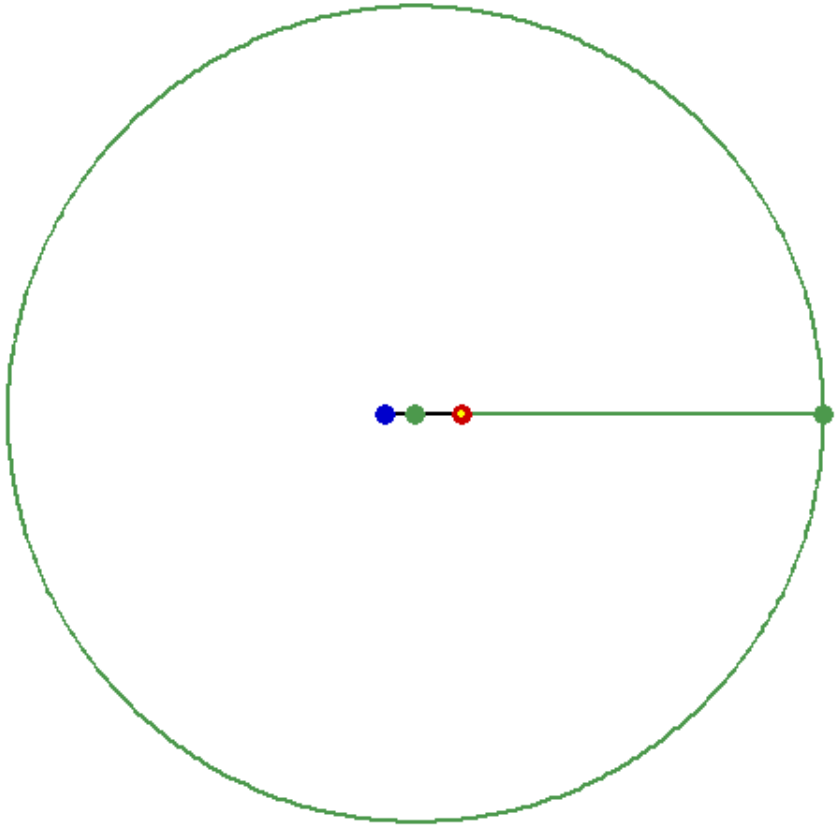




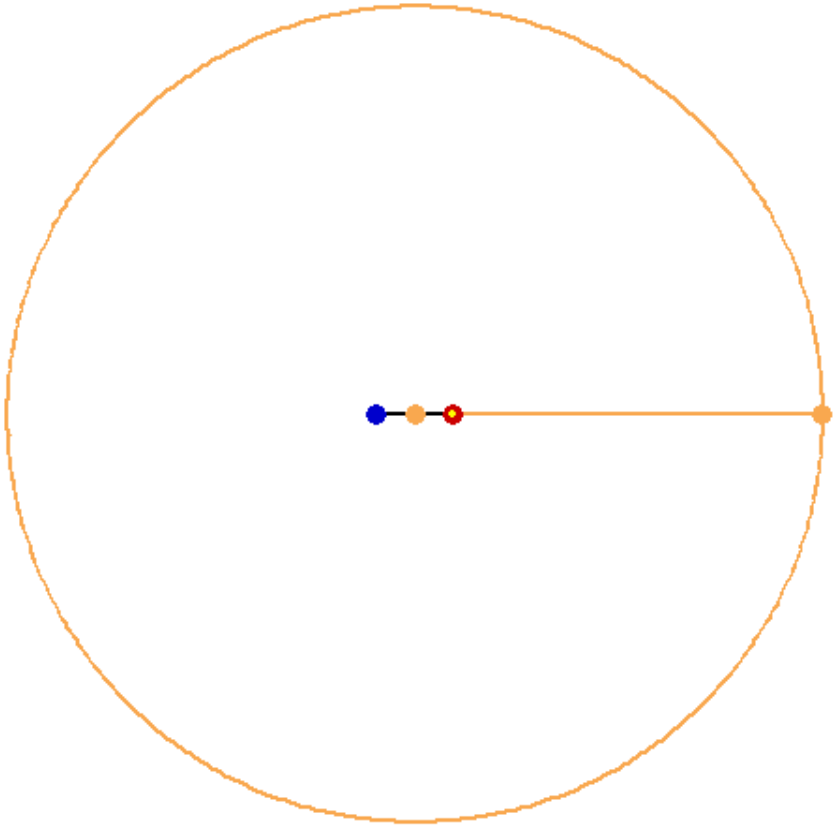
Kepler’s solution



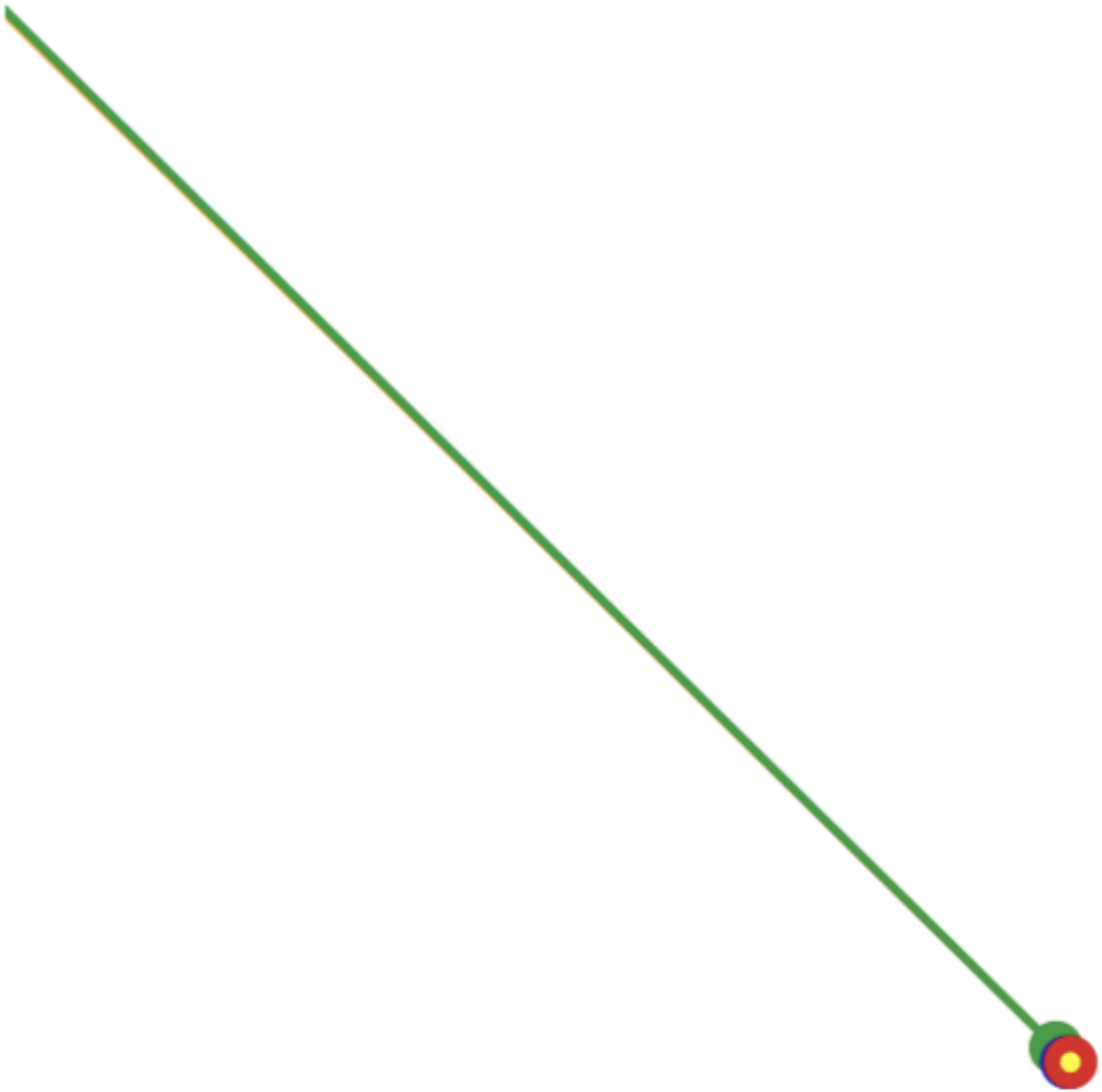
Best fit
(e_{Eq} != e_{Sun})



Best fit
(e_{Eq} = e_{Sun})

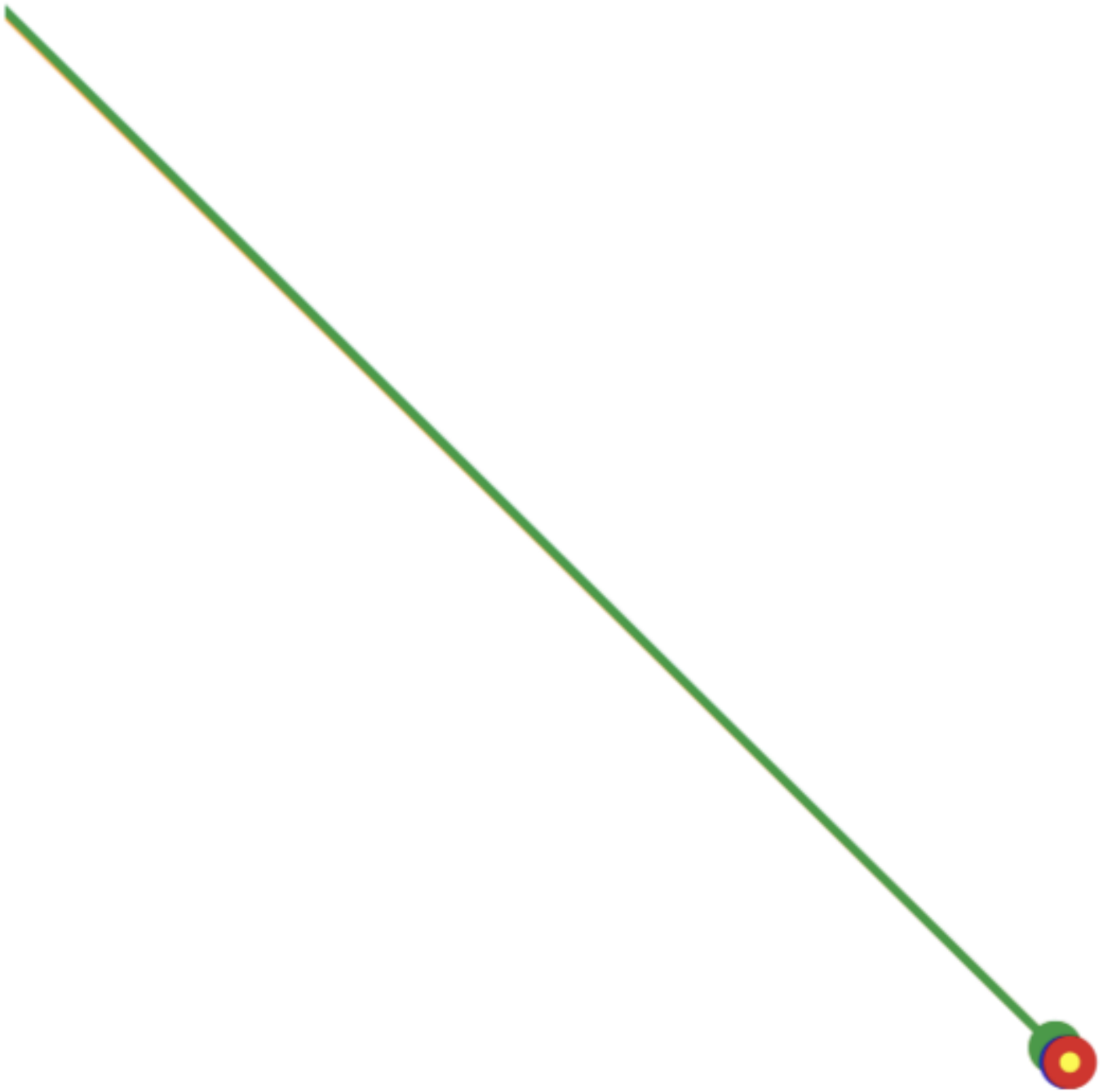


How big is the difference?



How big is the difference?

