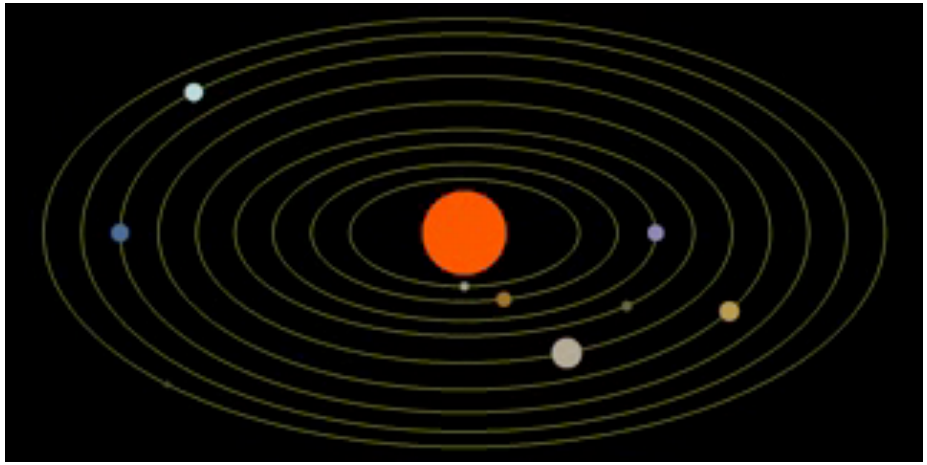
Celestial Mechanics

Learning Objectives

- Connect planetary motions with the causes of these motions
  - Orbits as a consequence of gravitation
- Connect orbital elements and conserved physical quantities

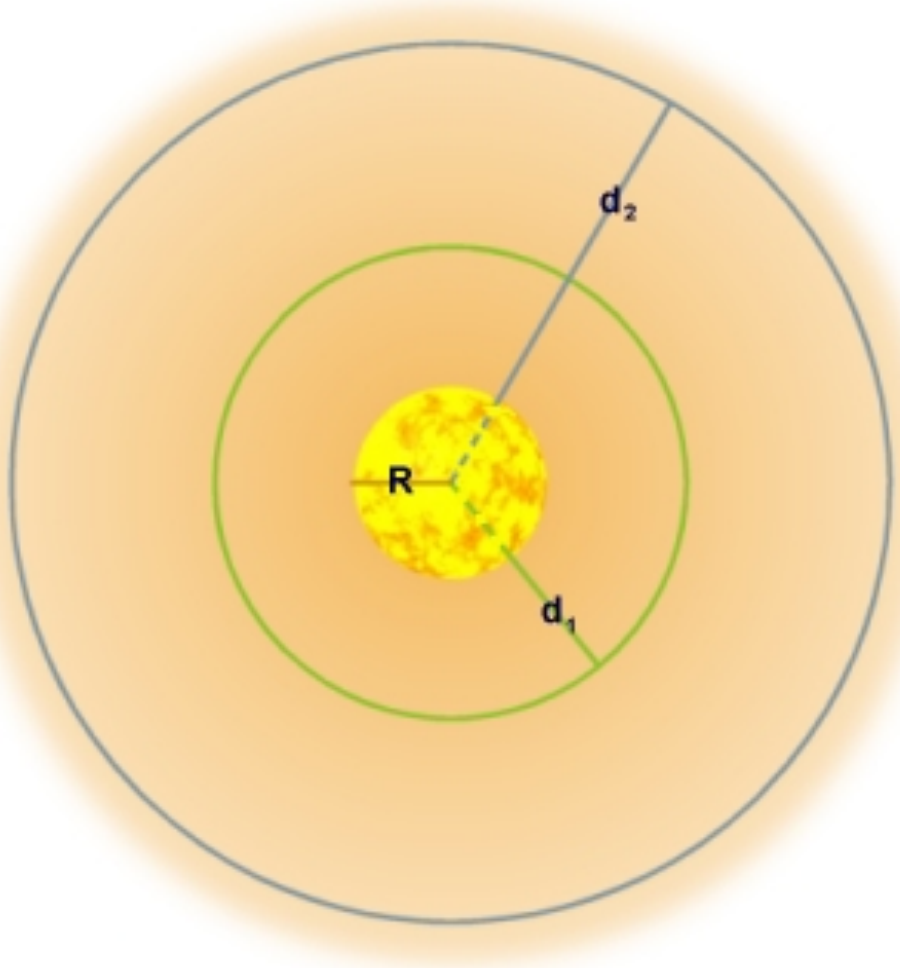
Hypothesis

What keeps the planets in motion
is *something* that
emanates from the Sun
and **dilutes with distance**.



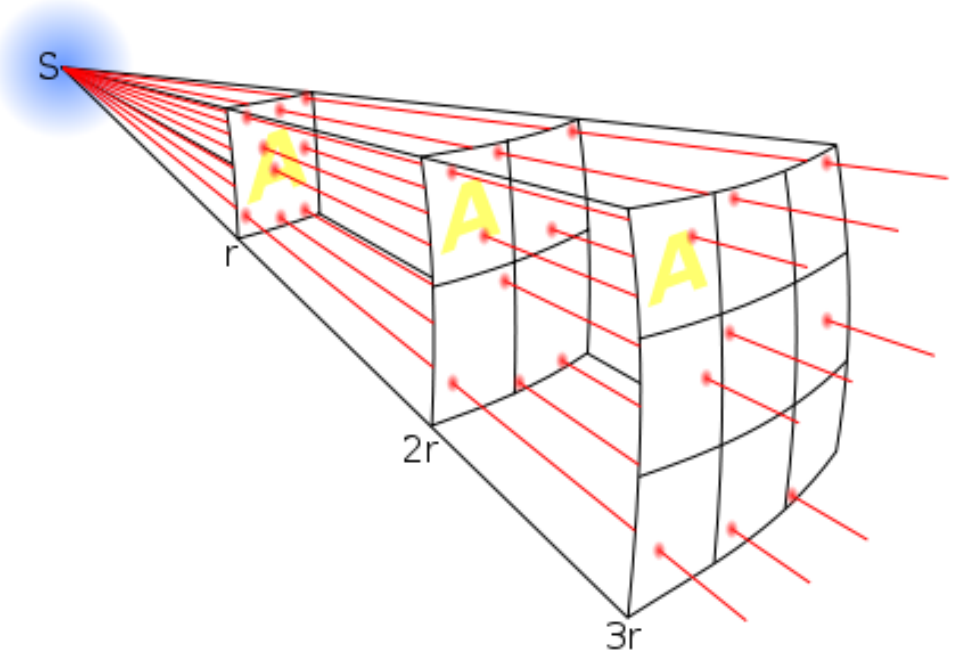
Stuff **close in** moves **fast**
stuff **far out** moves **slow**

Geometric dilution



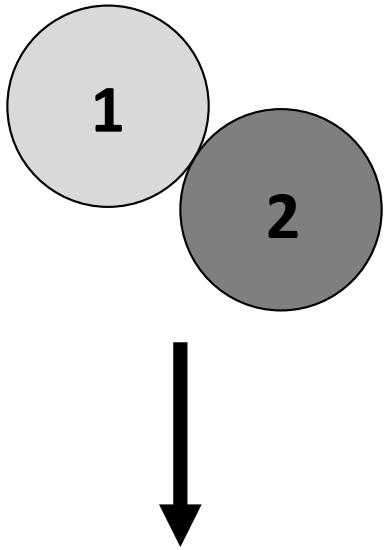
The same amount of light,
expanding spherically outwards,
crosses progressively
larger areas.

Inverse square

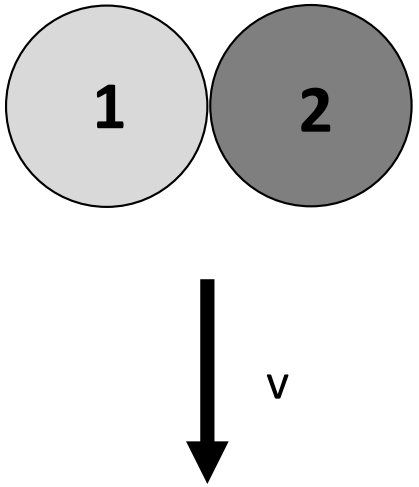


$$g \propto \frac{1}{r^2}$$

As distance increases, the same energy spreads through a larger area



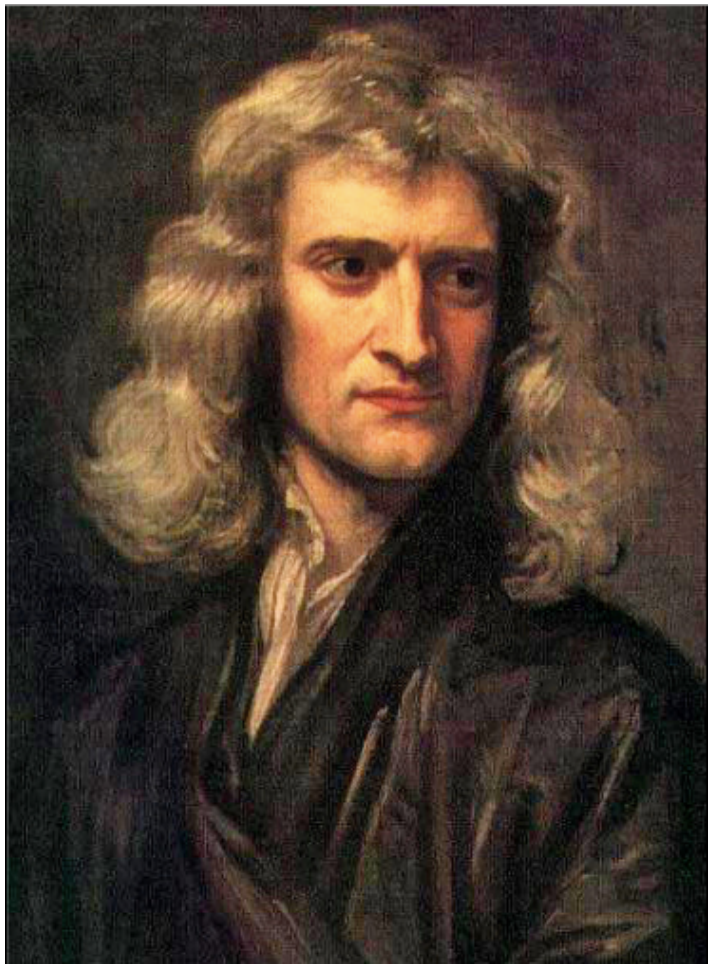
$v_1 < v < v_2$



$v_1 < v_2 < v$

” And the same year [1665] I began to think of gravity […] from Kepler’s rule of the periodical times of the planets being in a sesquialterate proportion of their distances from the centers of their orbs, I deduced that the forces which keep the planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found they answer pretty nearly.”

— *Isaac Newton, 1665-1666*



Isaac Newton, 1642-1727

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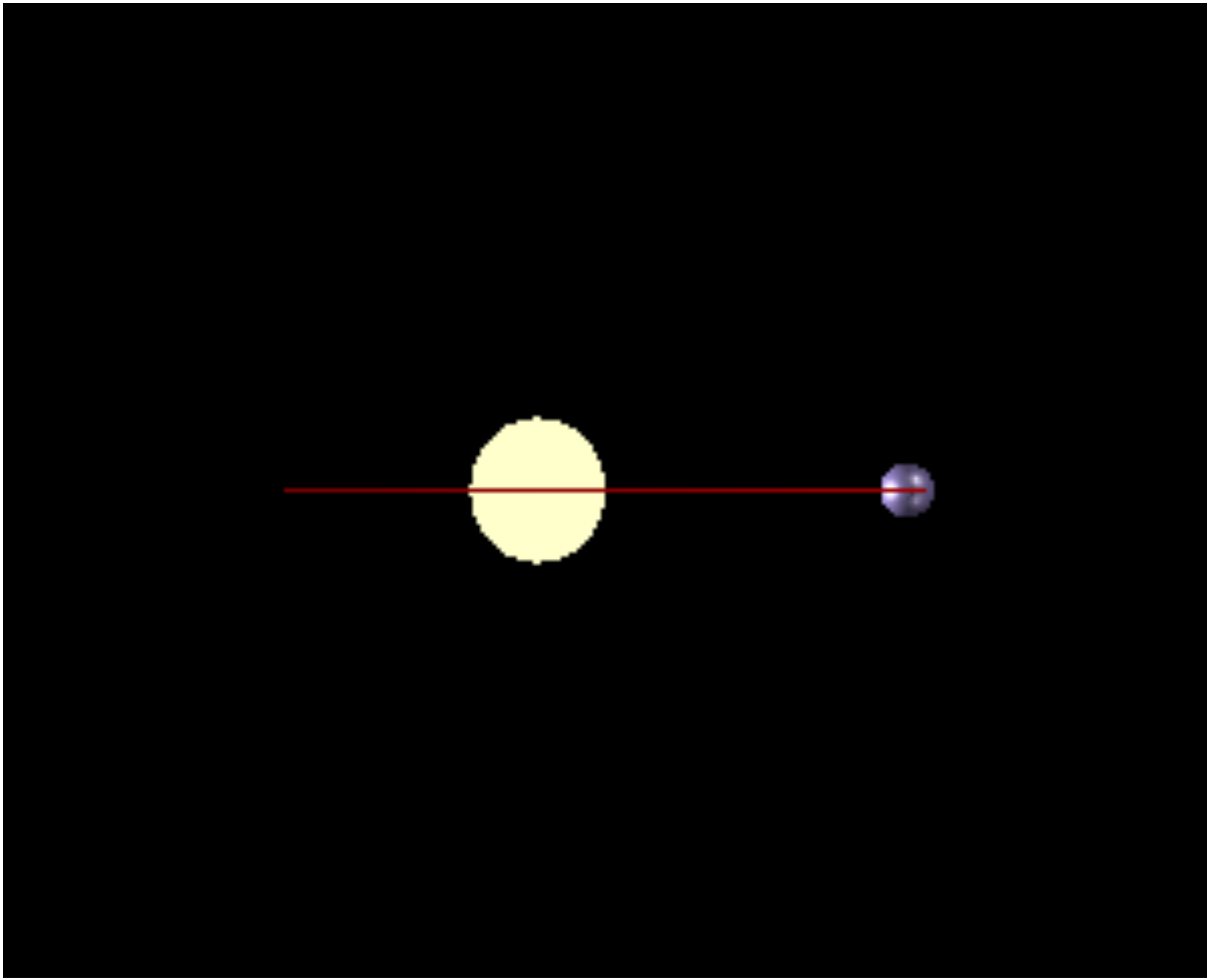
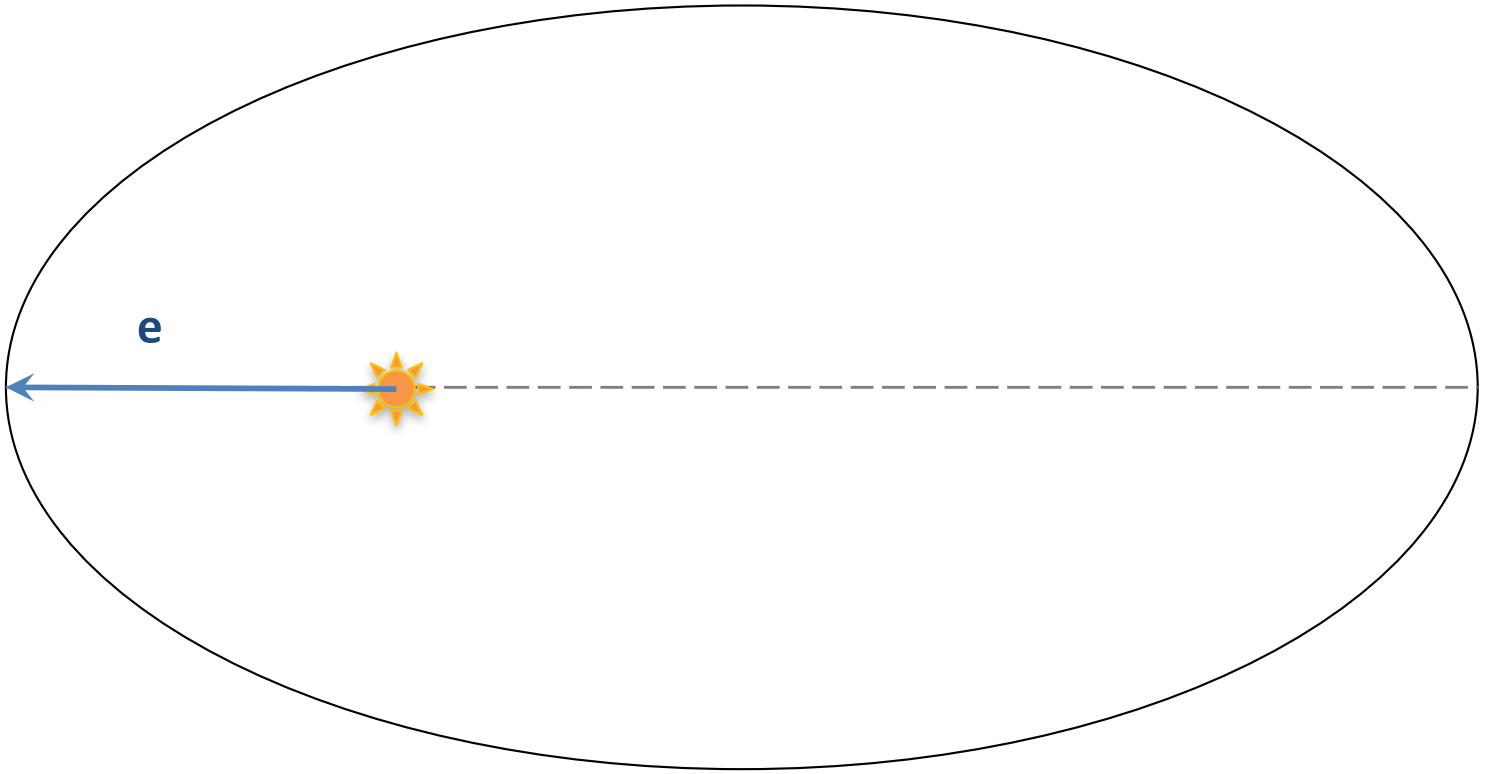
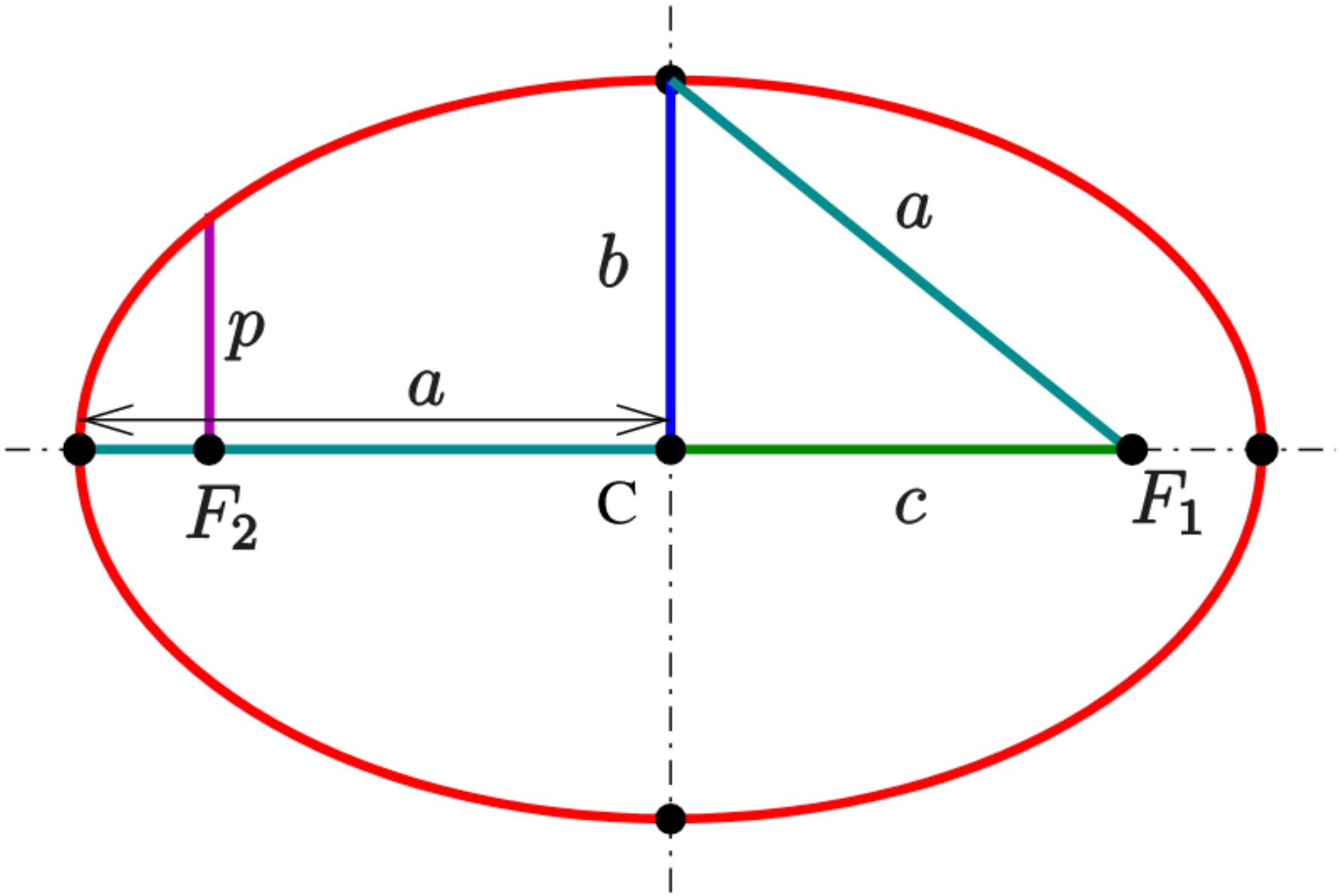
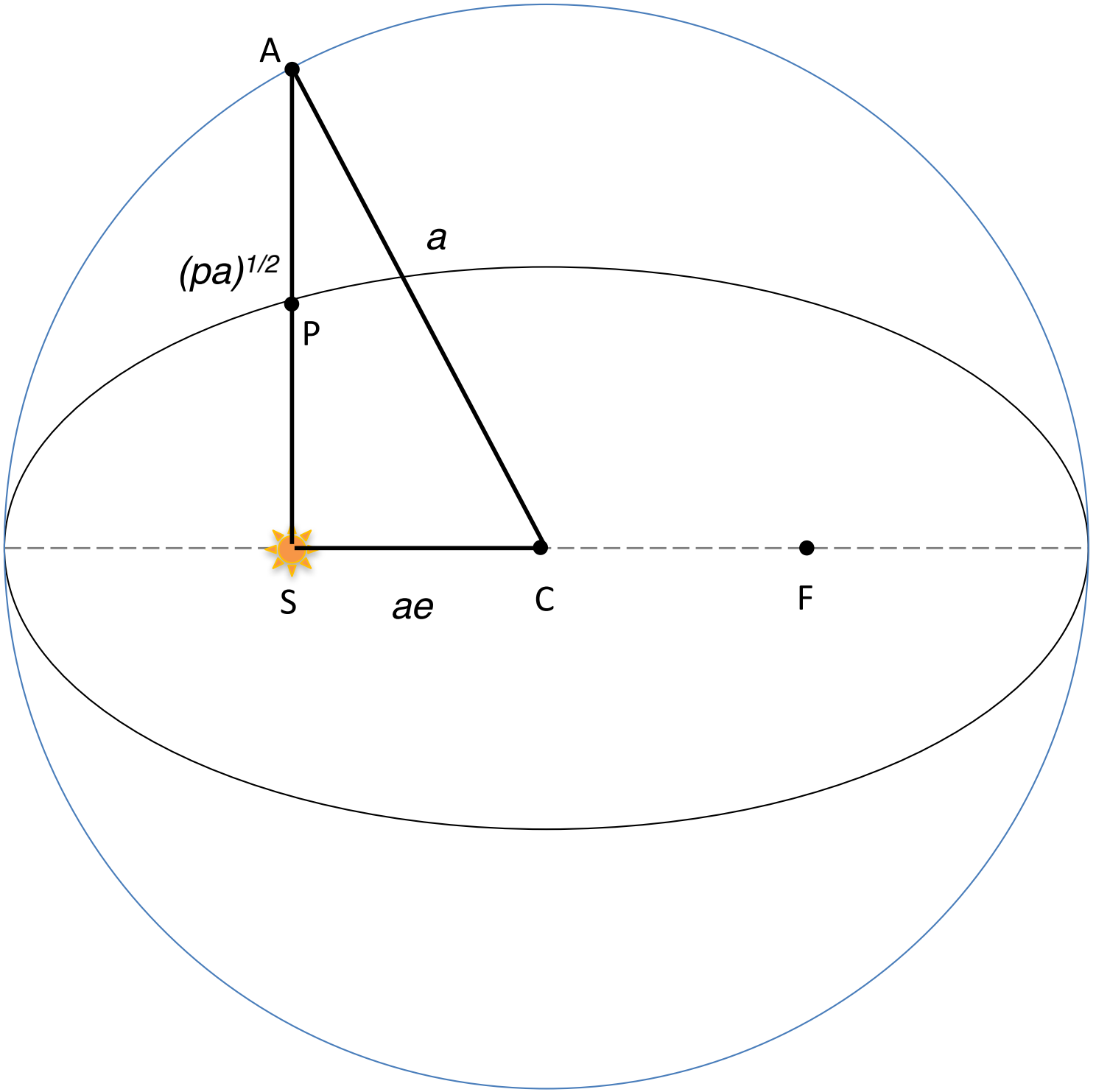
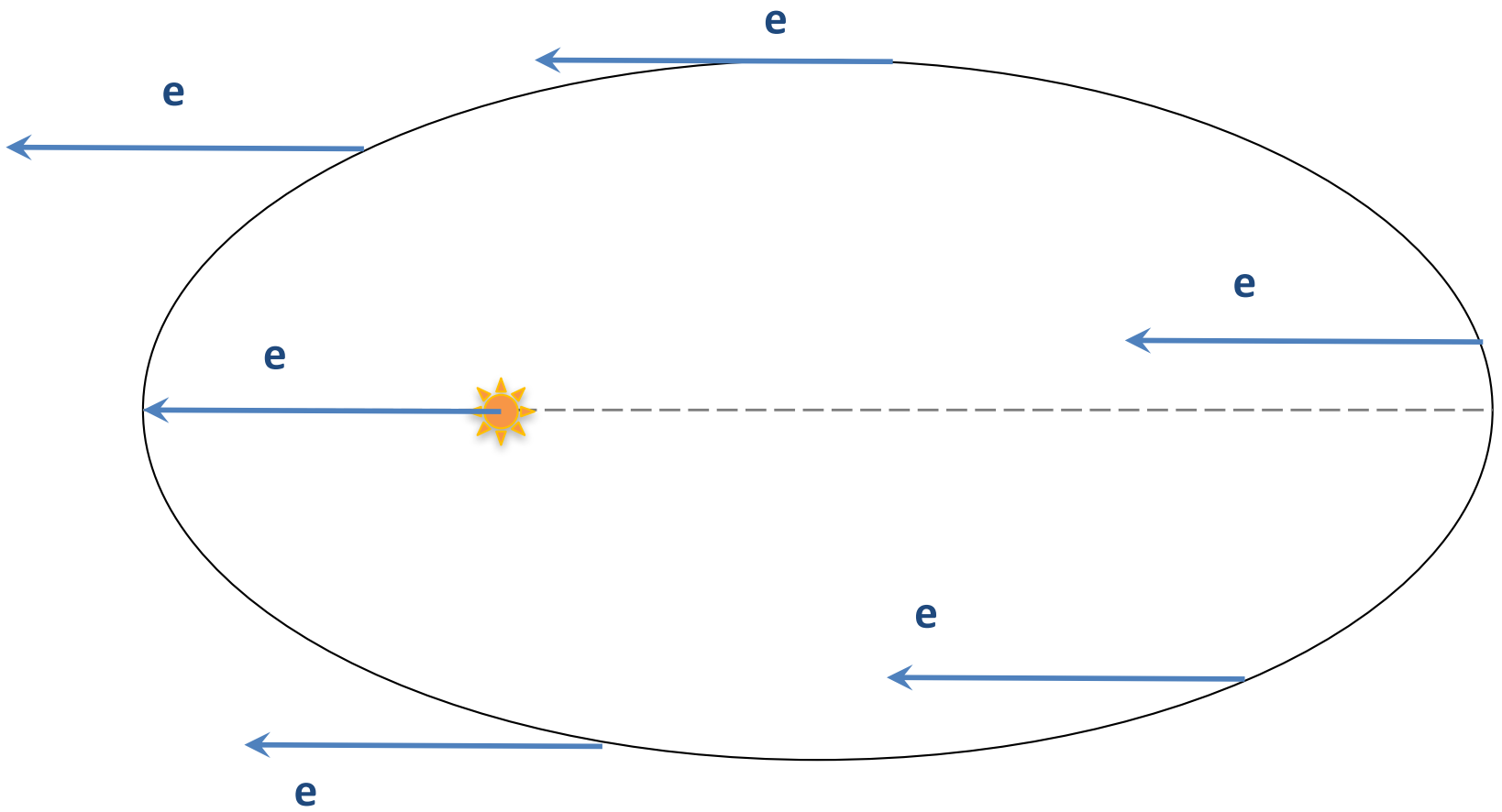
22-23 y-old Isaac Newton, 1642-1727