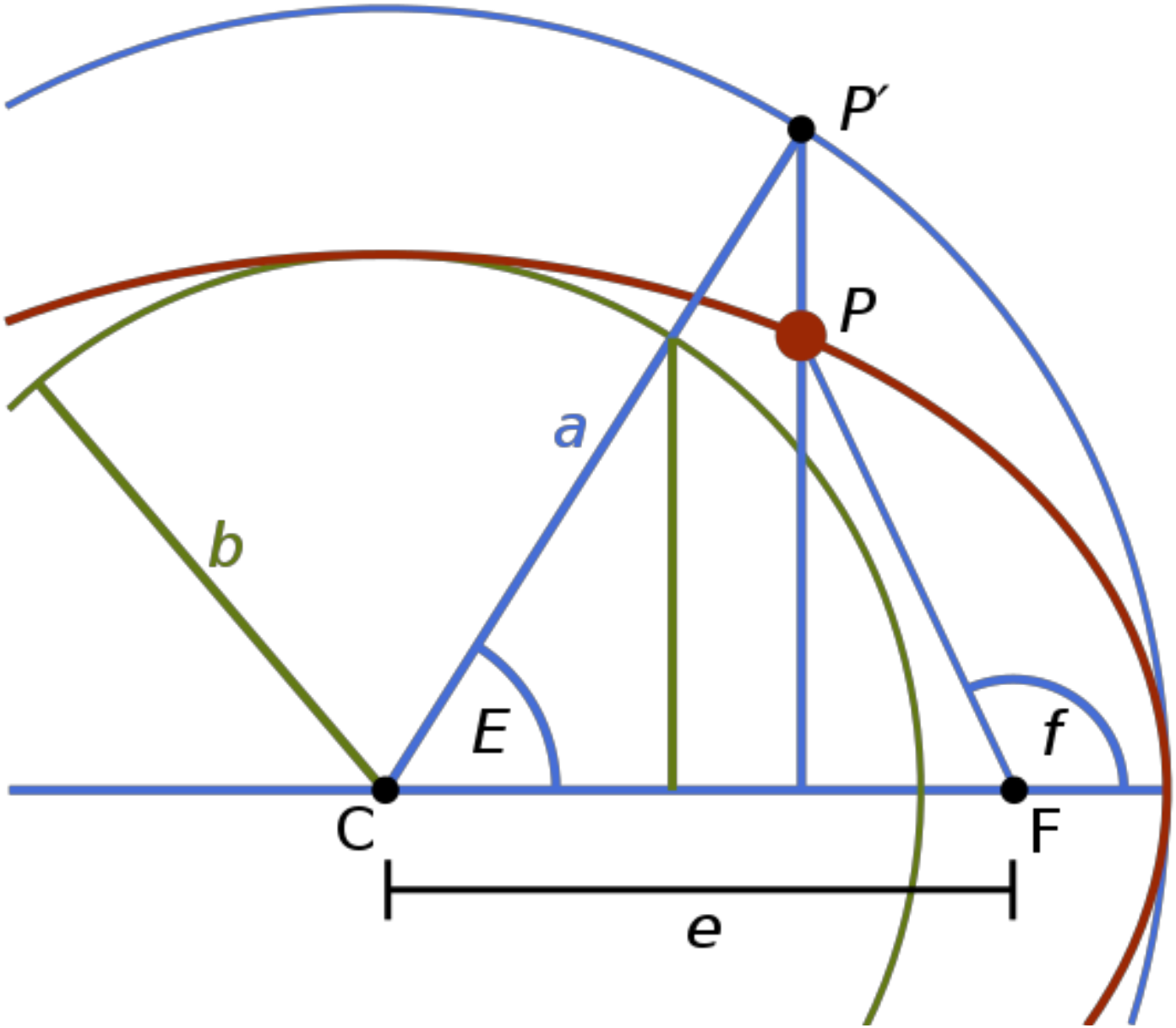
8 arcmin

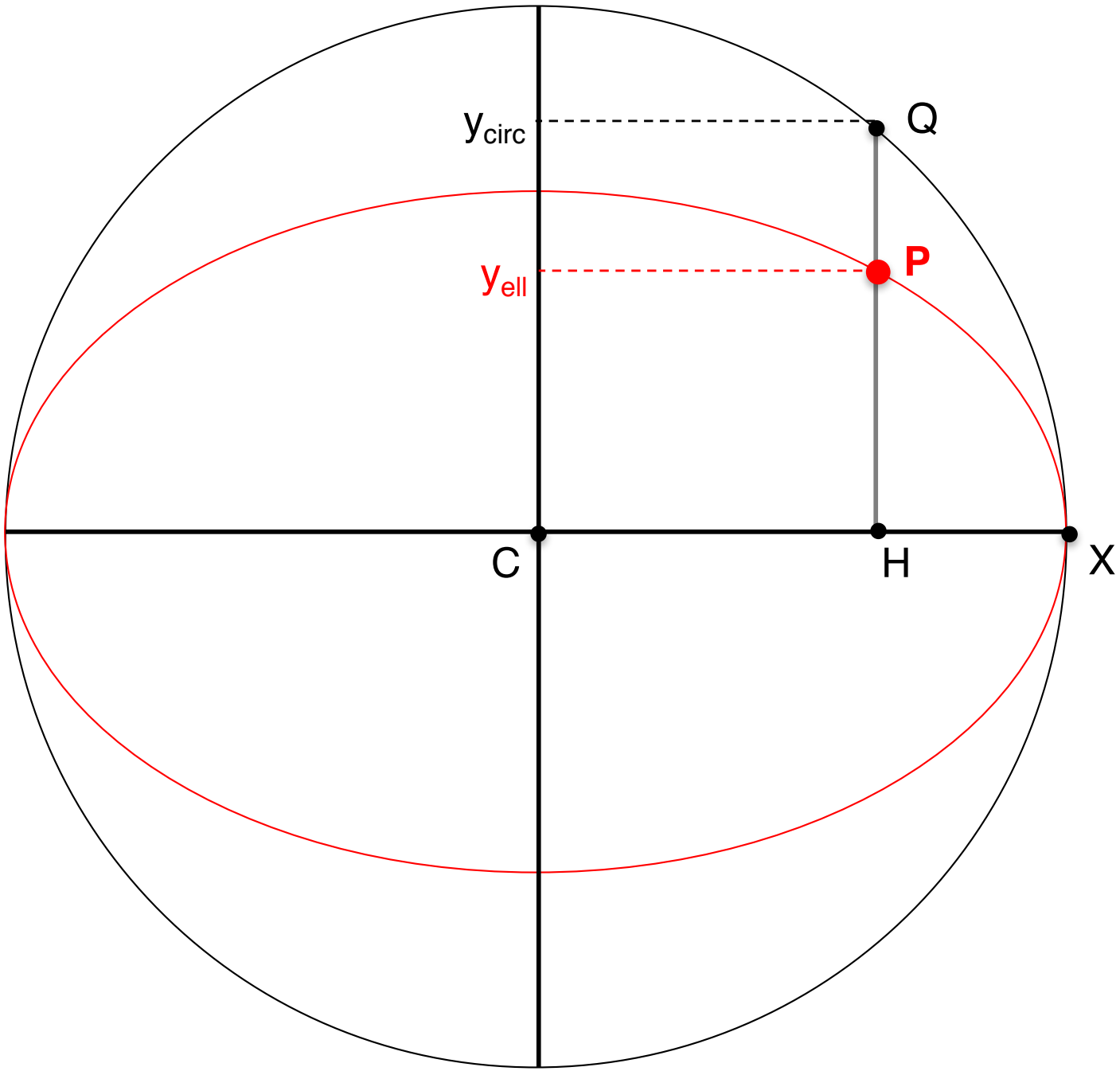


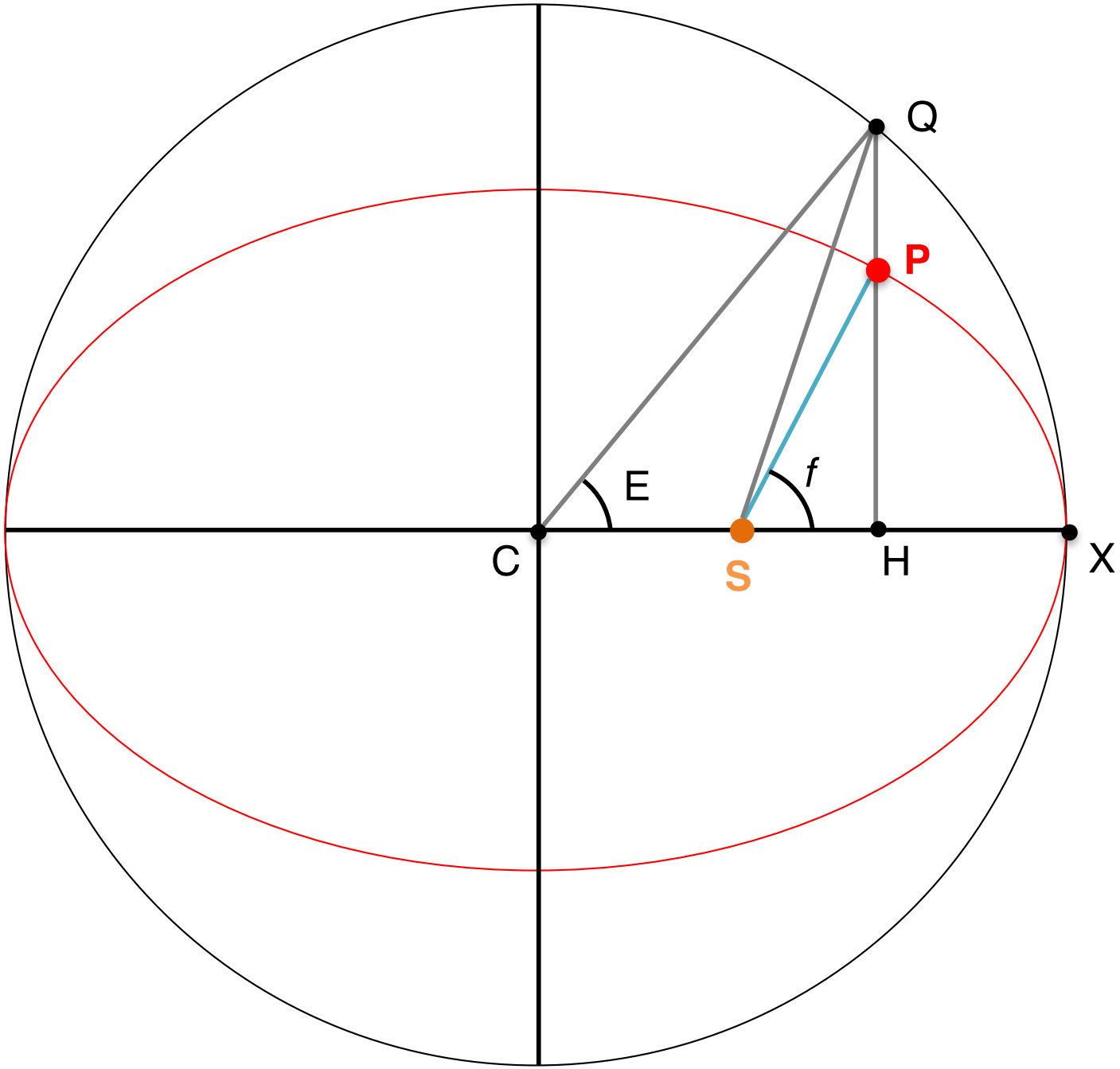
“If I had believed that we could ignore these eight arcminutes, I would have patched up my hypothesis accordingly. But since it was not permissible to ignore them, those eight arcminutes point the road to a complete reformation of astronomy.”

Kepler, Johannes, *Astronomia Nova*, 1609

Eccentric anomaly and True anomaly




y_{circ}
Q
y_{ell}
P
C
H
X


Q
P
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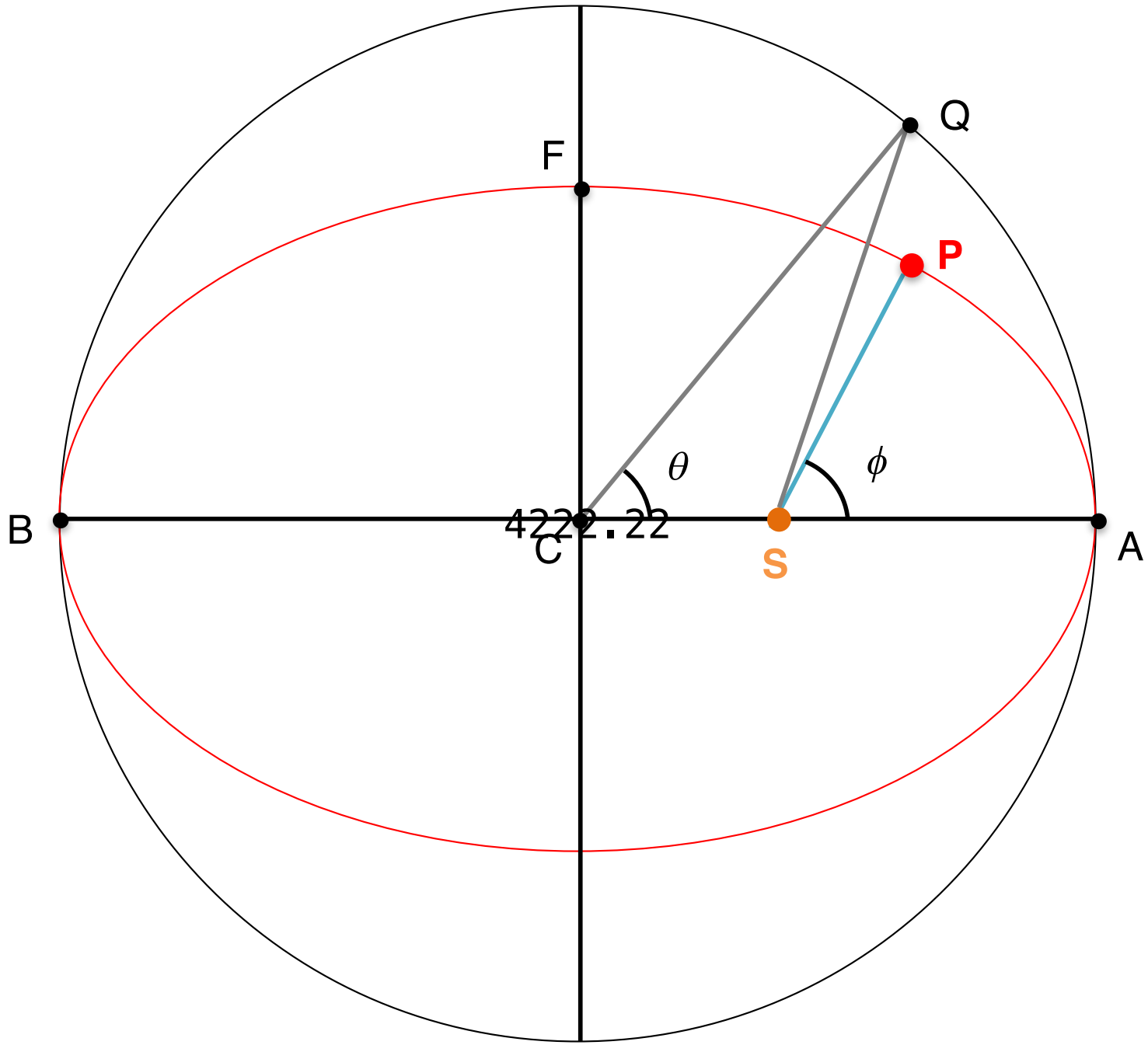
f
C
S
H
X