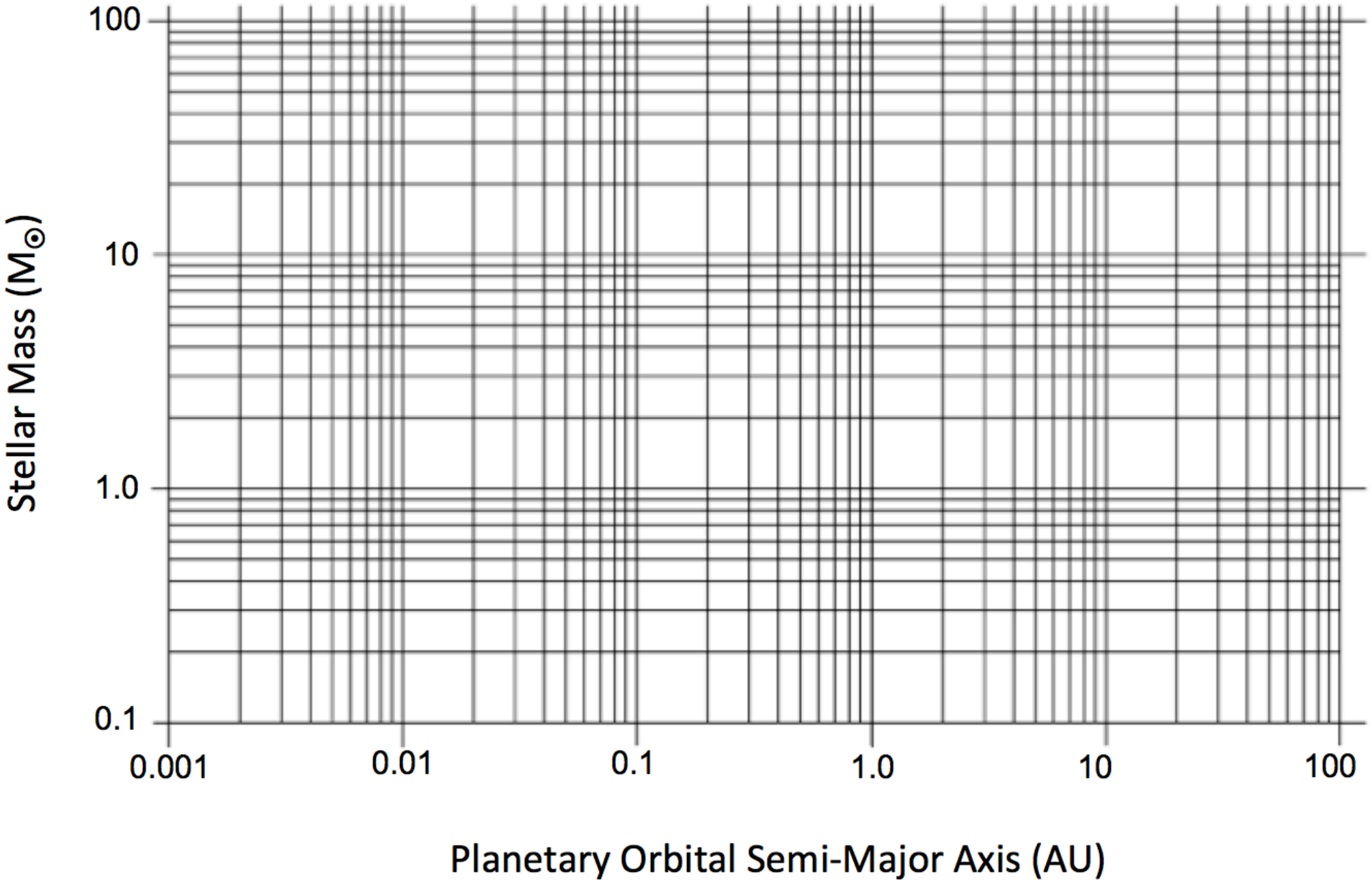


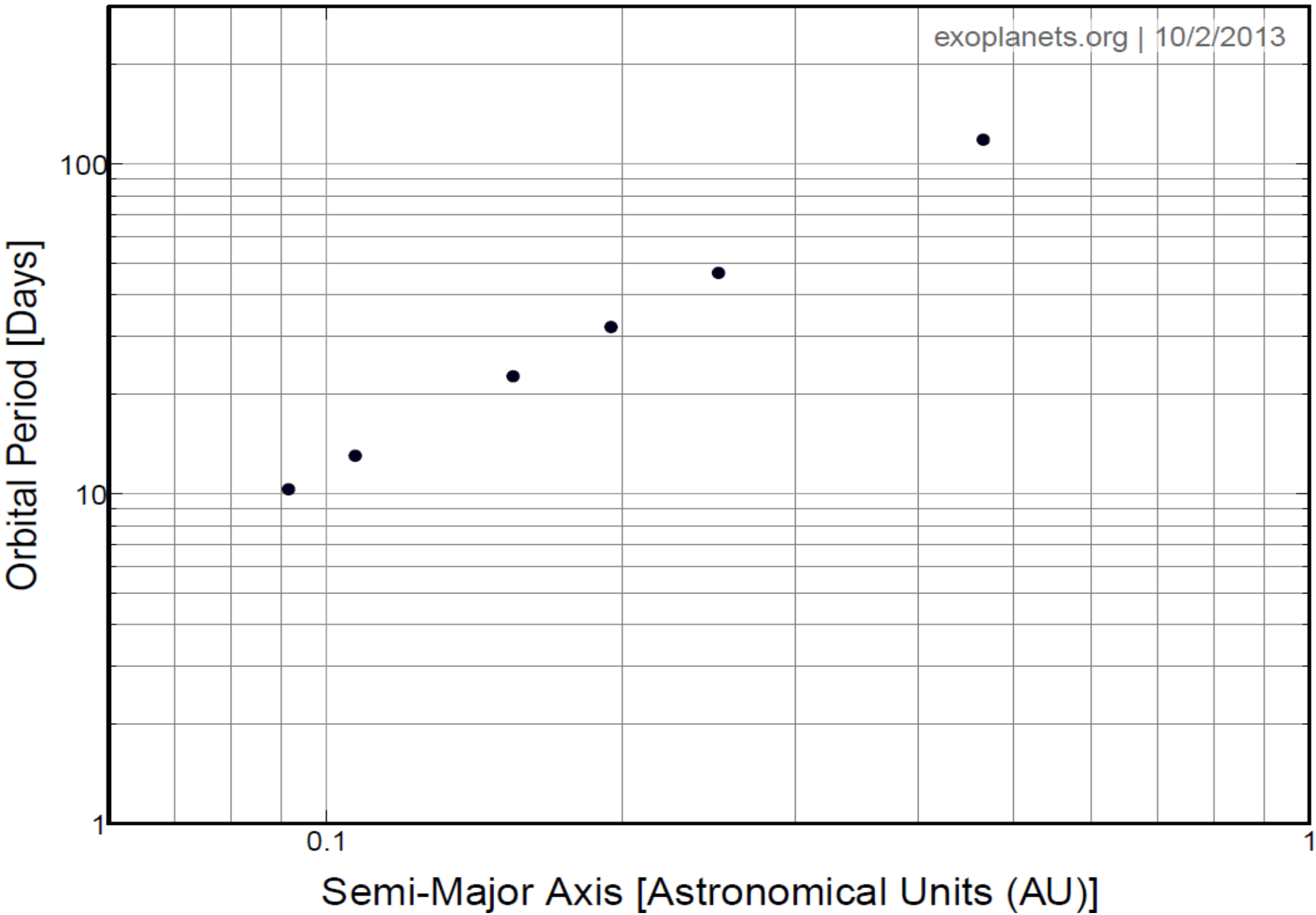

e

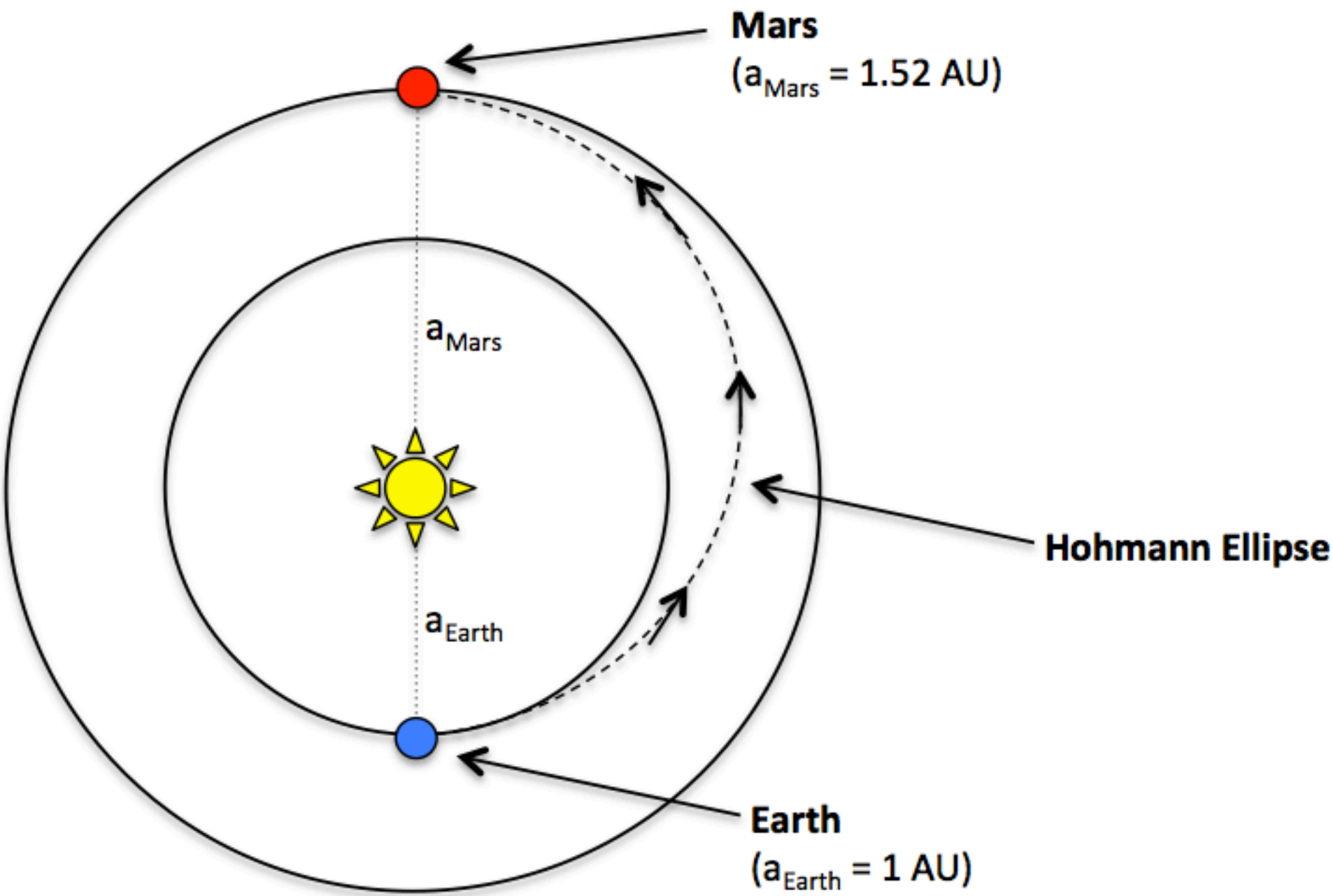

a
b
p
a
F2
C
c
F1

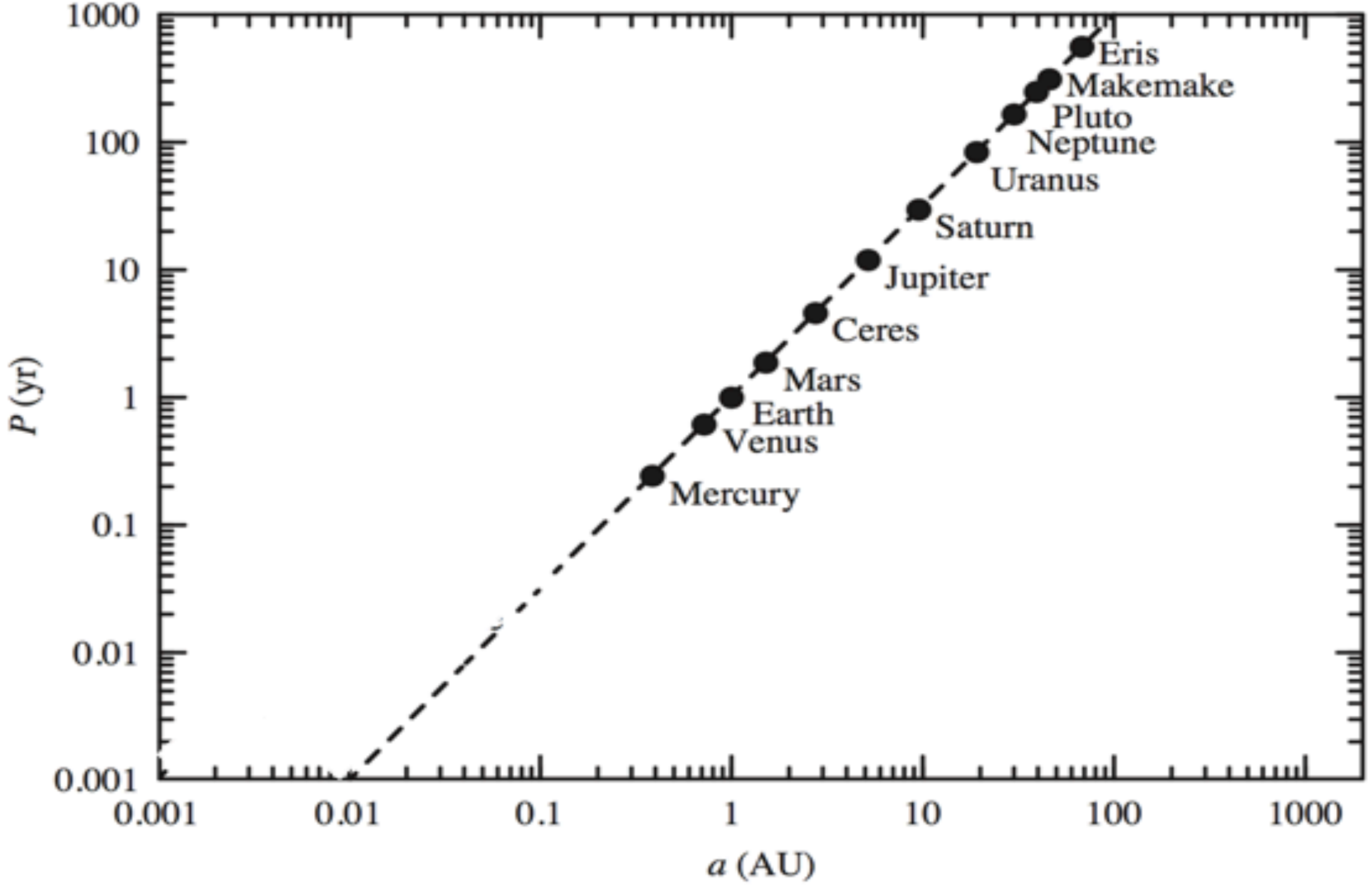

A
a
(pa)1/2
P
S
ae
C
F

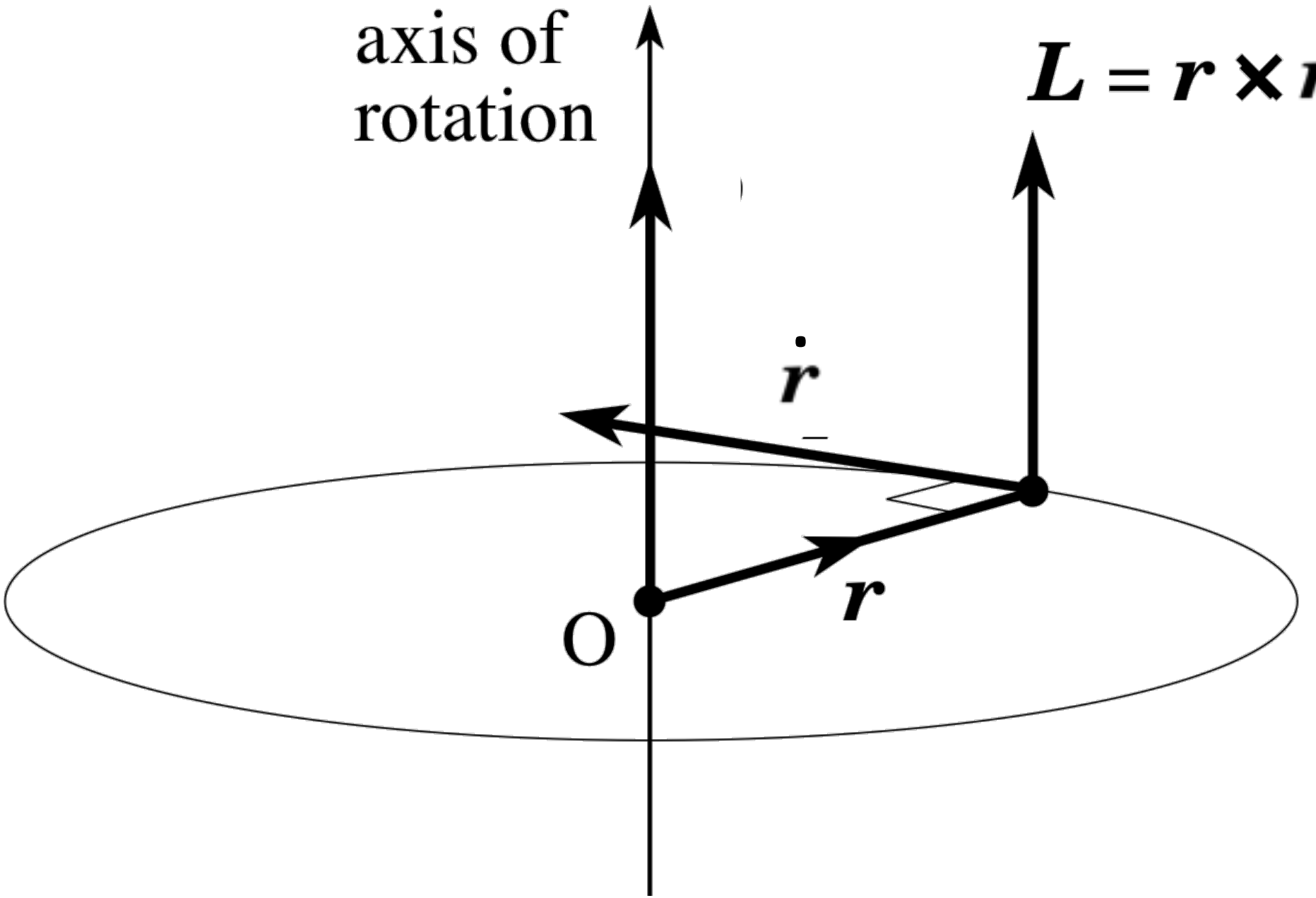

e
e
e
e
e
e
e


100
10
1.0
0.1
Stellar Mass ($M_\odot$)
0.001
0.01
0.1
1.0
10
100
Planetary Orbital Semi-Major Axis (AU)


exoplanets.org | 10/2/2013
100
10
1
0.1
1
Orbital Period [Days]
Semi-Major Axis [Astronomical Units (AU)]

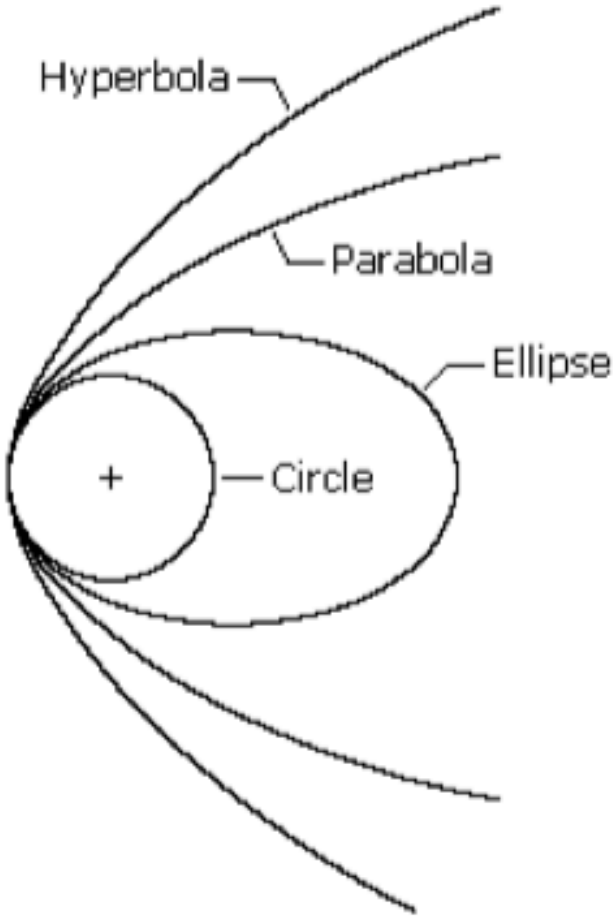