Q
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θ
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B
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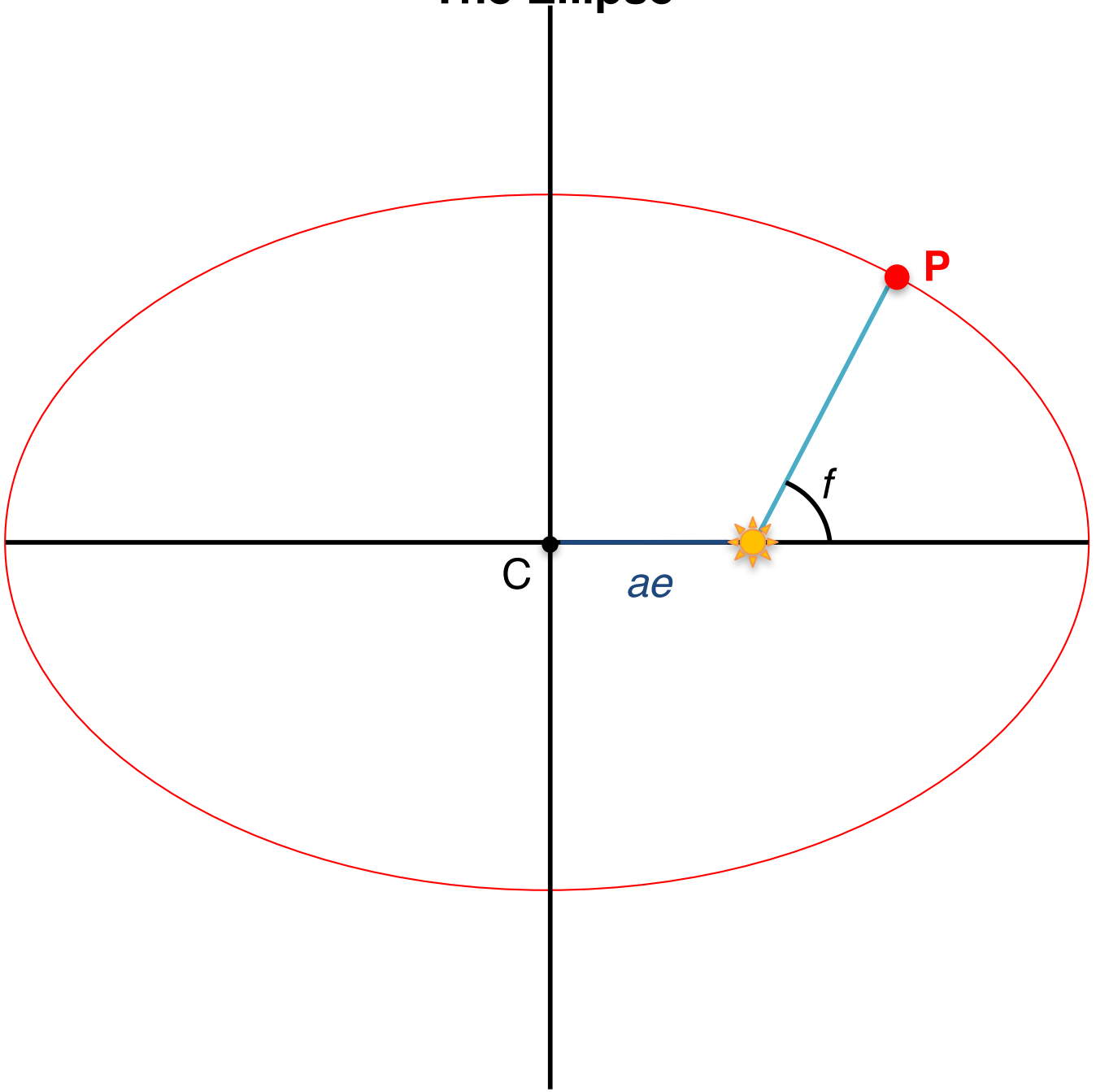
S
A

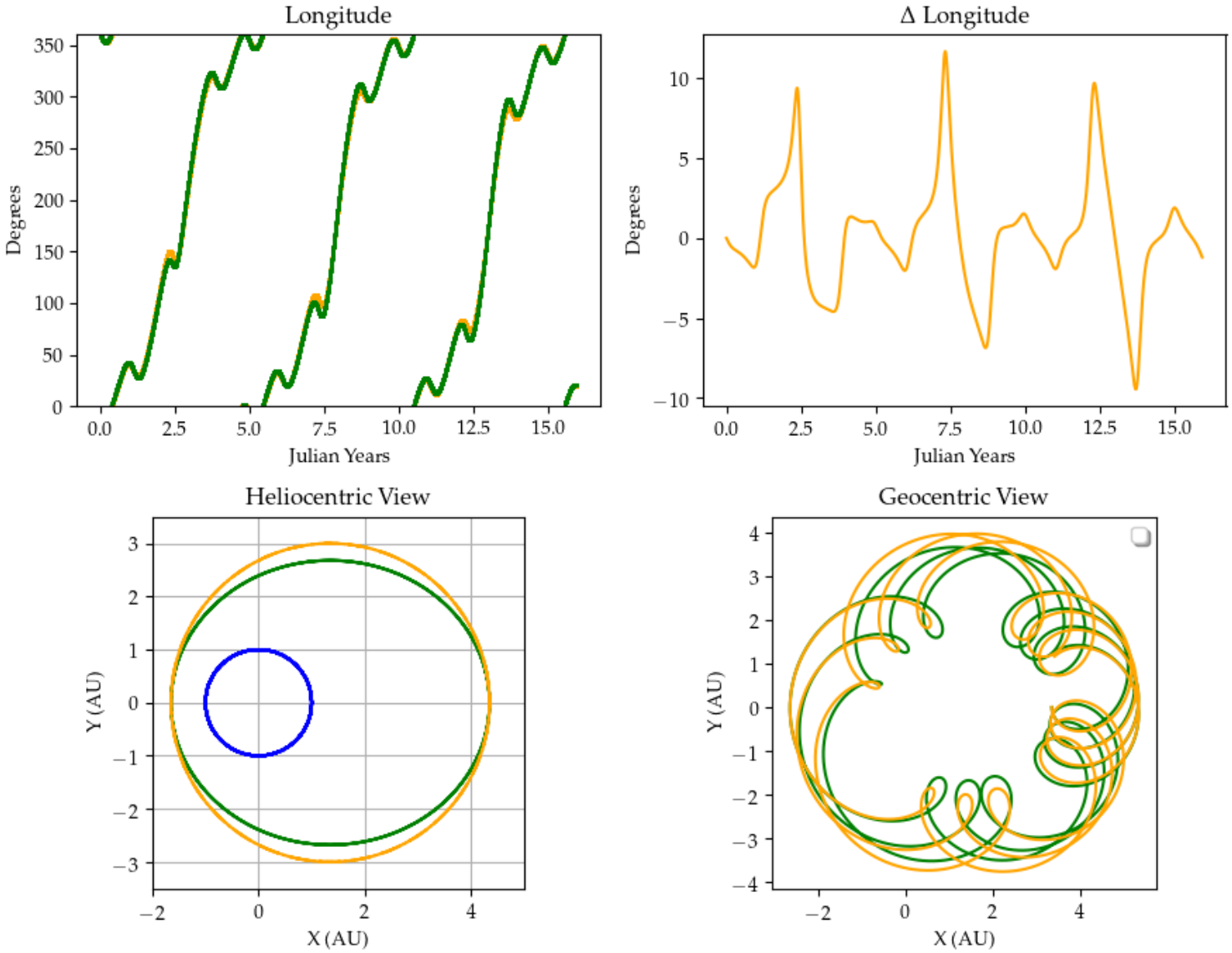
“If this wearisome method has filled you with loathing, it should more properly fill you with compassion for me as I have gone through it at least seventy times at the expense of a great deal of time.”