Mars
(a_{Mars} = 1.52 AU)
a_{Mars}
a_{Earth}
Hohmann Ellipse
Earth
(a_{Earth} = 1 AU)




axis of
rotation
$L = r \times \dot{r}$
$\dot{r}$
r
O

Types of orbit

Orbit Type	Eccentricity	Energy	
Circle	$e = 0$	$E = E_{min}$	*Bound*
Ellipse	$0 < e < 1$	$E_{min} < E < 0$	
Parabola	$e=1$	$E = 0$	*Unbound*
Hyperbola	$e > 1$	$E > 0$	



Conic Sections

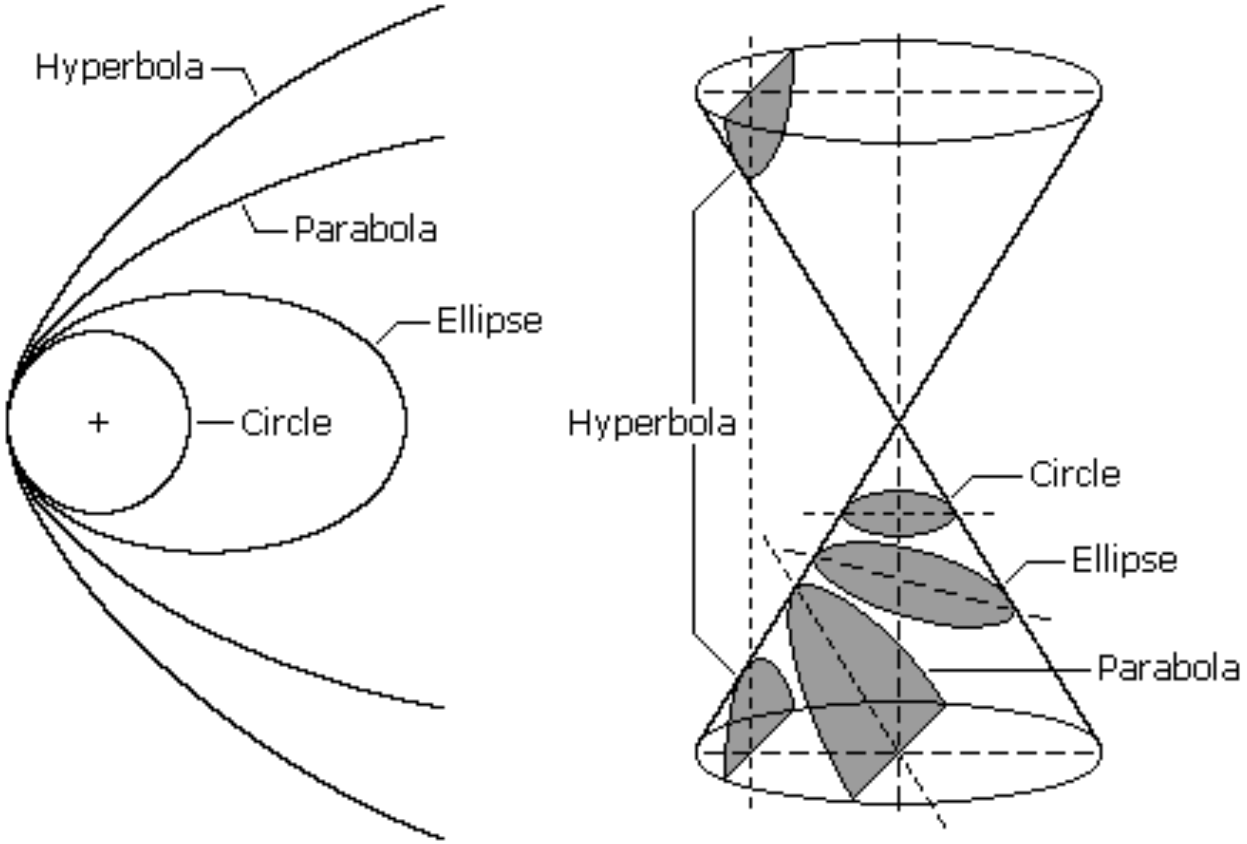
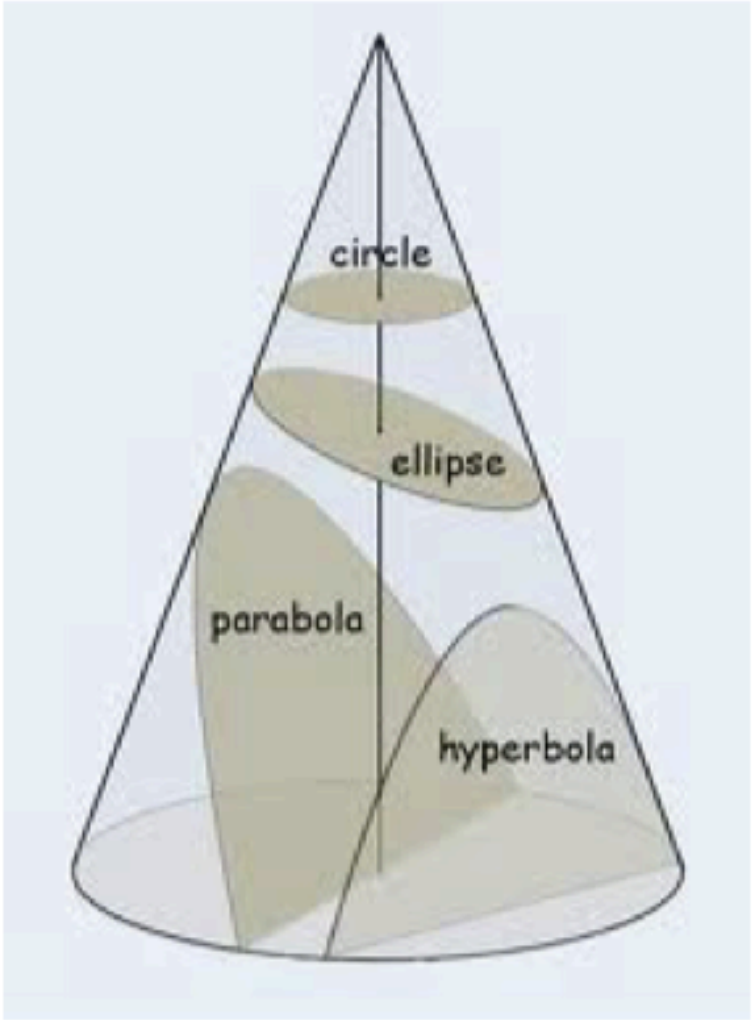
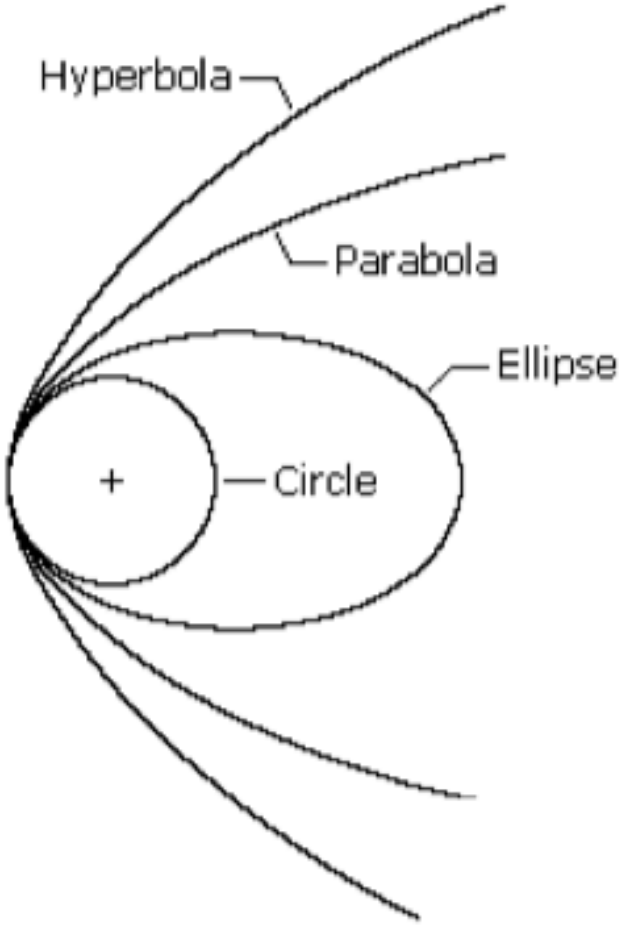
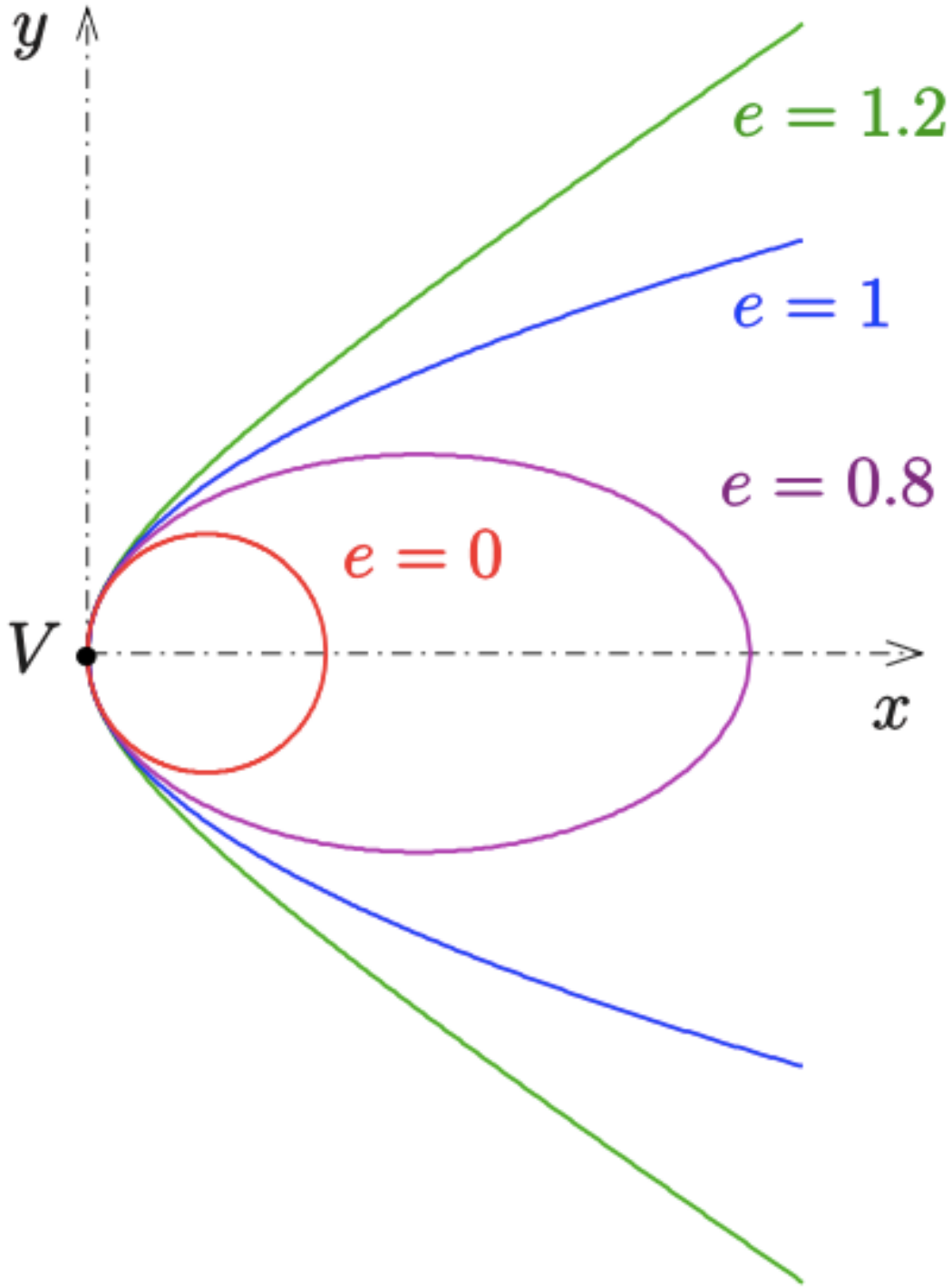