Kepler, Johannes, *Astronomia Nova*, 1609

The Ellipse

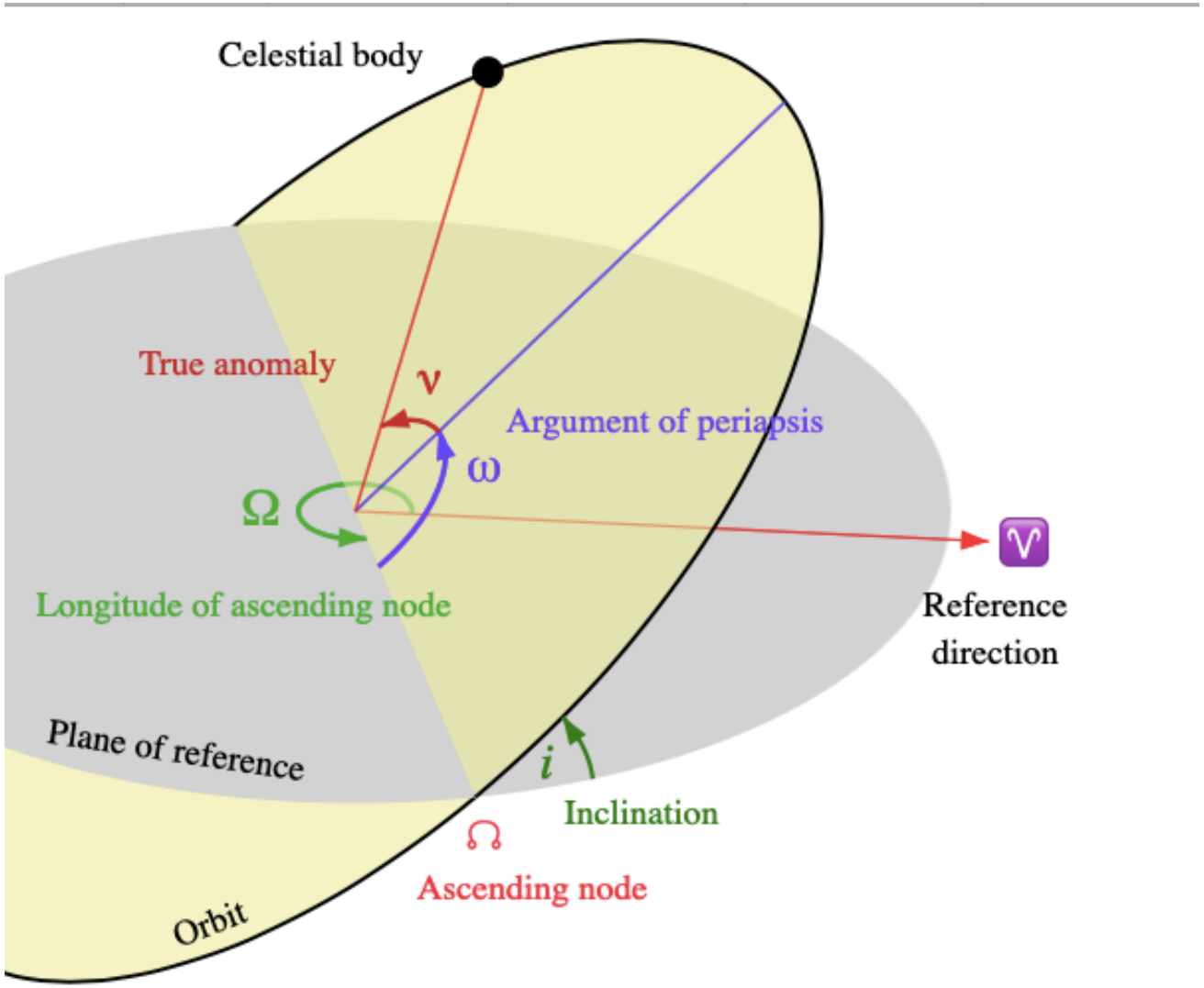


Equant fails at larger eccentricities

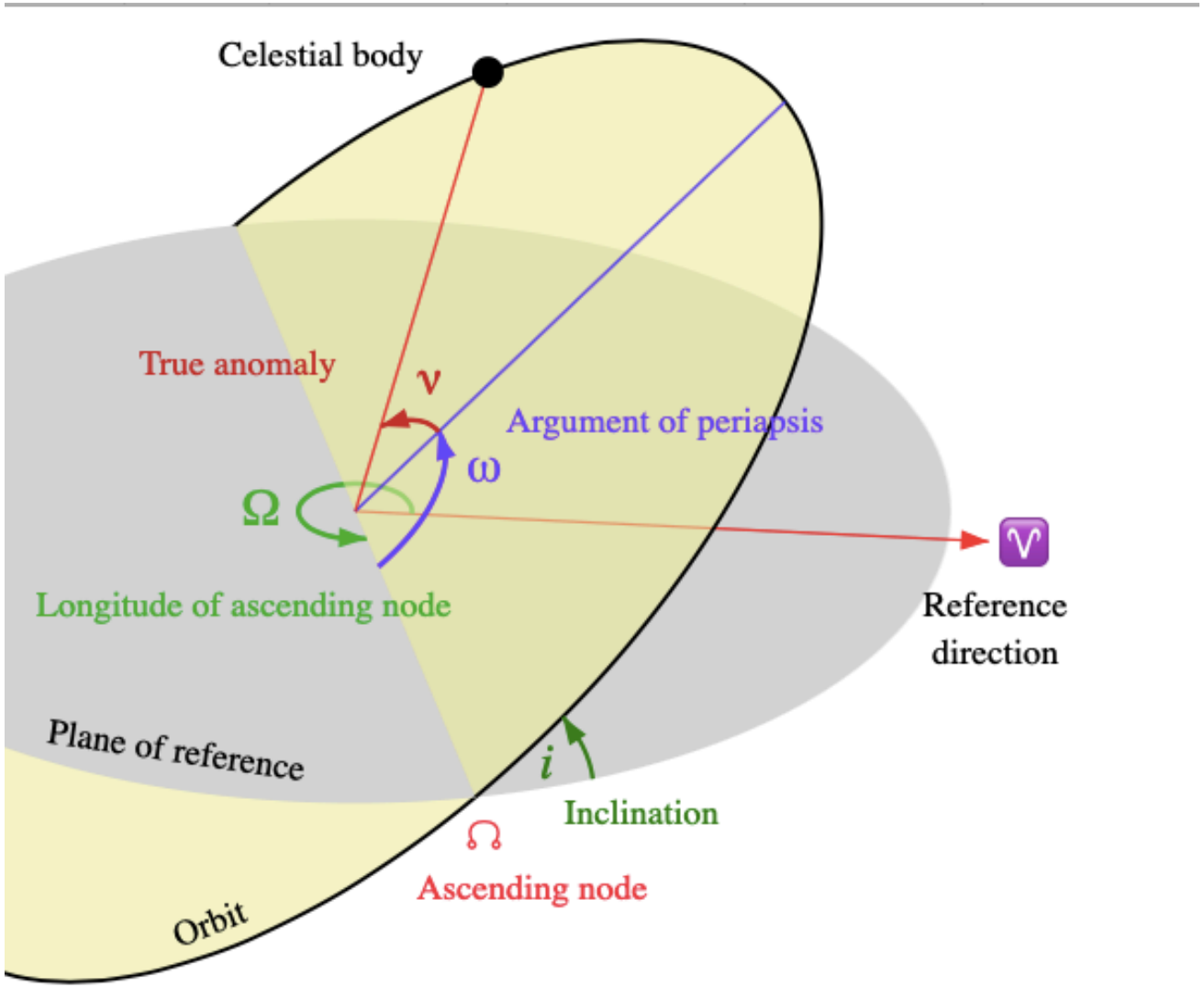


Hypothetical planet, *e=0.45*

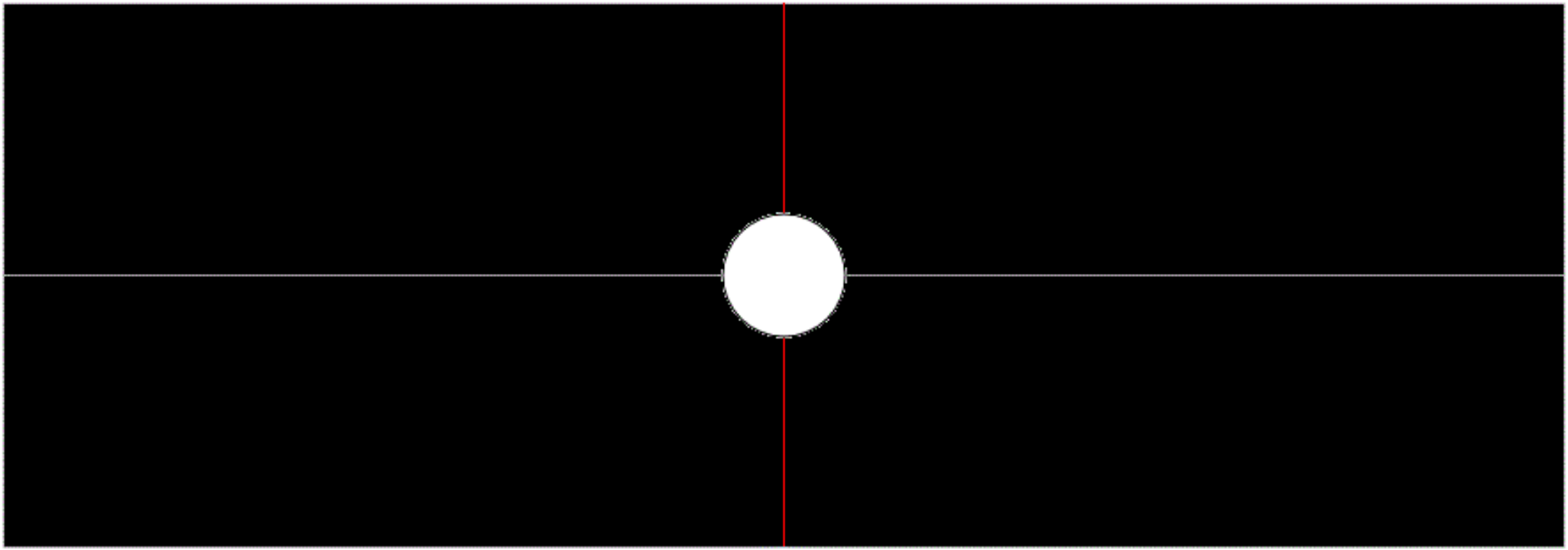
Orbital Elements



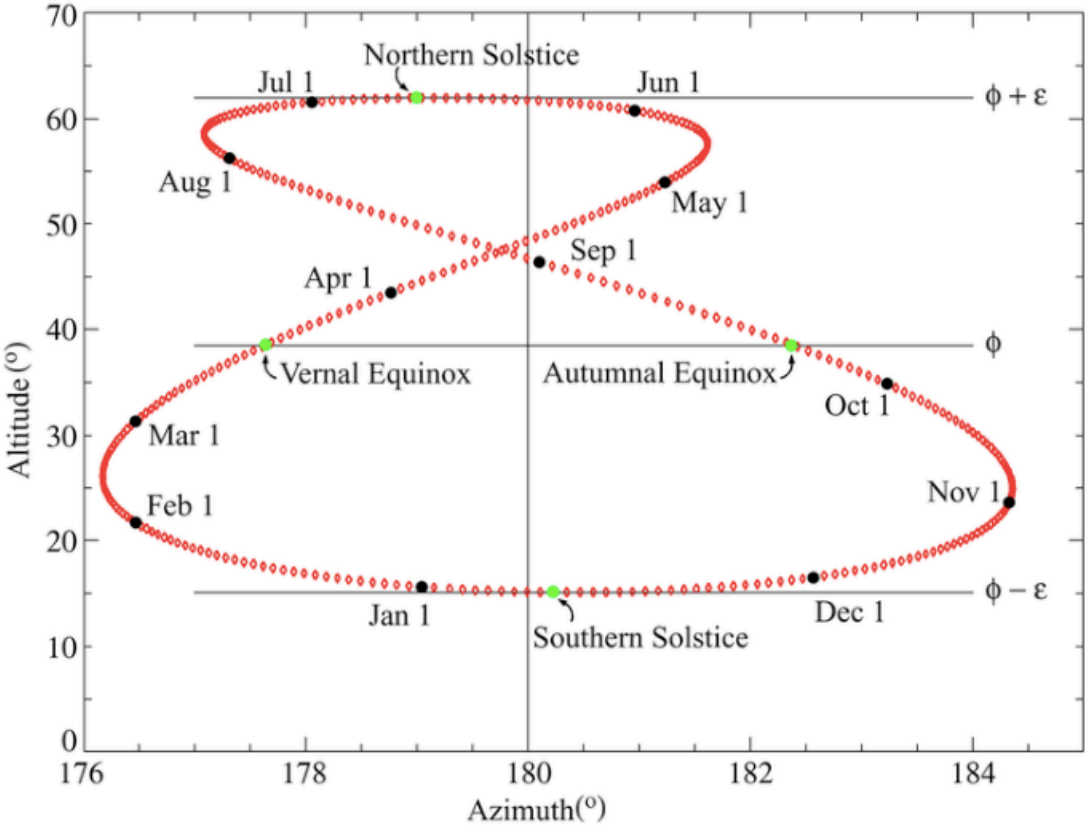
Orbital Elements

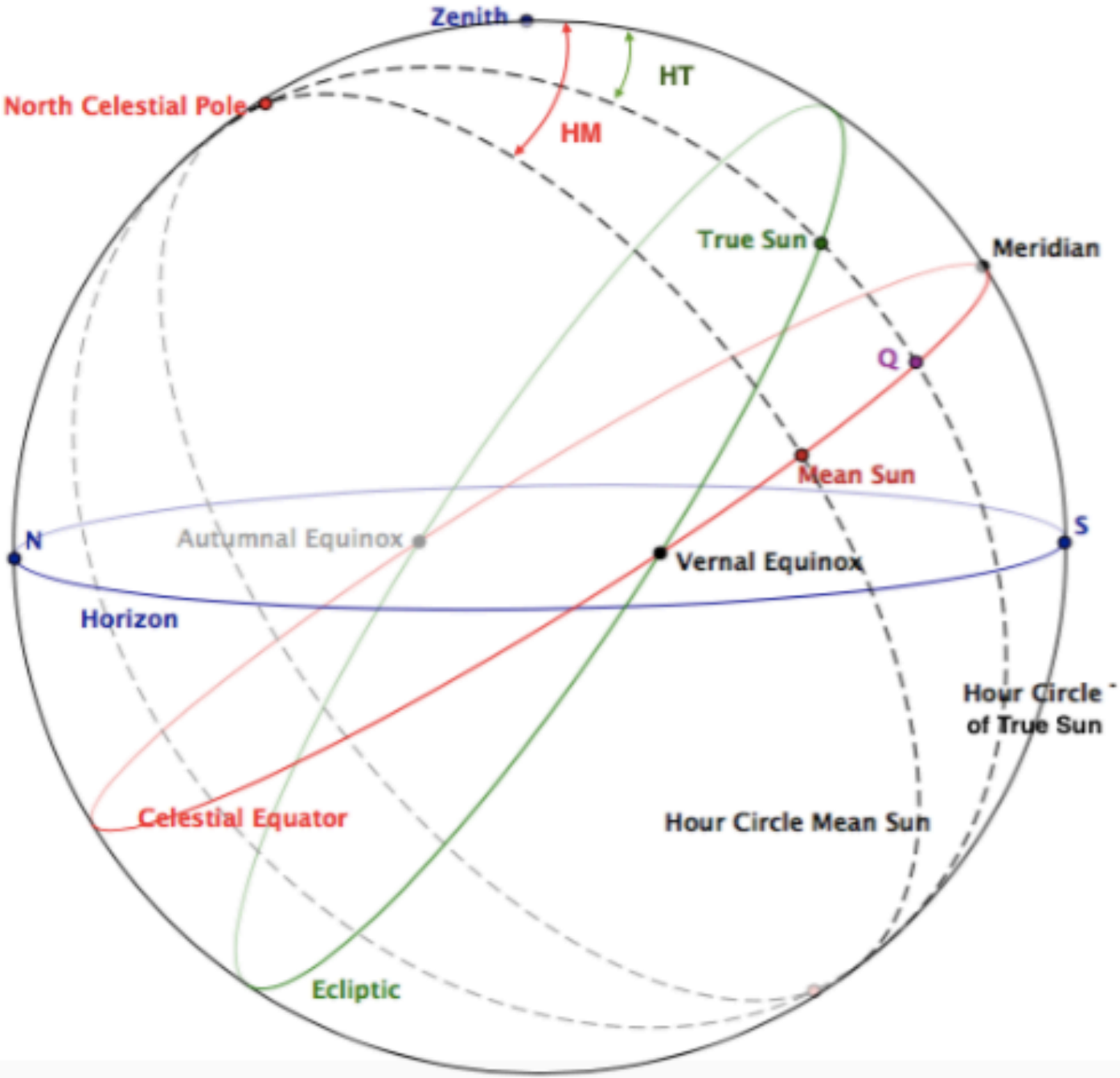


Deviation True and Mean Anomaly

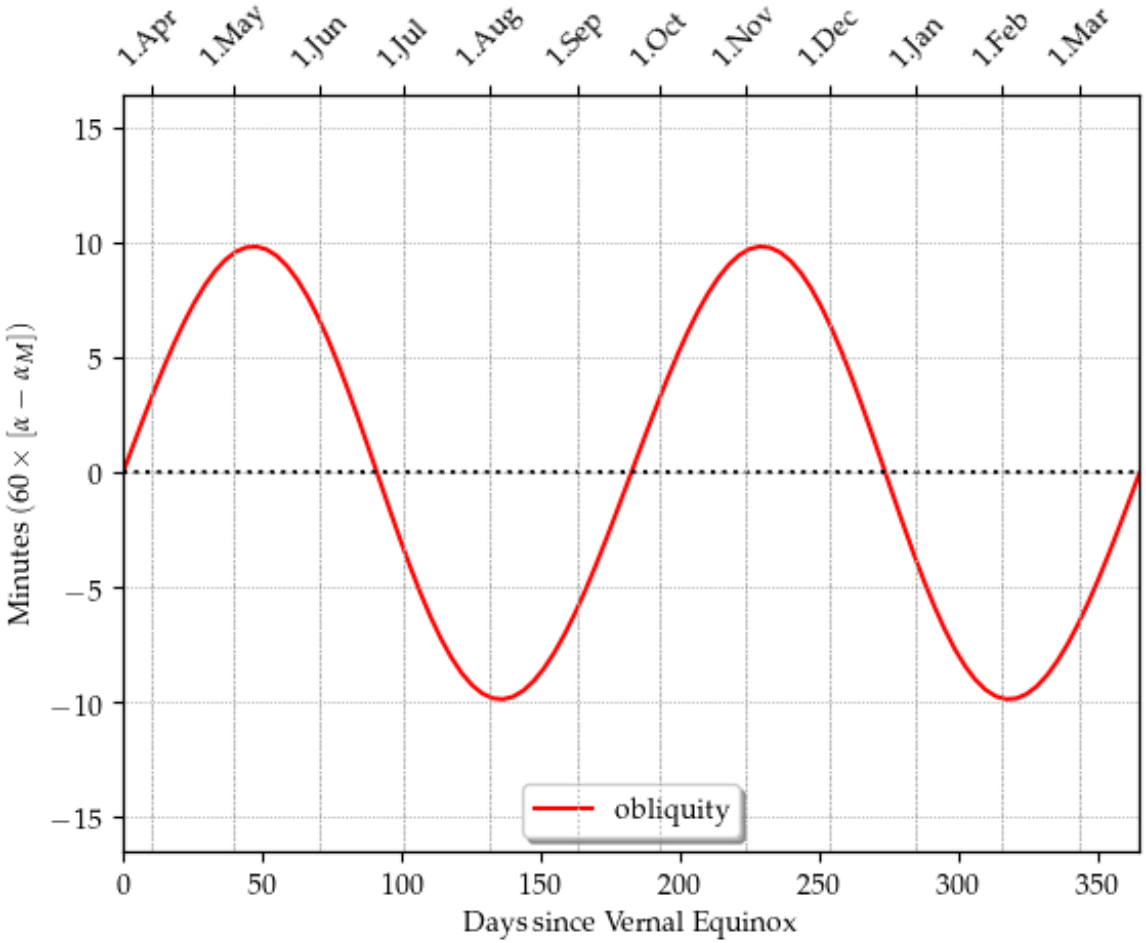


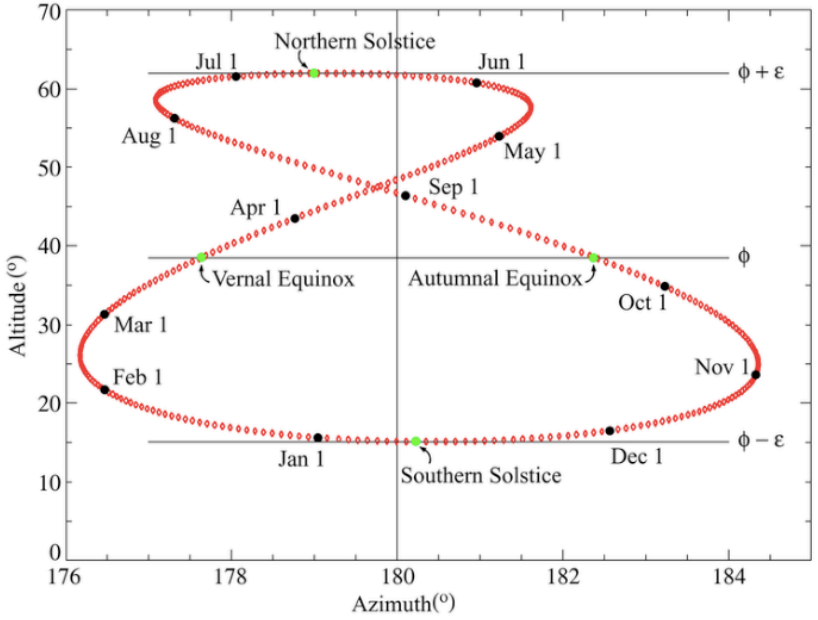
The analemma



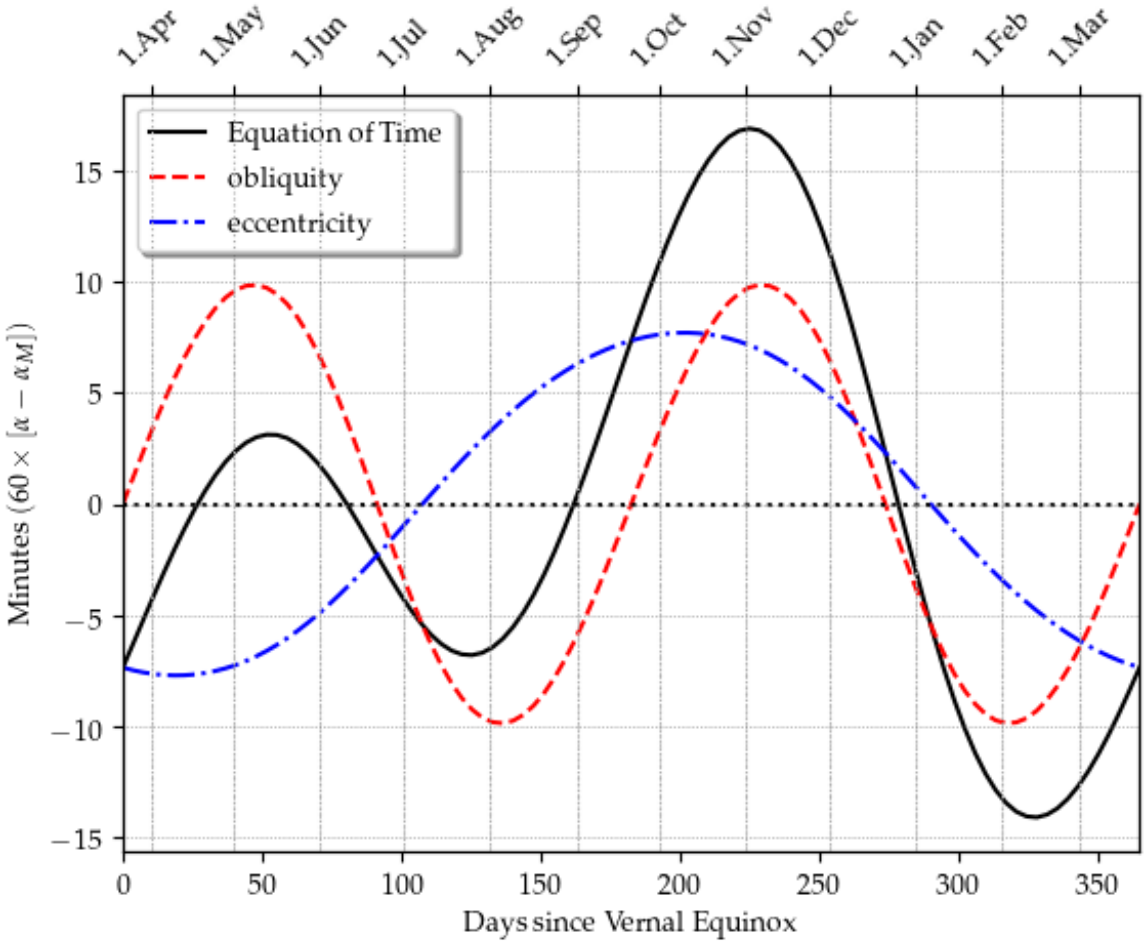

Zenith
North Celestial Pole
HT
HM
True Sun
Meridian
Q
Mean Sun
N
Autumnal Equinox
Vernal Equinox
S
Horizon
Hour Circle
of True Sun
Celestial Equator
Hour Circle Mean Sun
Ecliptic