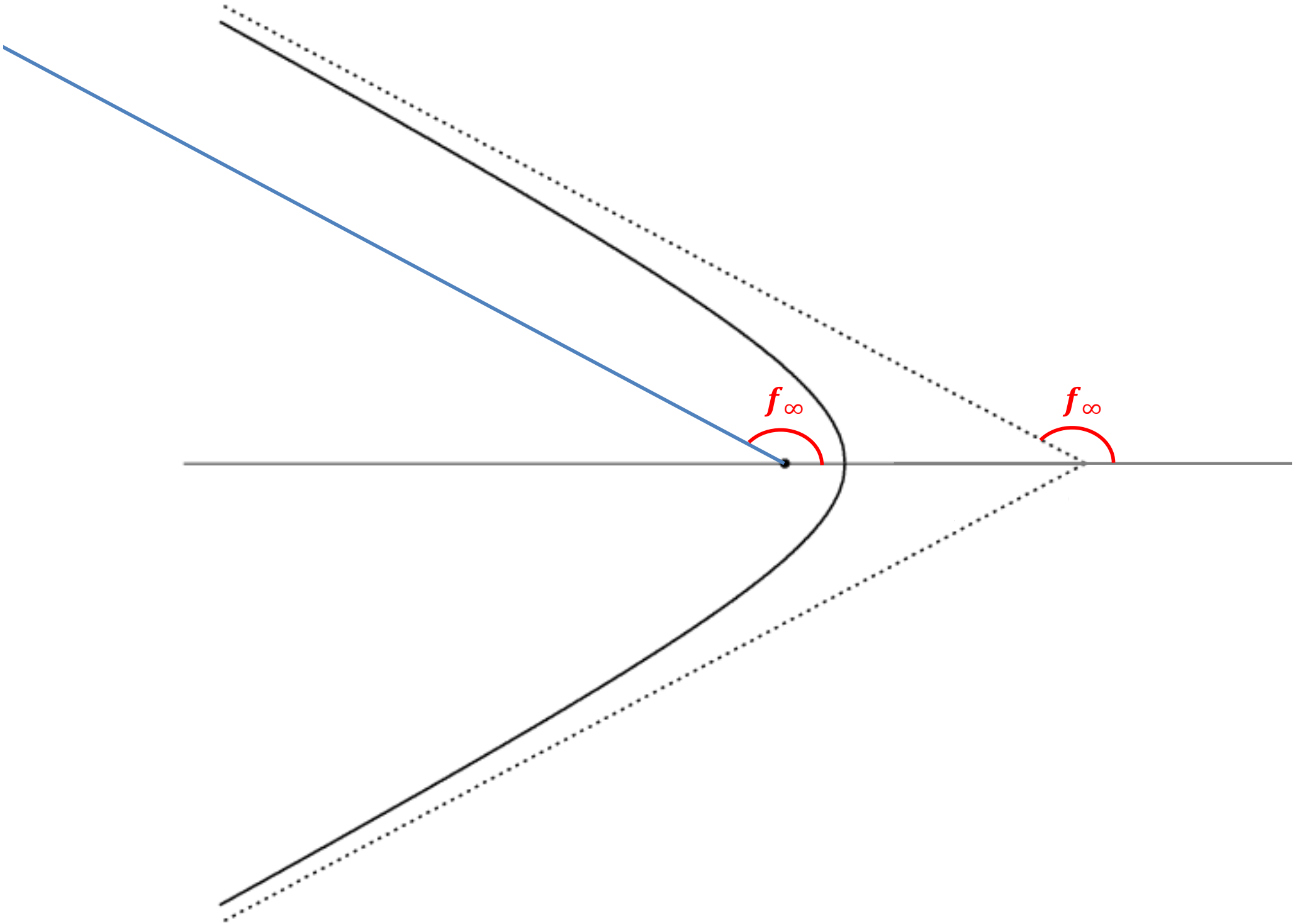
Figure 4.1

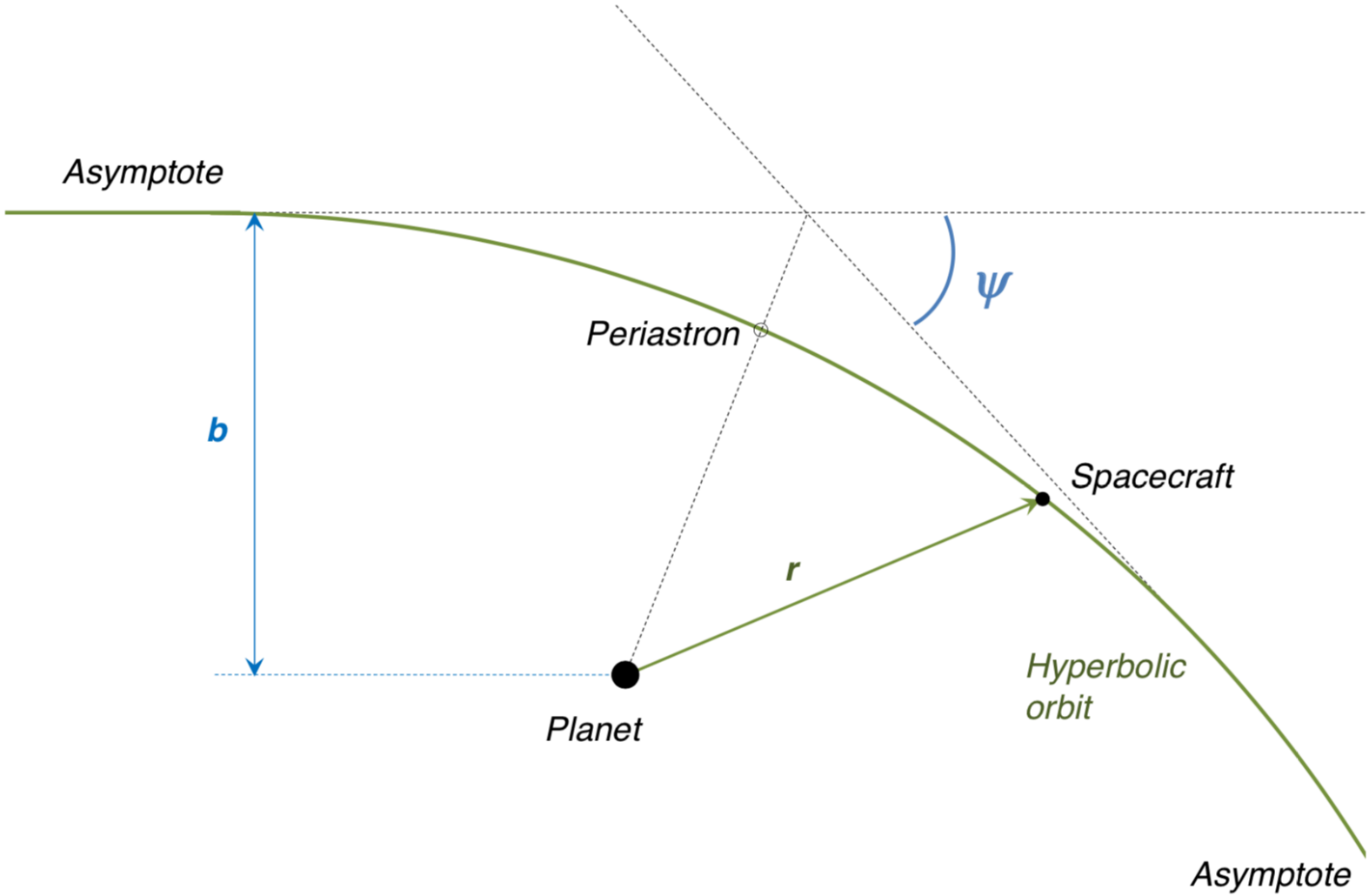
Types of orbit: Conic Sections

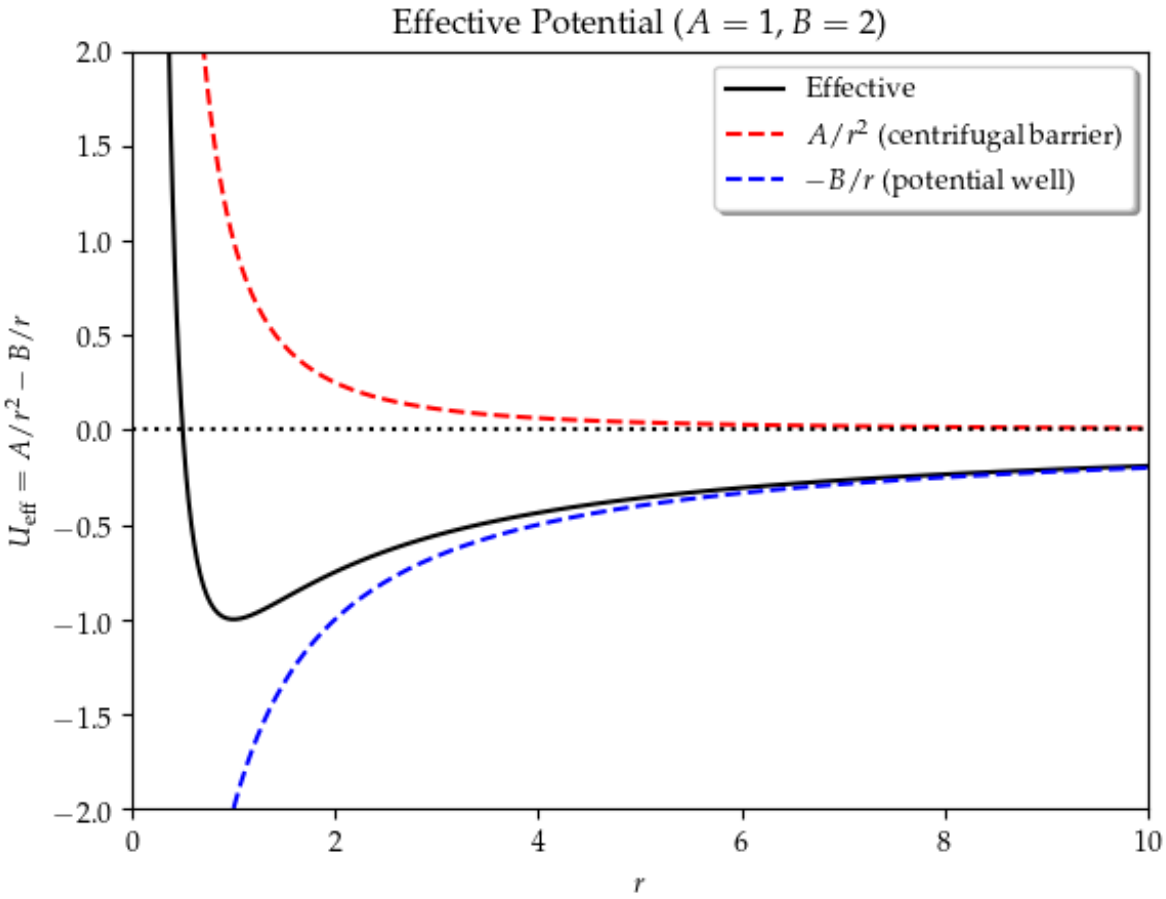


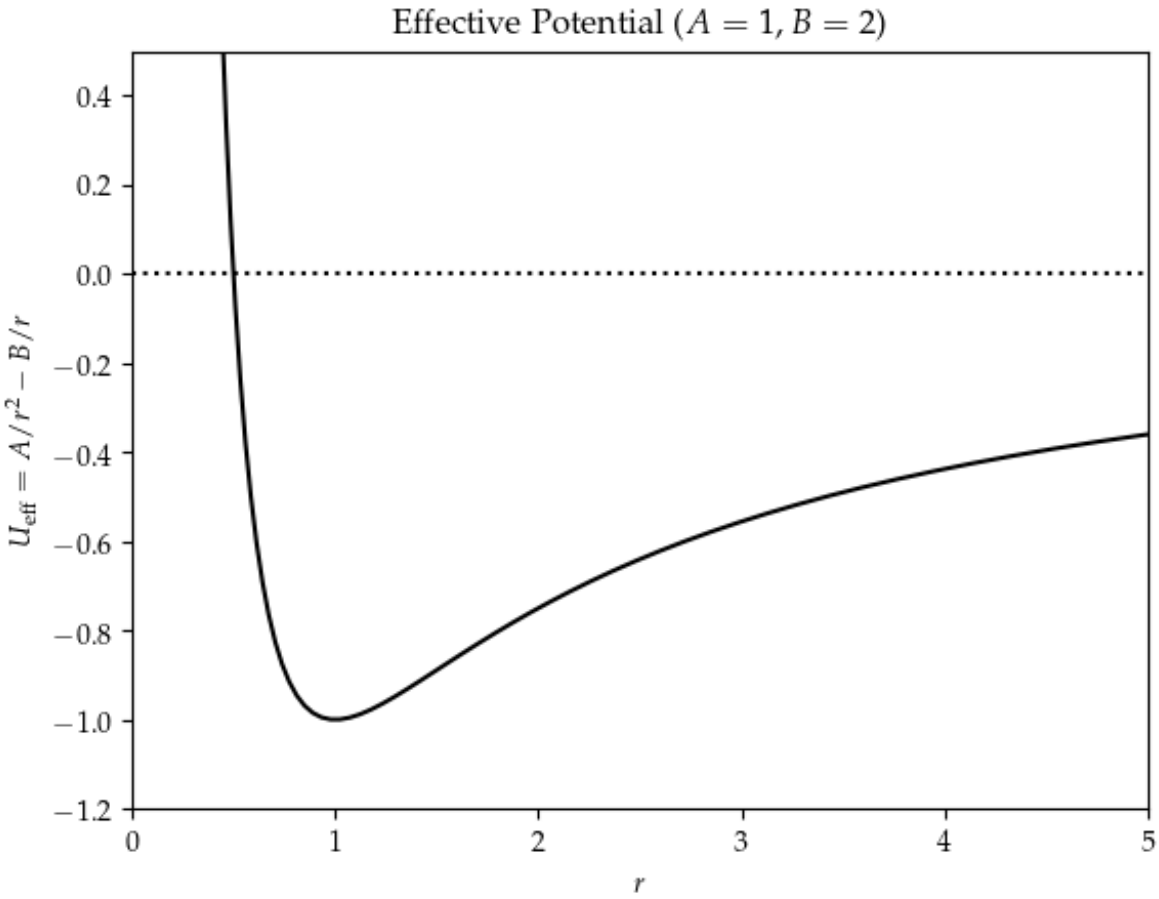


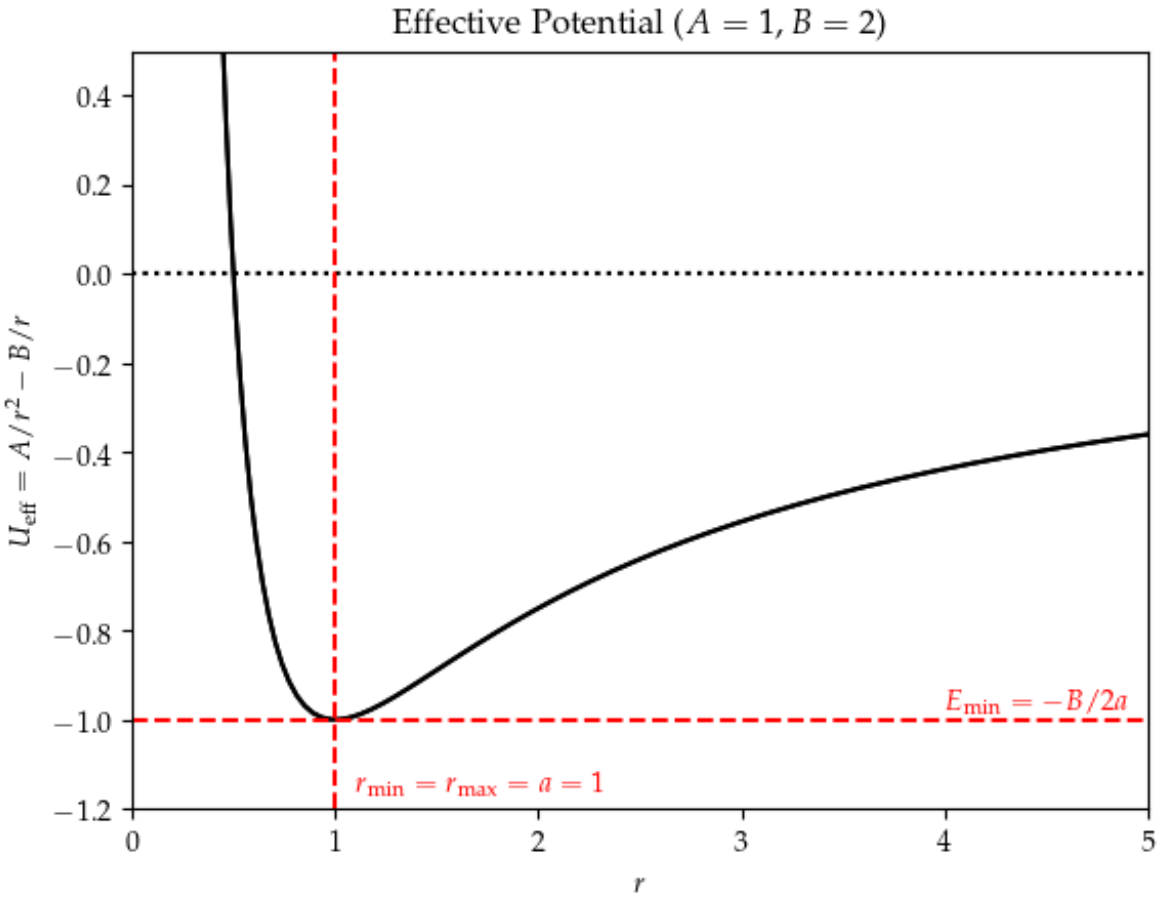

y
e = 1.2
e = 1
e = 0.8
e = 0
V
x

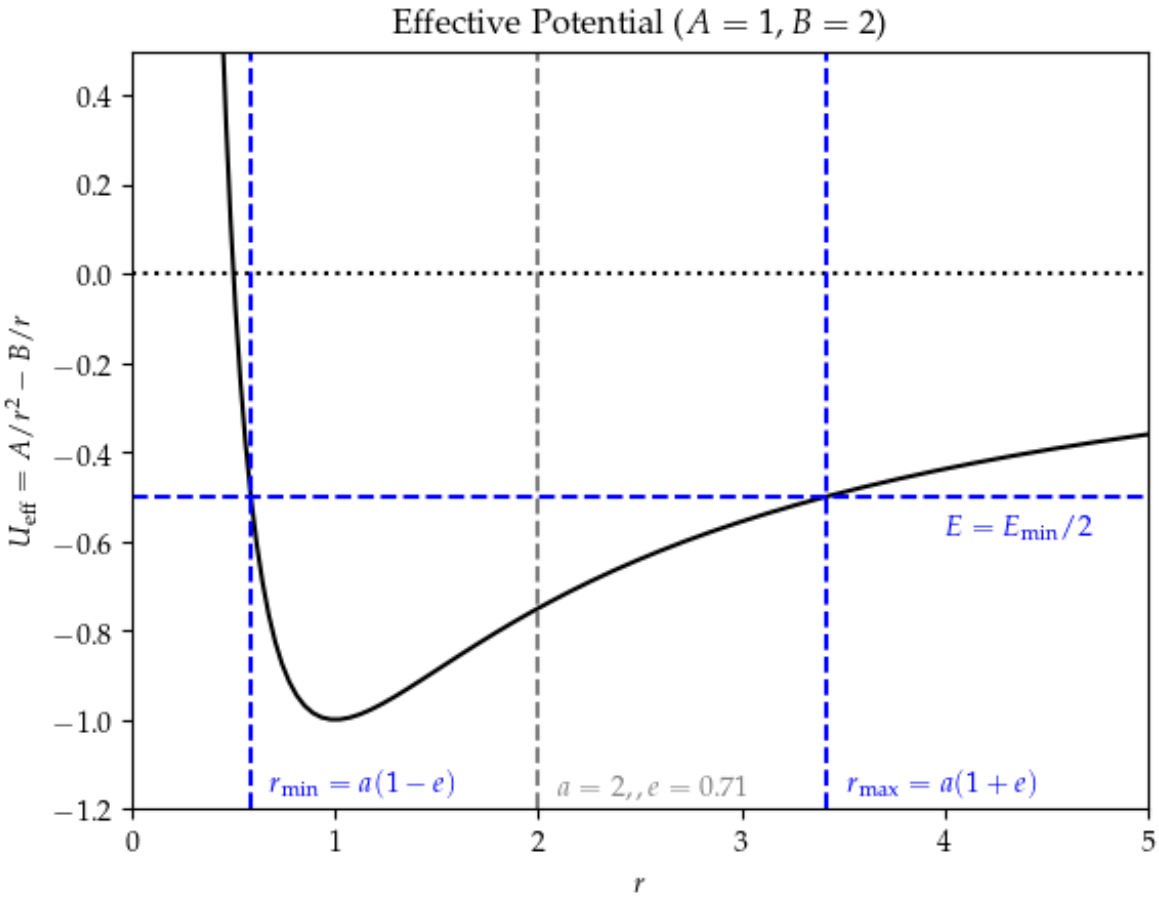

f_∞
f_∞

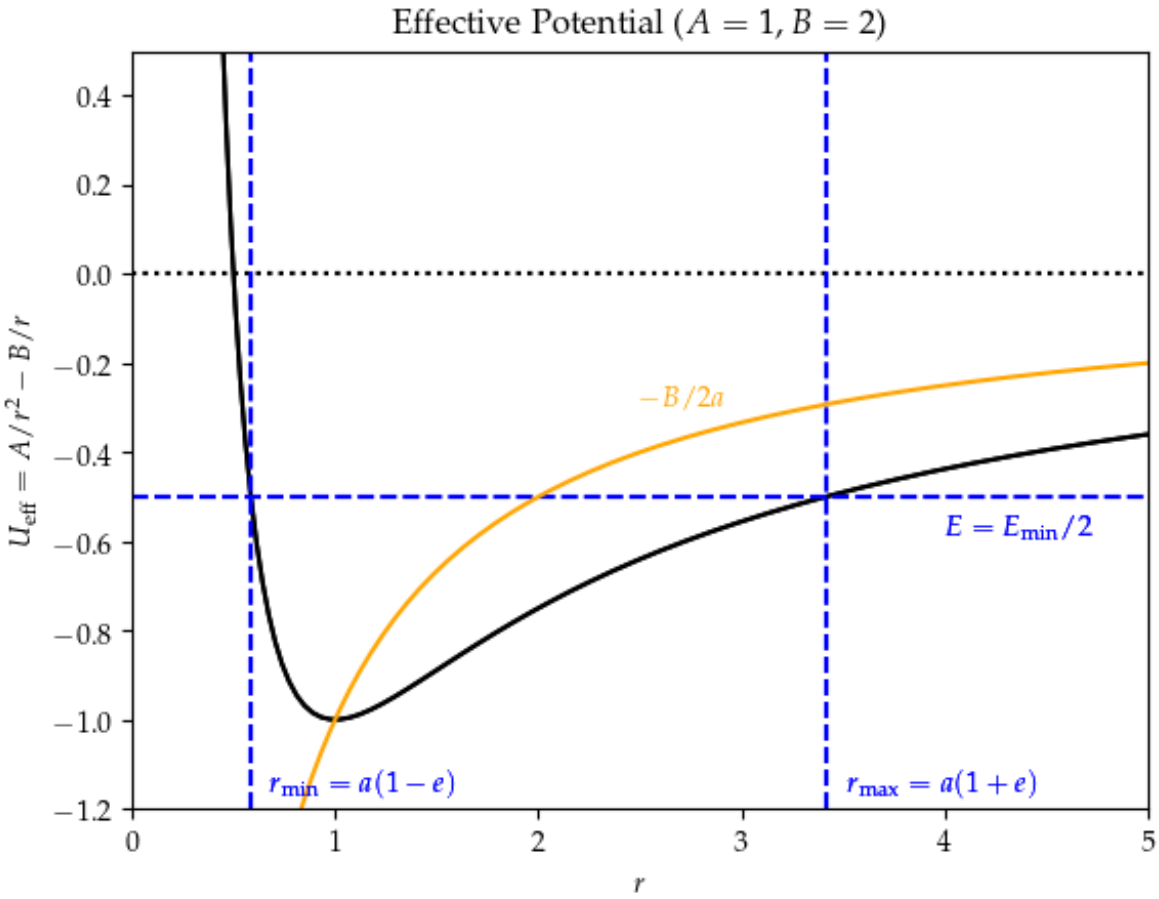

Asymptote
ψ
Periastron
b
Spacecraft
r
Hyperbolic
orbit
Planet
Asymptote

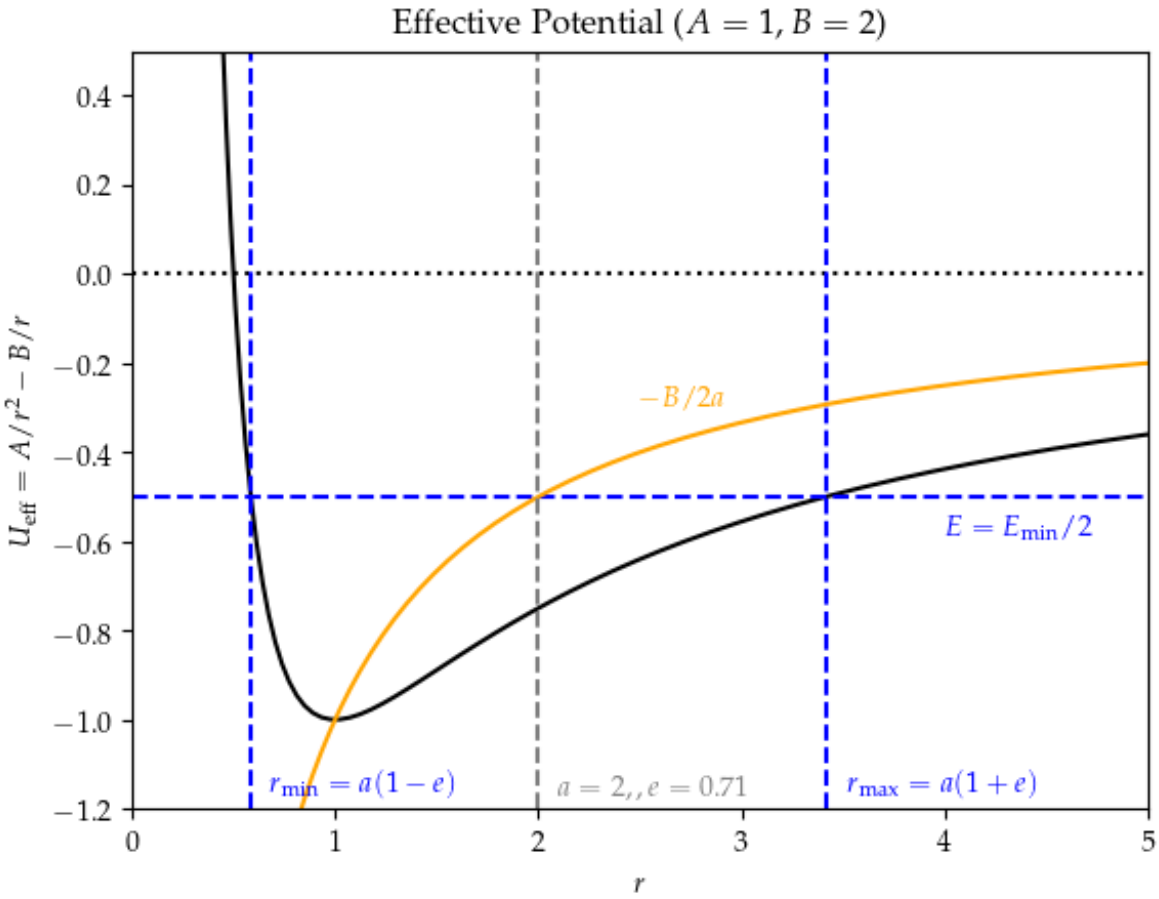

Effective Potential ($A = 1, B = 2$)
Effective
A/r^2 (centrifugal barrier)
$-B/r$ (potential well)
$U_{\text{eff}} = A/r^2 - B/r$
r
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
2
4
6
8
10


Effective Potential ($A = 1, B = 2$)
$U_{\text{eff}} = A/r^2 - B/r$
r
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0
1
2
3
4
5

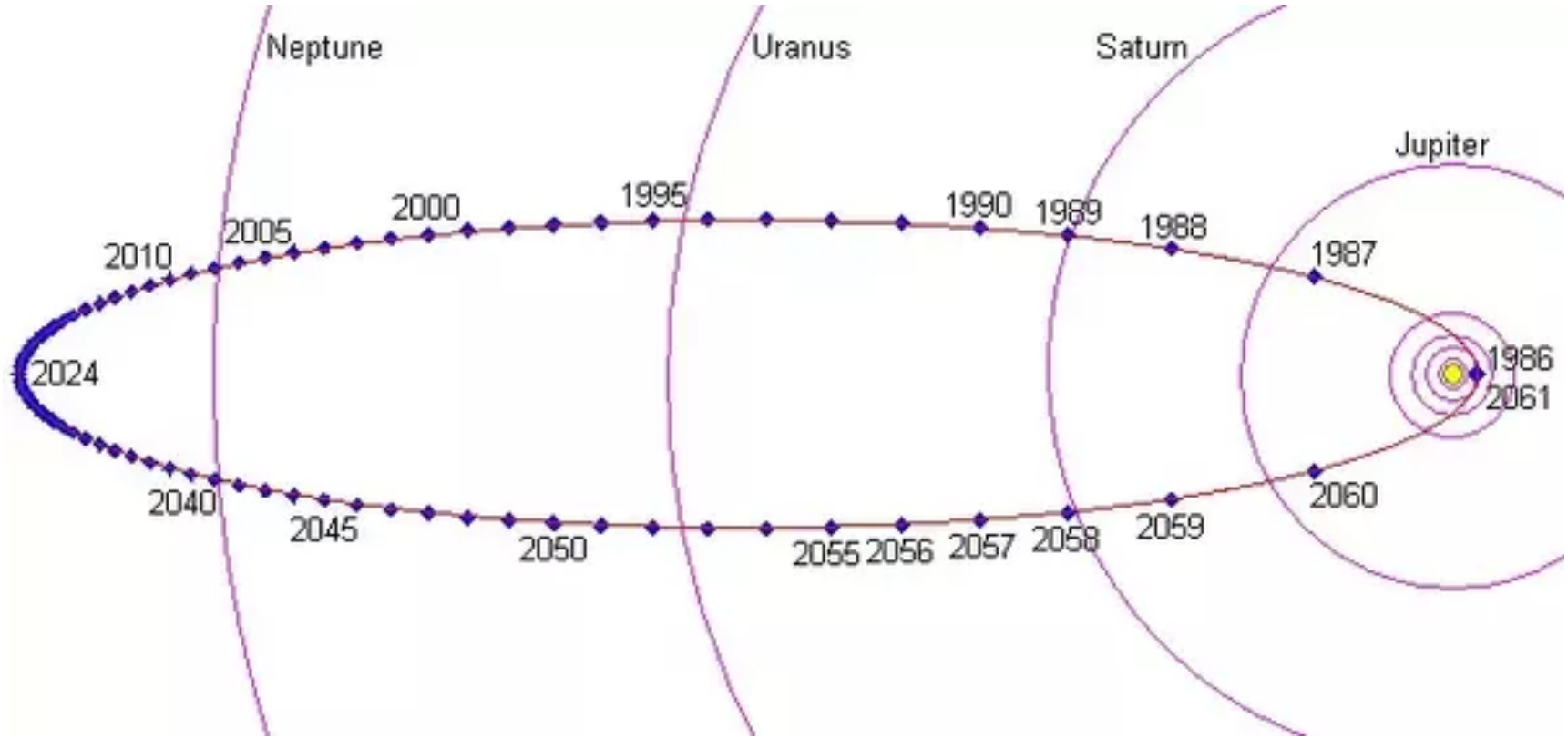

Effective Potential ($A = 1, B = 2$)
$U_{\text{eff}} = A/r^2 - B/r$
r
$E_{\min} = -B/2a$
$r_{\min} = r_{\max} = a = 1$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0
1
2
3
4
5


Effective Potential ($A = 1, B = 2$)
$U_{\text{eff}} = A/r^2 - B/r$
r
$E = E_{\text{min}}/2$
$r_{\text{min}} = a(1-e)$
$a = 2,, e = 0.71$