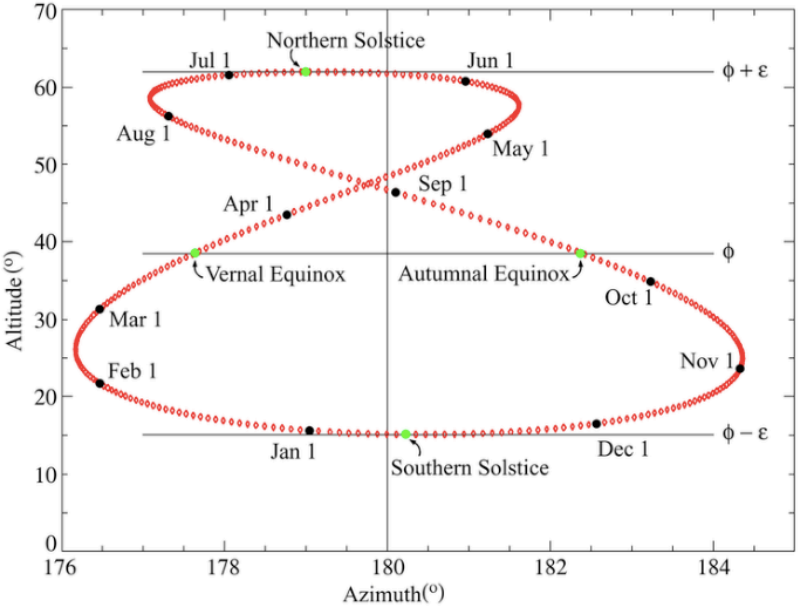
Equation of Time

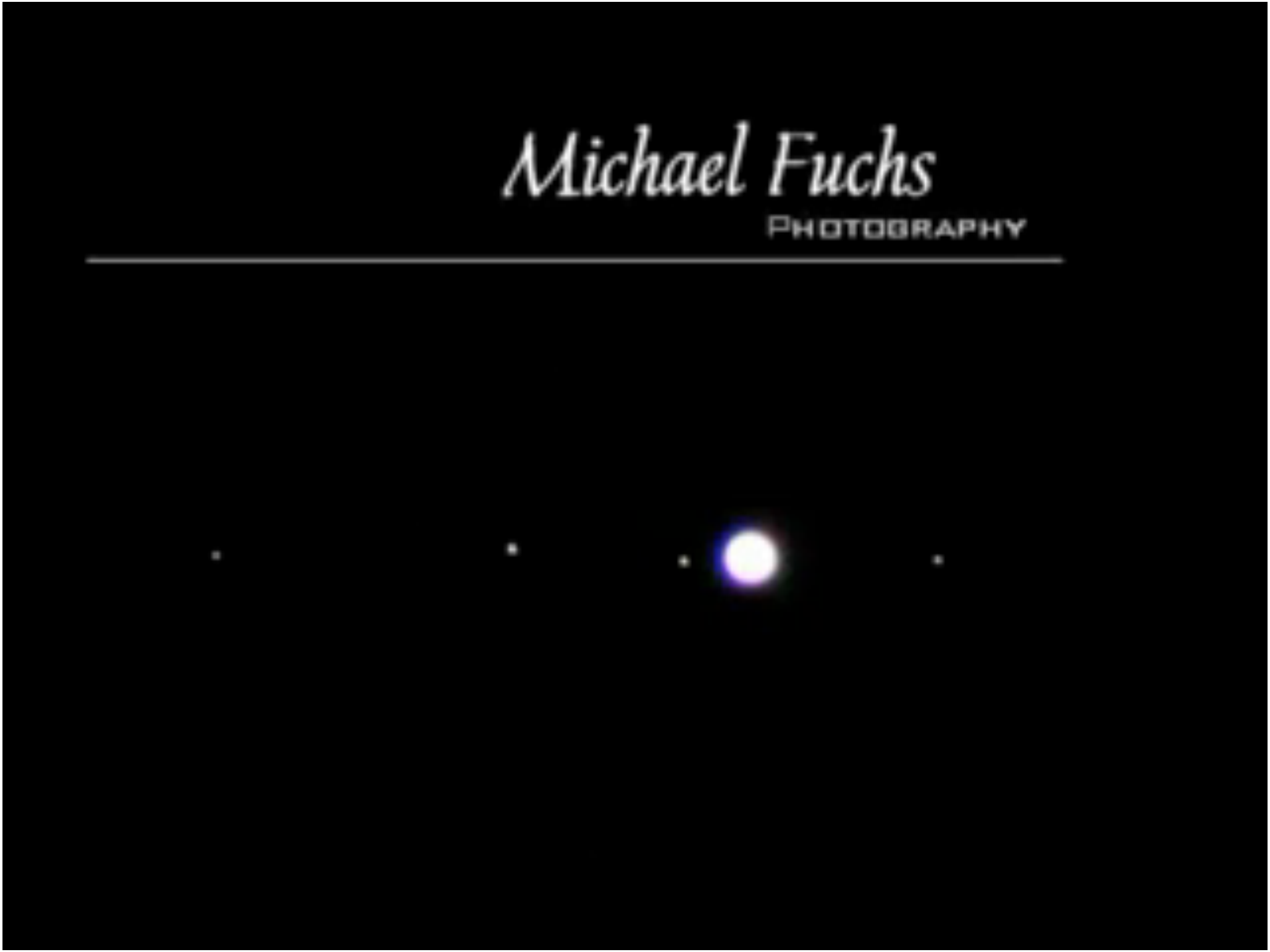


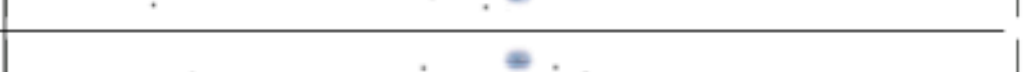


Equation of Time

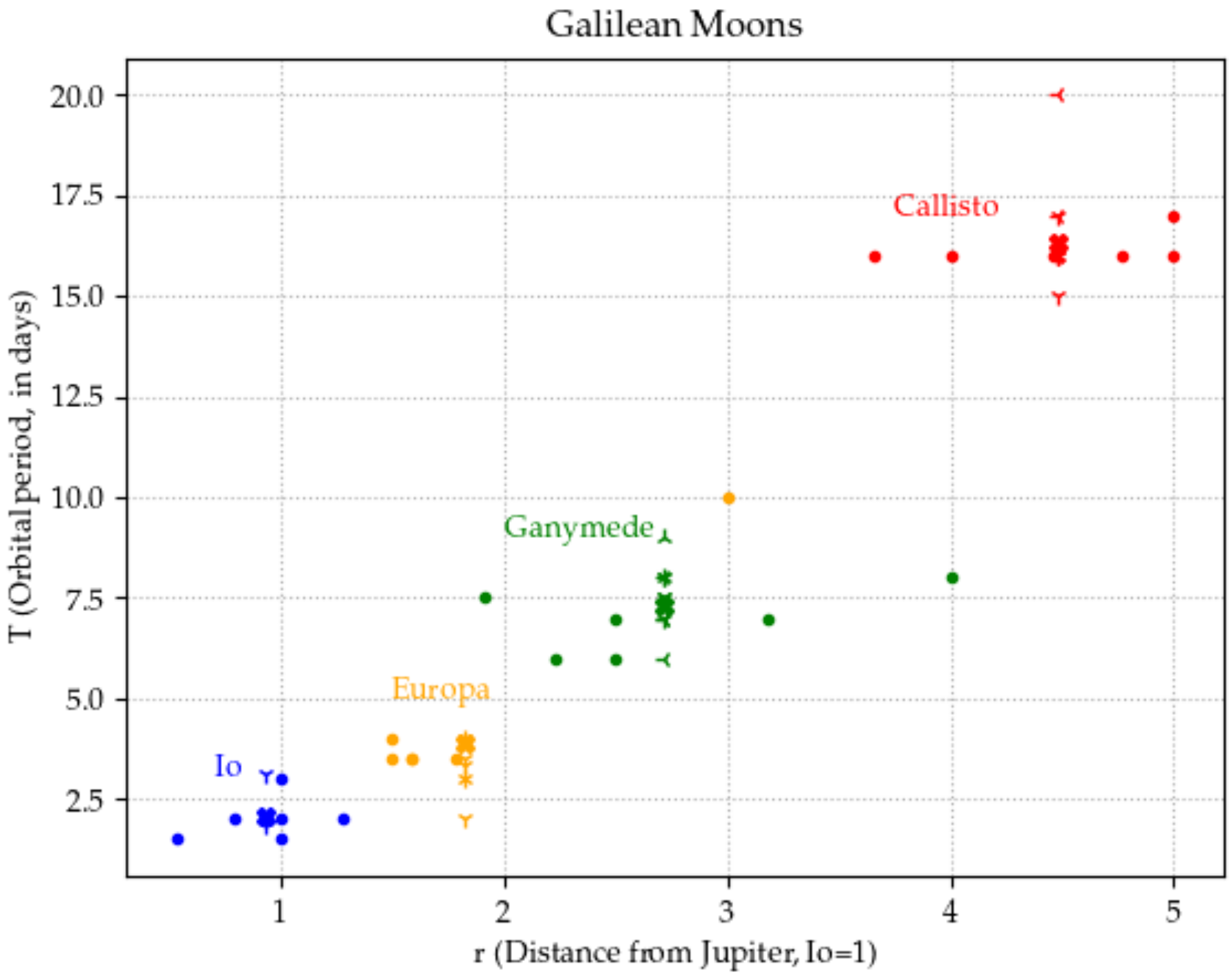


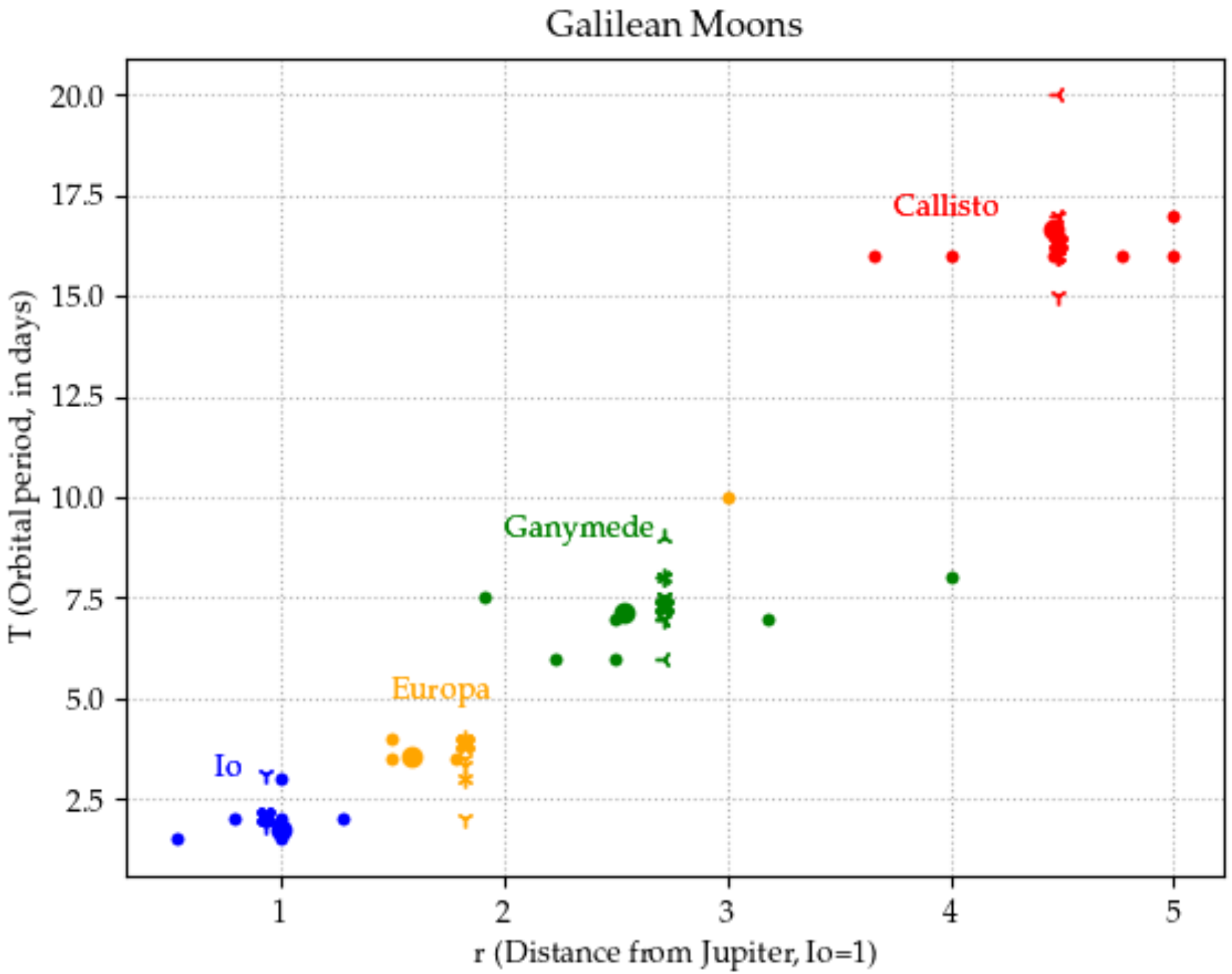


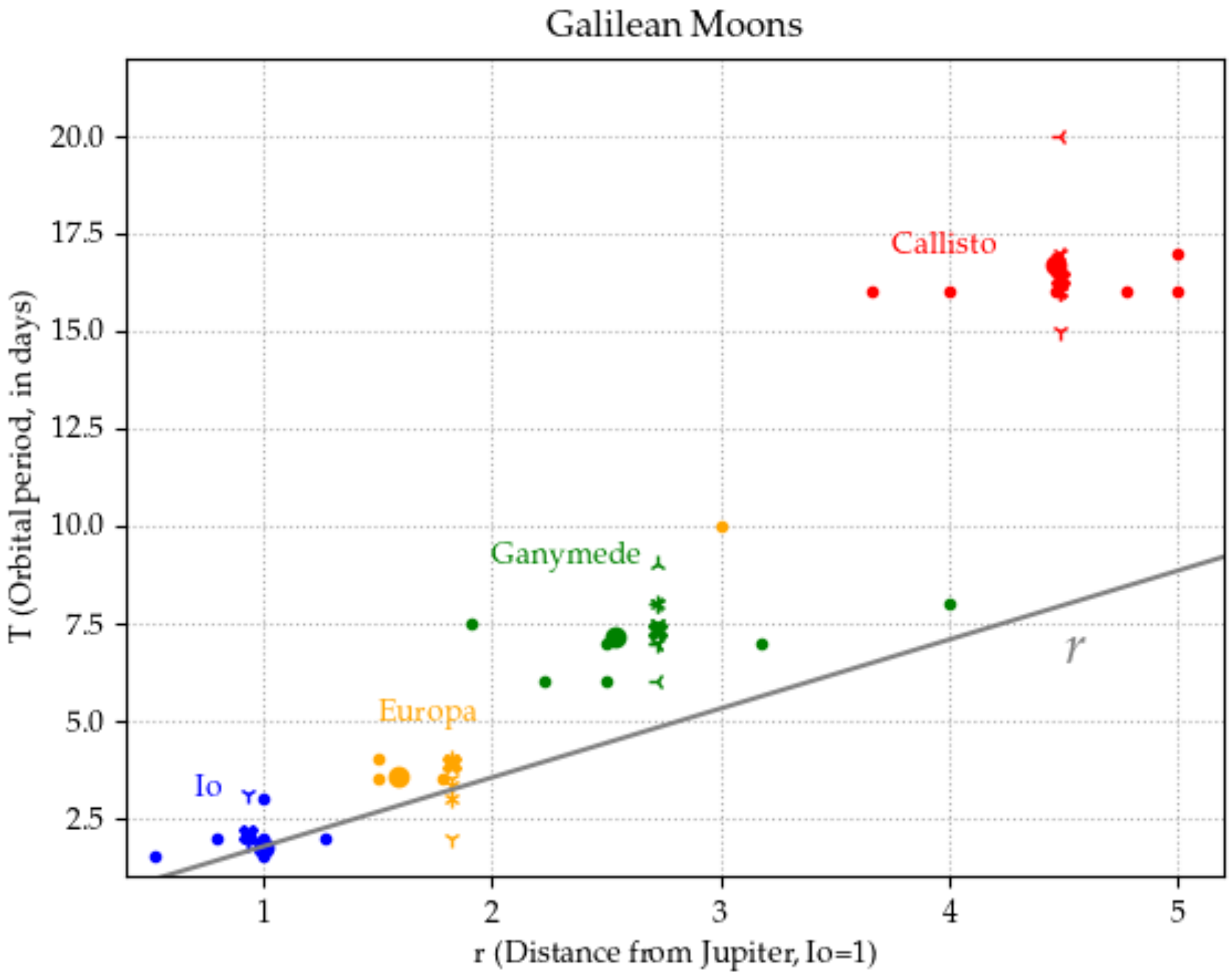

Michael Fuchs
PHOTOGRAPHY

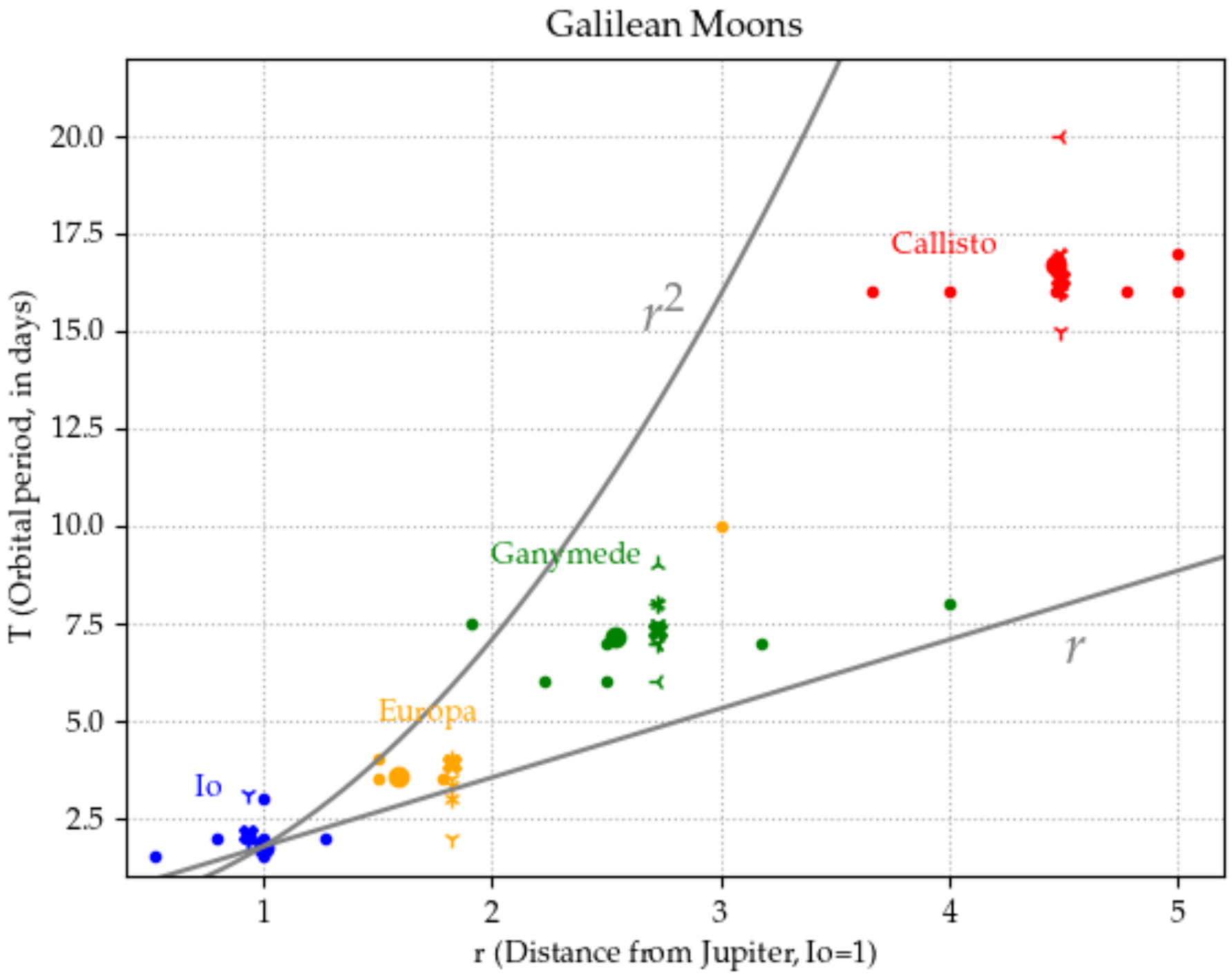
d	h	
0	0	
0	12	
1	0	
1	12	
2	0	
2	12	
3	0	
3	12	
4	0	
4	12	
5	0	
5	12	
6	0	
6	12	
7	0	
7	12	
8	0	
8	12	
9	0	
9	12	
10	0	

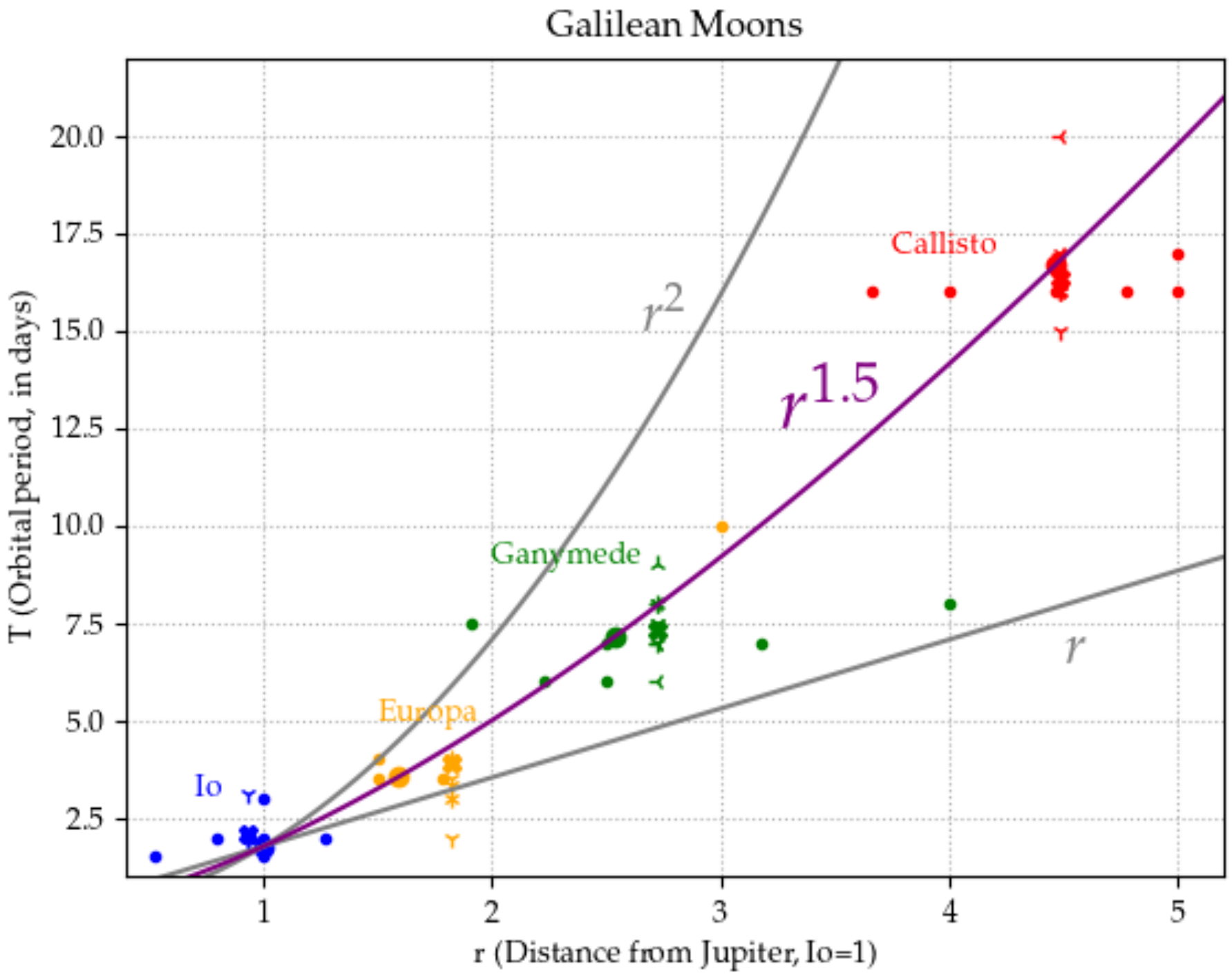
Images captured from http://www.shallowsky.com/jupiter/