$r_{\text{max}} = a(1+e)$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0
1
2
3
4
5

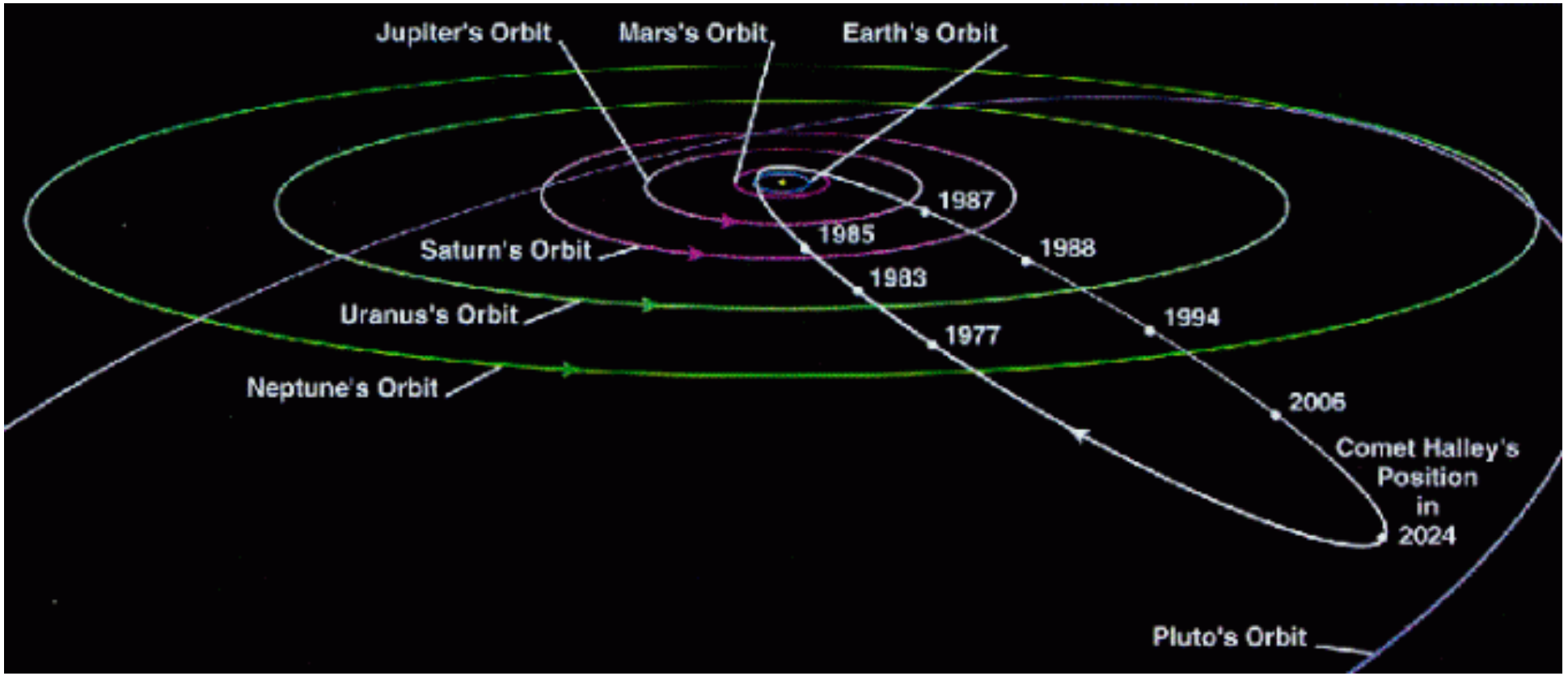

Effective Potential ($A = 1, B = 2$)
$U_{\text{eff}} = A/r^2 - B/r$
r
$-B/2a$
$E = E_{\text{min}}/2$
$r_{\text{min}} = a(1-e)$
$r_{\text{max}} = a(1+e)$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0
1
2
3
4
5

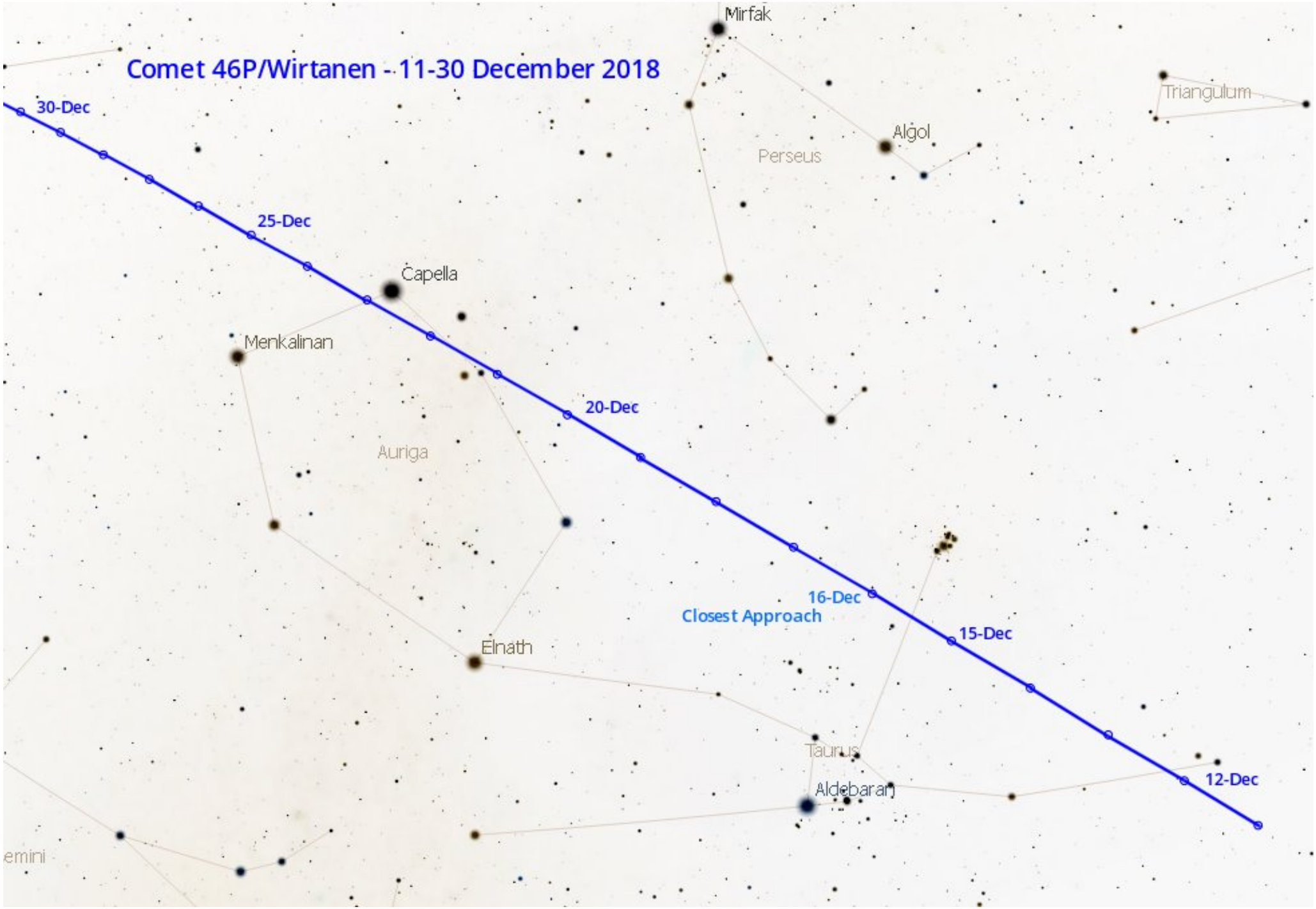

Effective Potential ($A = 1, B = 2$)
$U_{\text{eff}} = A/r^2 - B/r$
r
$-B/2a$
$E = E_{\text{min}}/2$
$r_{\text{min}} = a(1-e)$
$a = 2,, e = 0.71$
$r_{\text{max}} = a(1+e)$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0
1
2
3
4
5