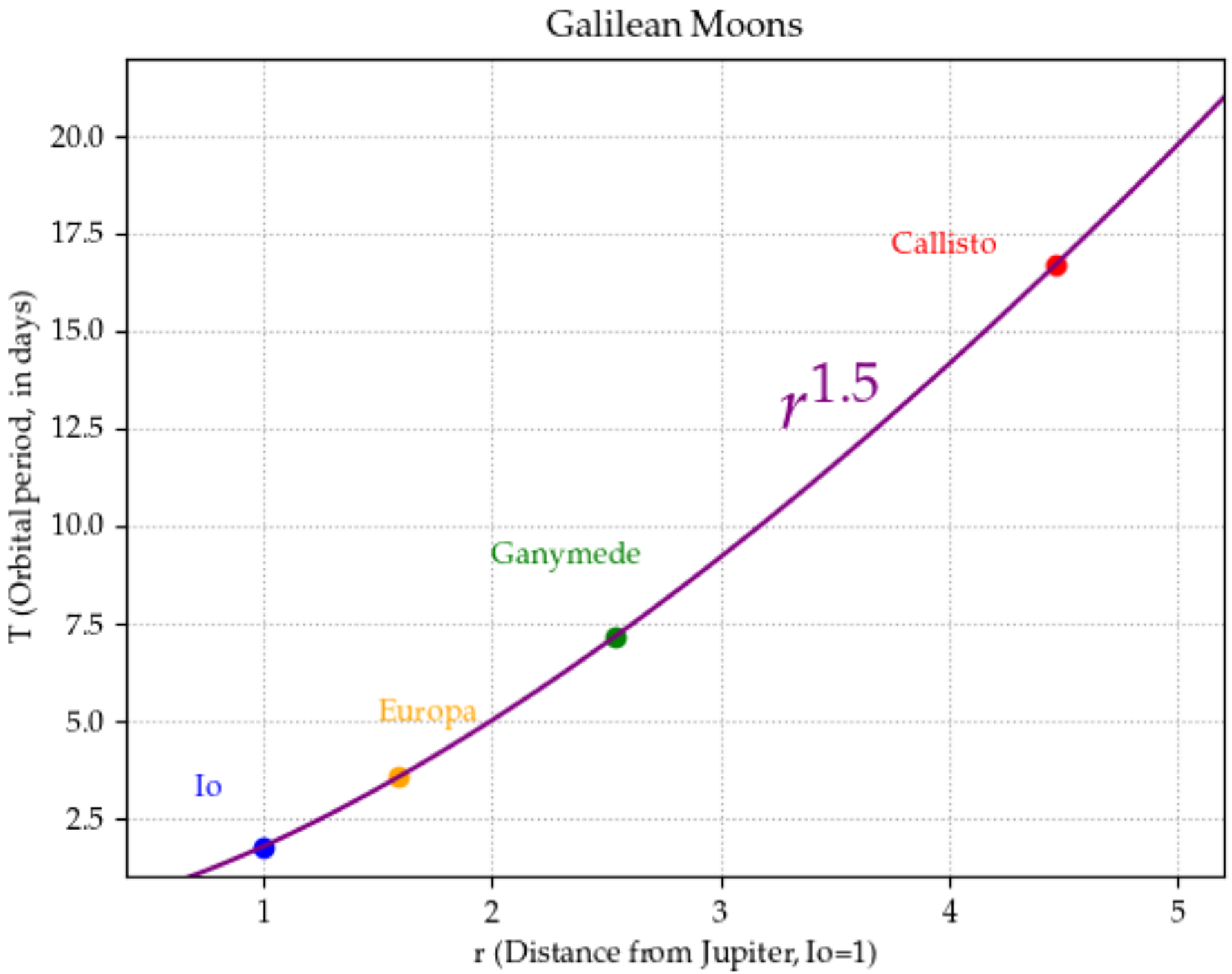

Galilean Moons
T (Orbital period, in days)
r (Distance from Jupiter, Io=1)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1
2
3
4
5
Callisto
Ganymede
Europa
Io


Galilean Moons
T (Orbital period, in days)
r (Distance from Jupiter, Io=1)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1
2
3
4
5
Callisto
Ganymede
Europa
Io

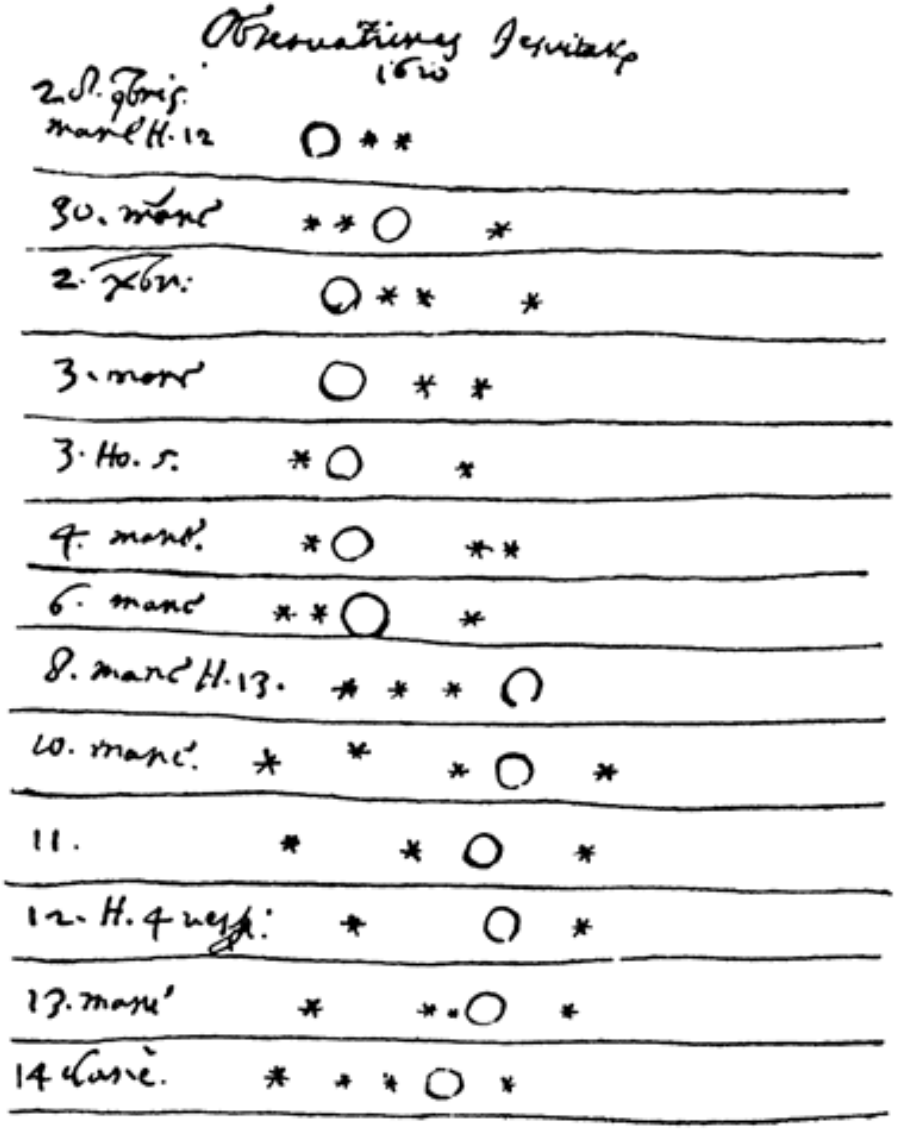

Galilean Moons
T (Orbital period, in days)
r (Distance from Jupiter, Io=1)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1
2
3
4
5
Callisto
Ganymede
Europa
Io
r

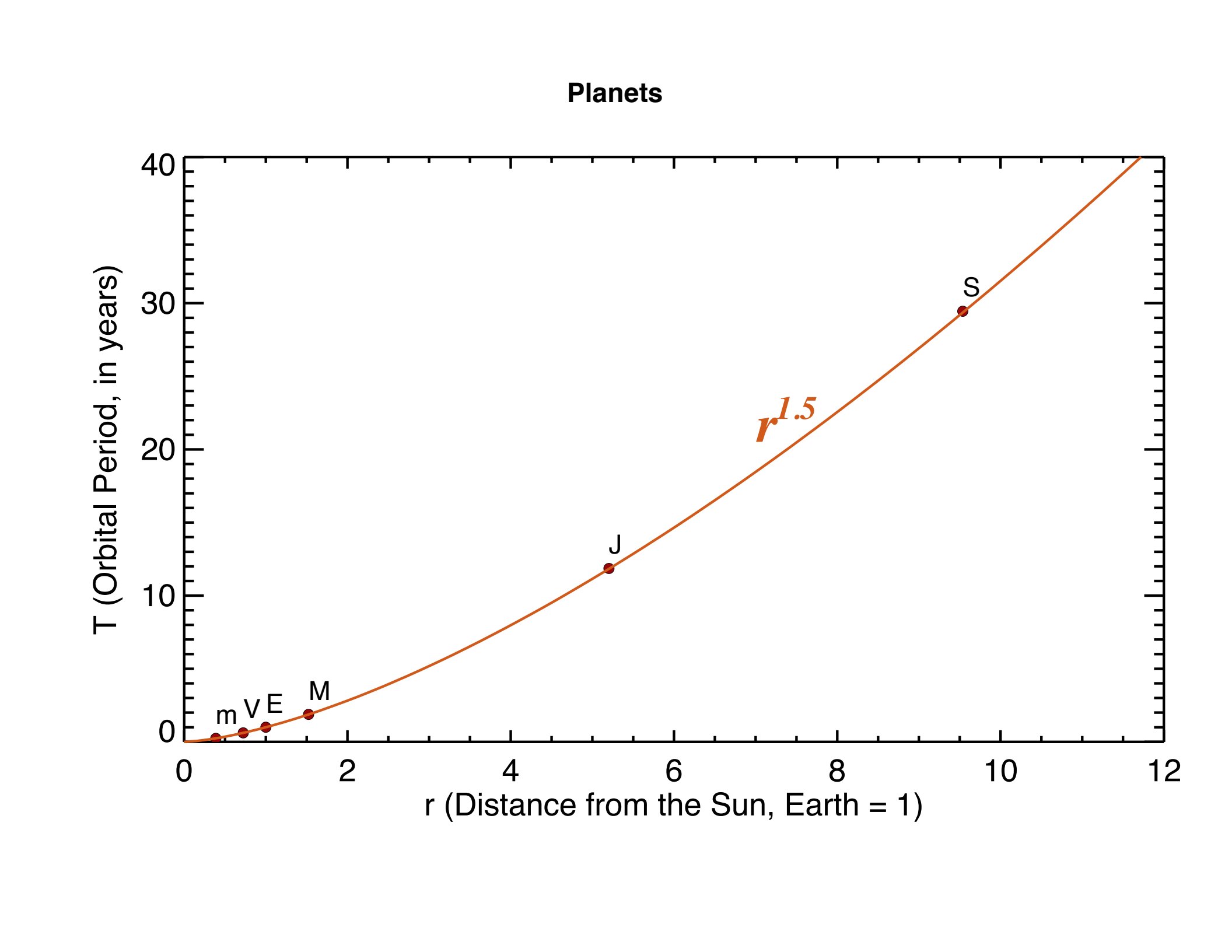

Galilean Moons
T (Orbital period, in days)
r (Distance from Jupiter, Io=1)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1
2
3
4
5
Callisto
Ganymede
Europa
Io
r^2
r


Galilean Moons
T (Orbital period, in days)
r (Distance from Jupiter, Io=1)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1
2
3
4
5
Callisto
Ganymede
Europa
Io
r^2
$r^{1.5}$
r

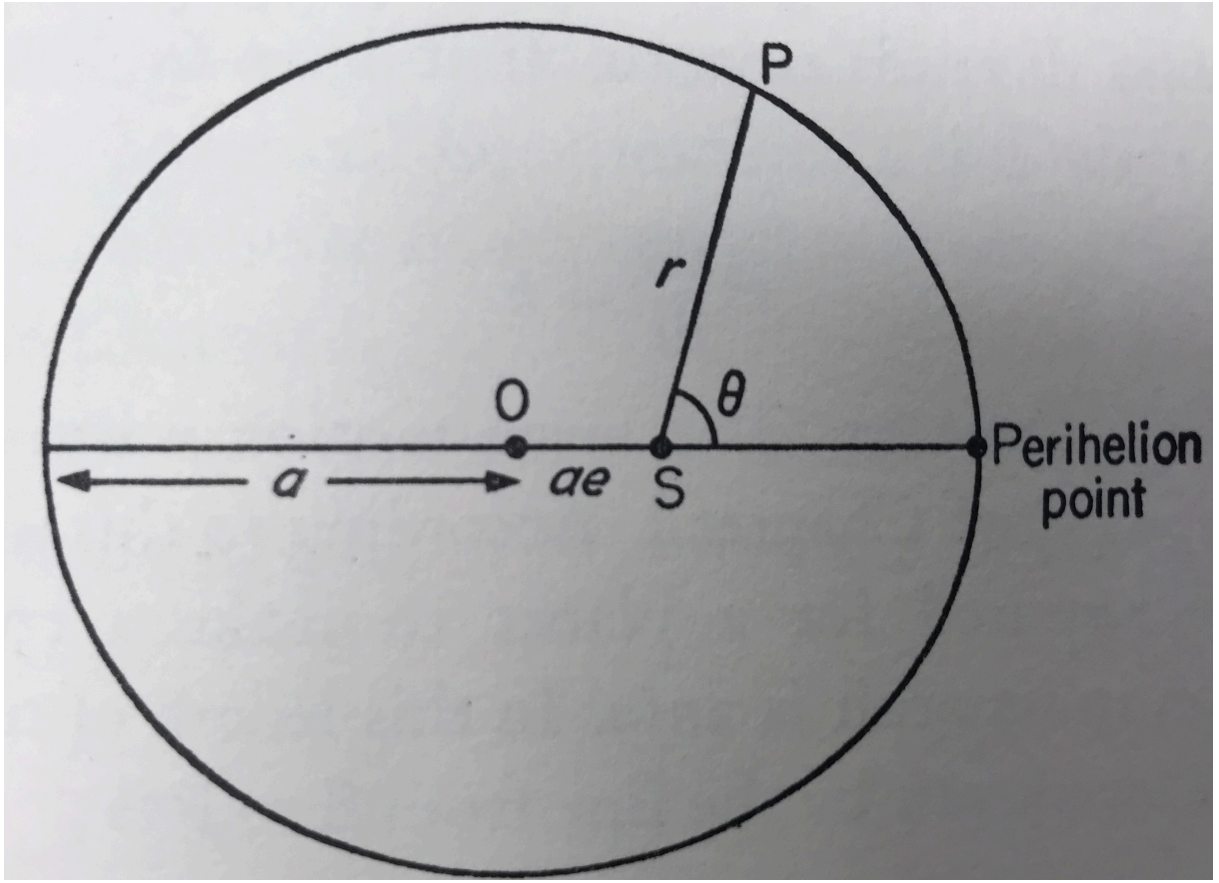

Galilean Moons
T (Orbital period, in days)
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
1
2
3
4
5
r (Distance from Jupiter, Io=1)
Callisto
$r^{1.5}$
Ganymede
Europa
Io

A page from Galileo's *Sidereus Nuncius*

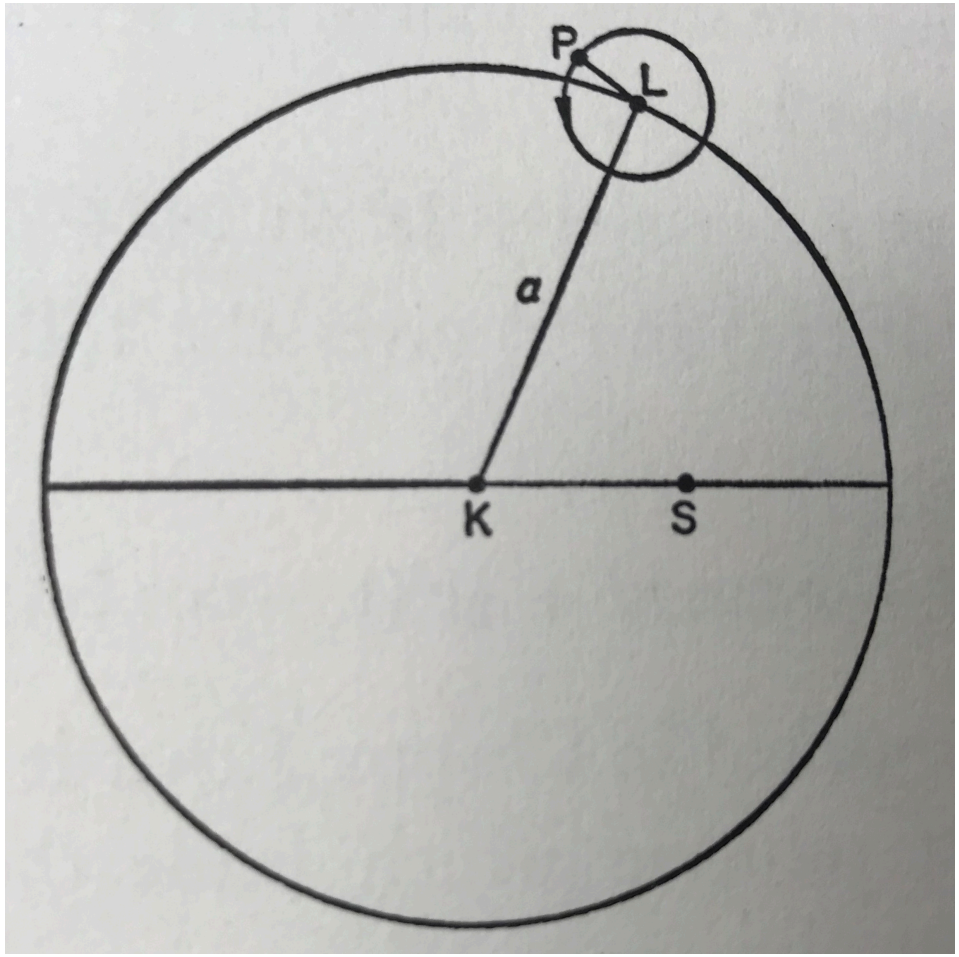



Planets
T (Orbital Period, in years)
r (Distance from the Sun, Earth = 1)
0
10
20
30
40
0
2
4
6
8
10
12
m
V
E
M
J
S
r^1.5

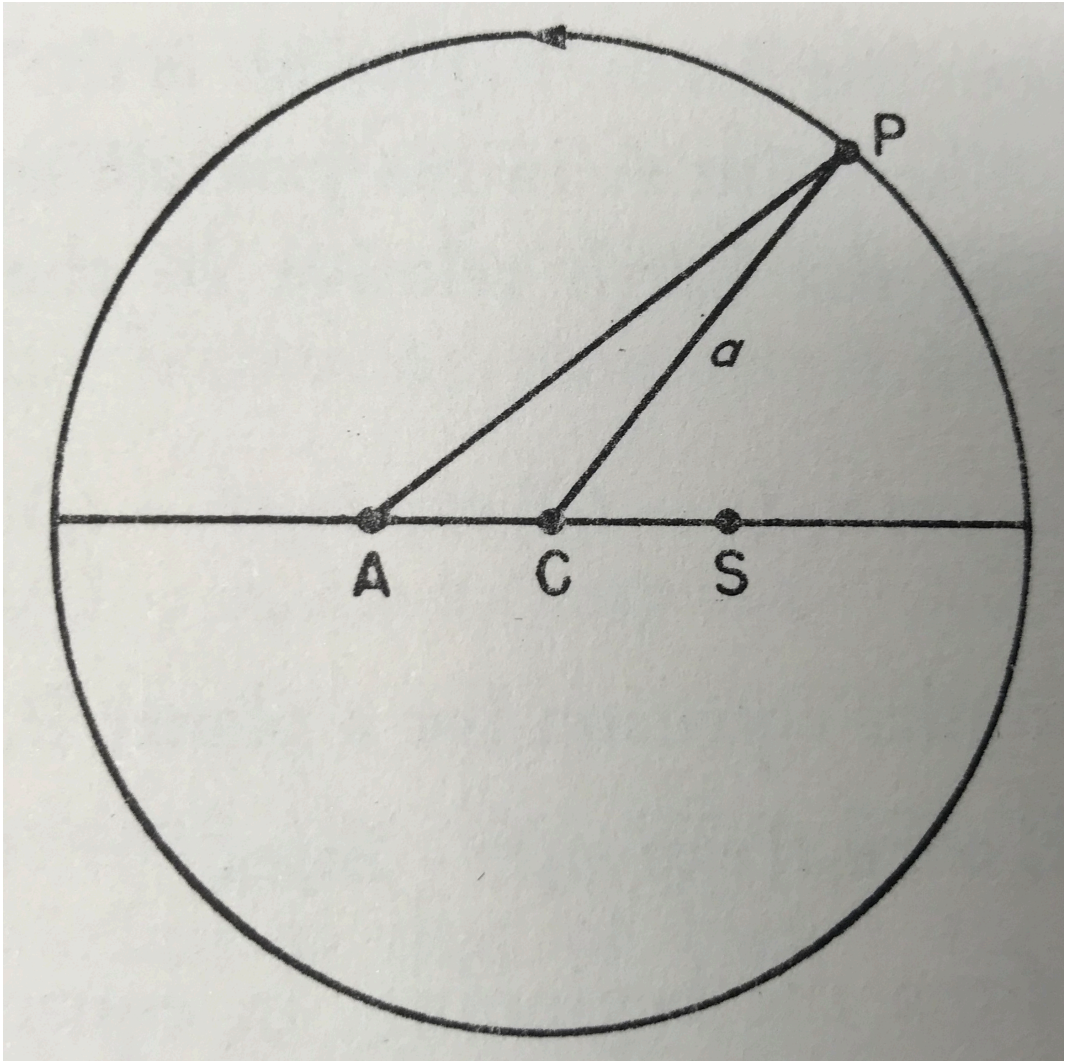
Elliptical orbit



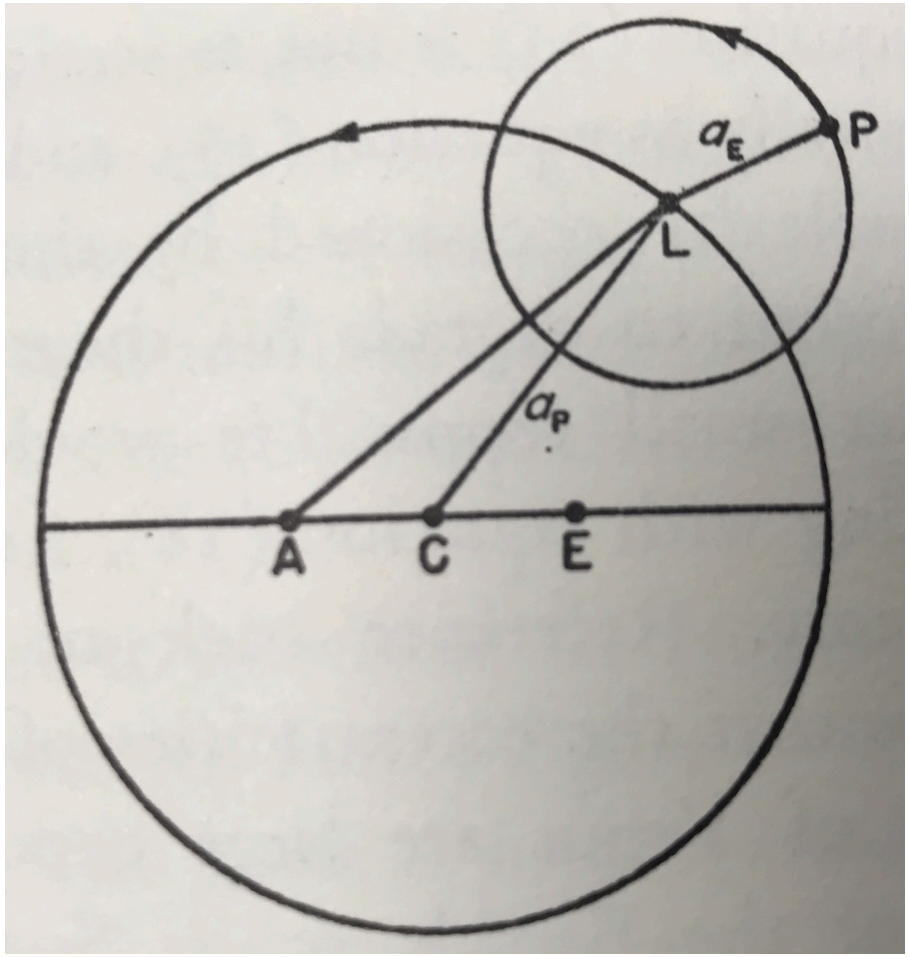
Copernicus' model



Ptolemy's model (in heliocentric frame)

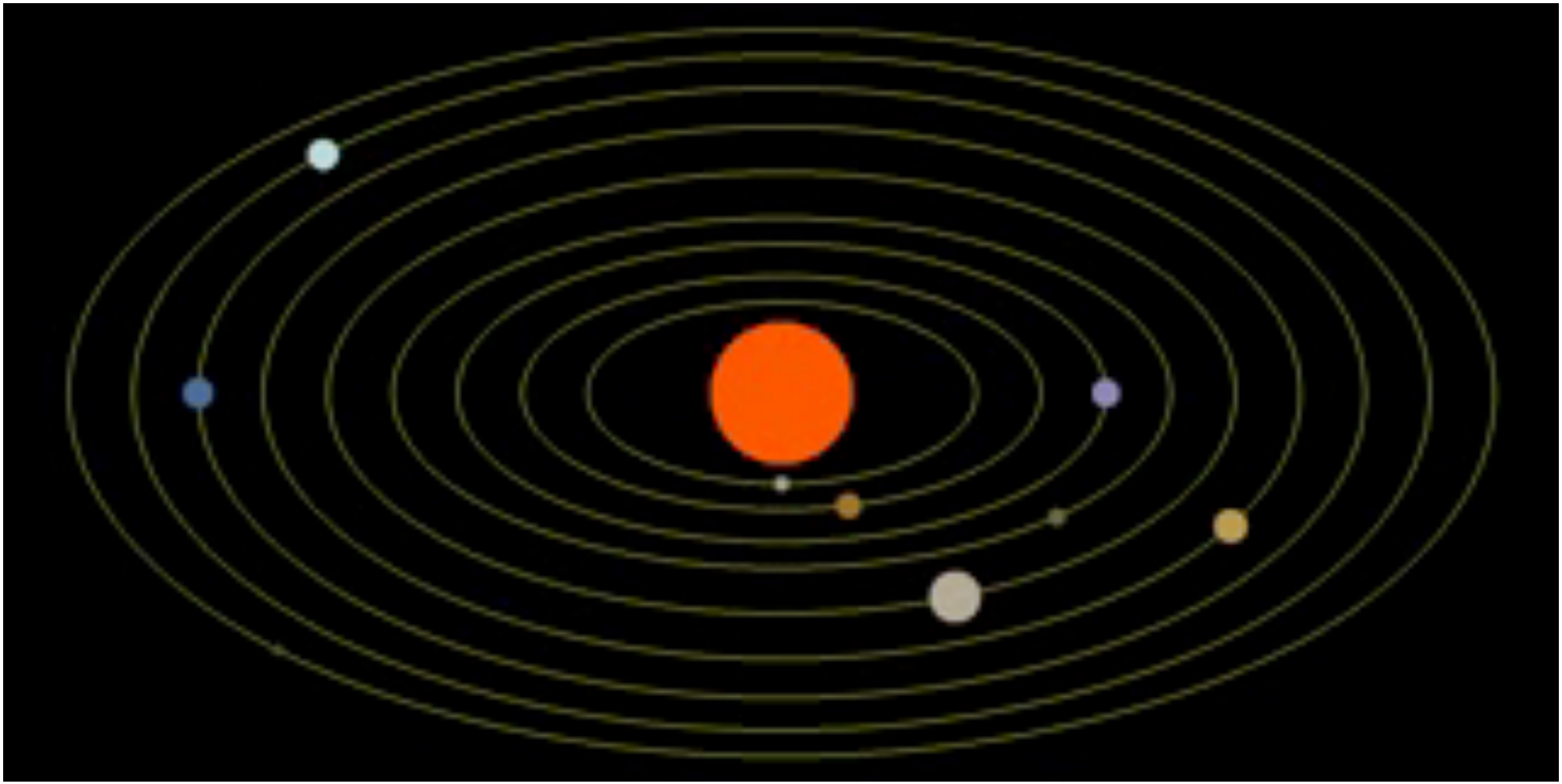


Ptolemy's model



Kepler's 3rd law

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.



Stuff **close in** moves **fast**

stuff **far out** moves **slow**