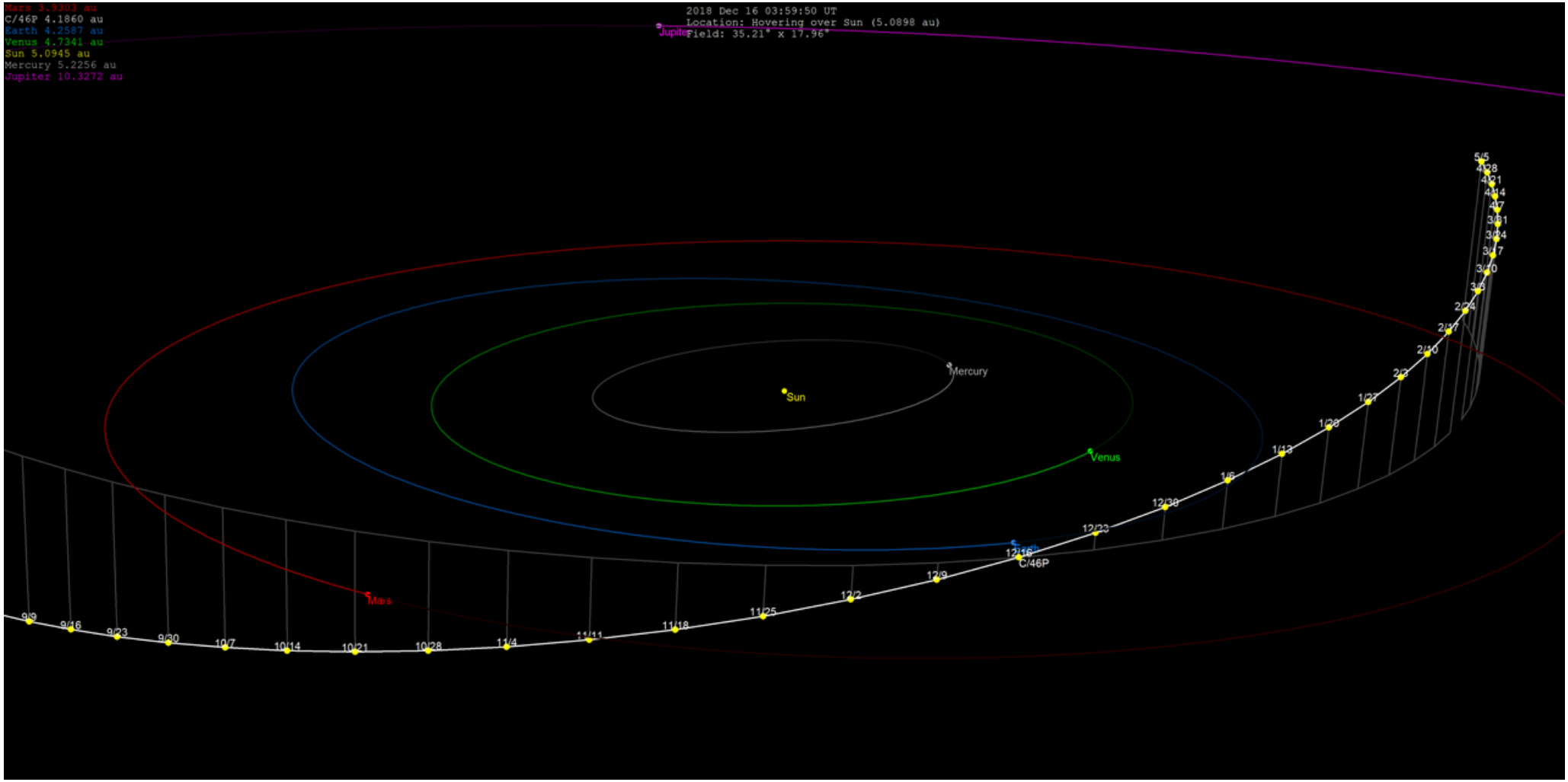
Halley's comet



Halley's comet




Comet 46P/Wirtanen - 11-30 December 2018
30-Dec
25-Dec
20-Dec
16-Dec
Closest Approach
15-Dec
12-Dec
Mirfak
Algol
Perseus
Triangulum
Capella
Menkalinan
Auriga
Elnath
Taurus
Aldebaran
emini


Mars 3.9303 au
C/46P 4.1860 au
Earth 4.2587 au
Venus 4.7341 au
Sun 5.0945 au
Mercury 5.2256 au
Jupiter 10.3272 au
2018 Dec 16 03:59:50 UT
Location: Hovering over Sun (5.0898 au)
Field: 35.21° x 17.96°
Jupiter
Mercury
Sun
Venus
Mars
C/46P
9/9
9/16
9/23
9/30
10/7
10/14
10/21
10/28
11/4
11/11
11/18
11/25
12/2
12/9
12/16
12/23
12/30
1/6
1/13
1/20
1/27
2/3
2/10
2/17
2/24
3/3
3/10
3/17
3/24
3/31
4/7
4/14
4/21
4/28
5/5

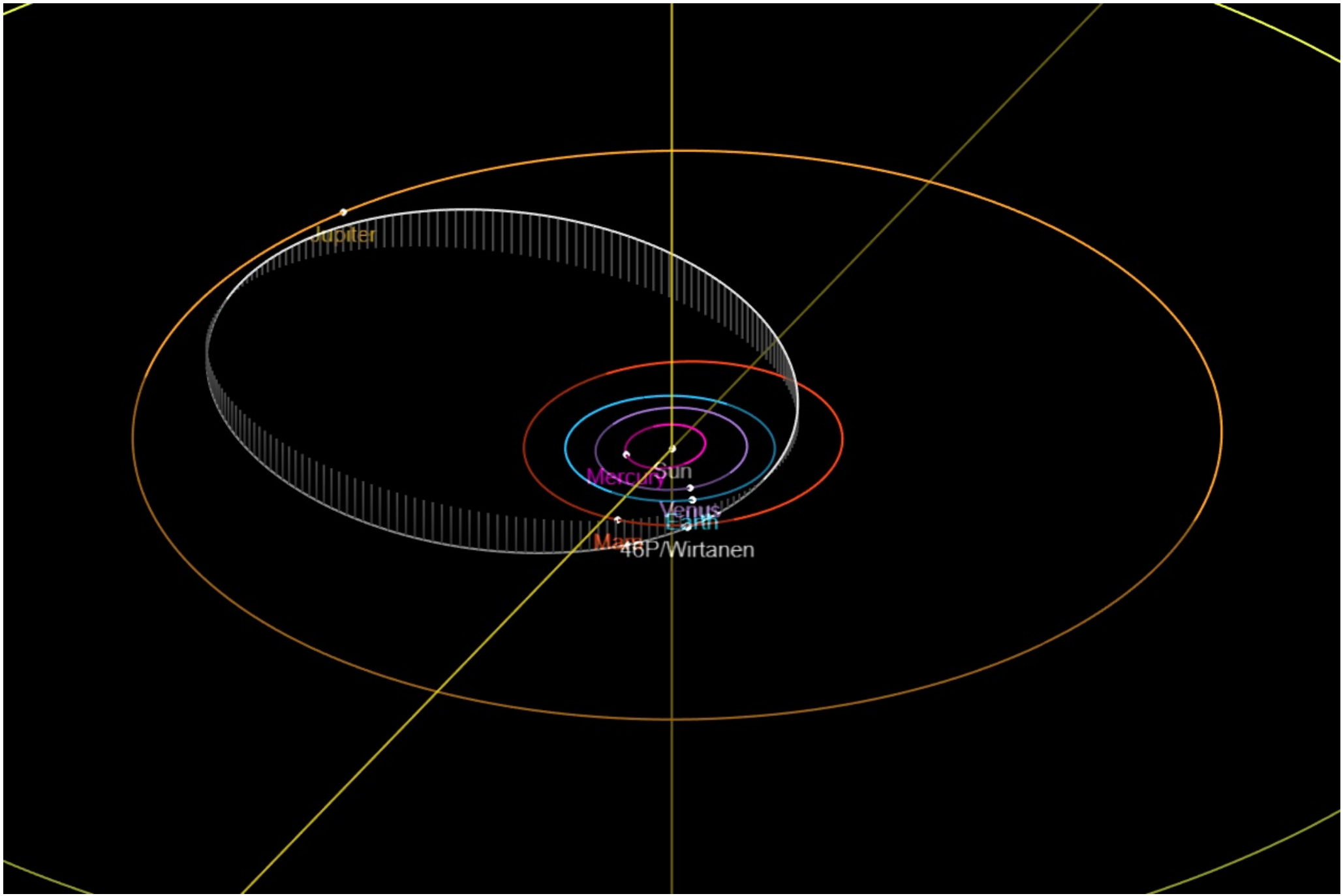

Jupiter
Mercury
Sun
Venus
Earth
Mars
46P/Wirtanen

Figure 1a: Comet Semimajor Axis Distribution

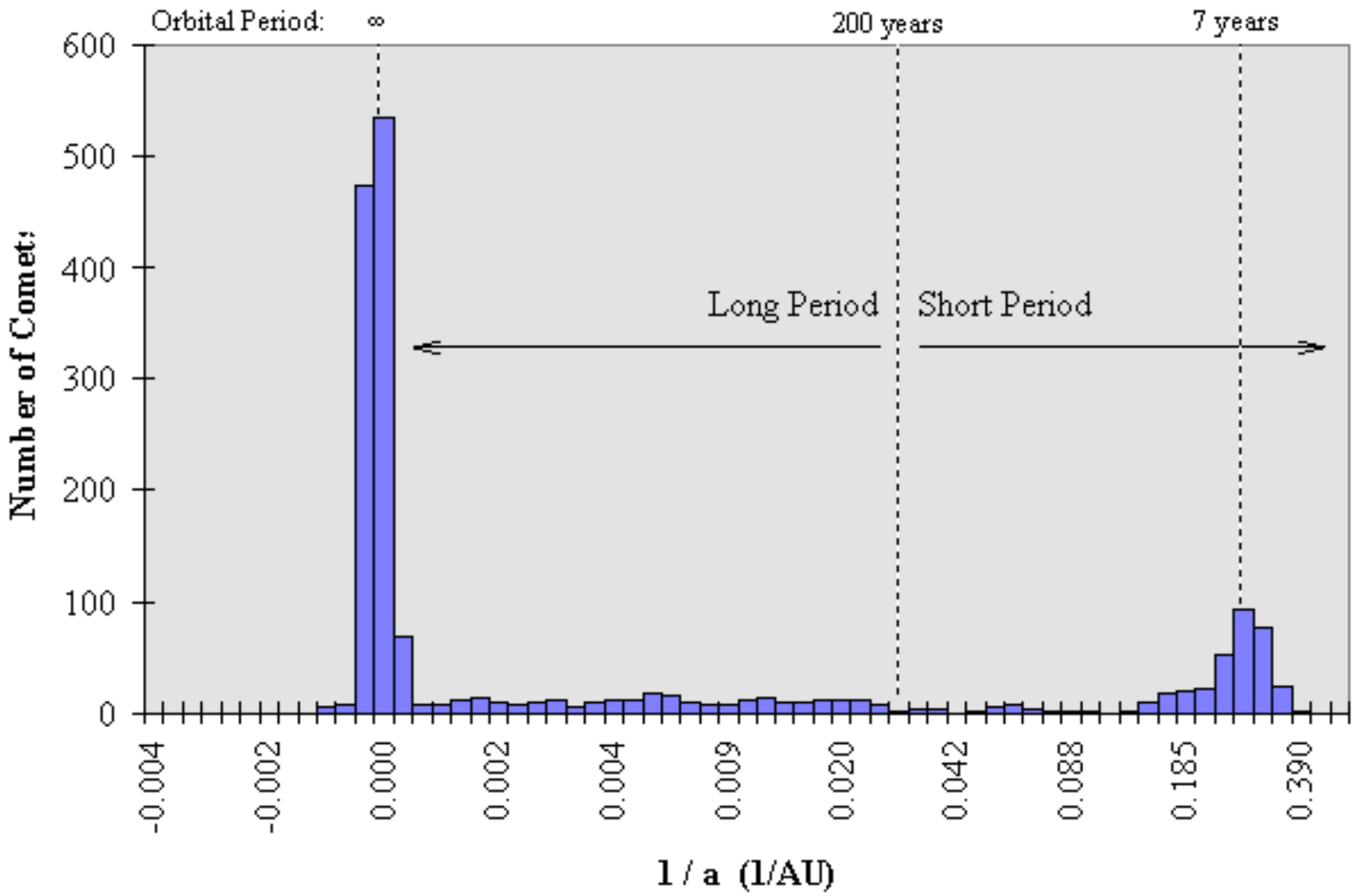
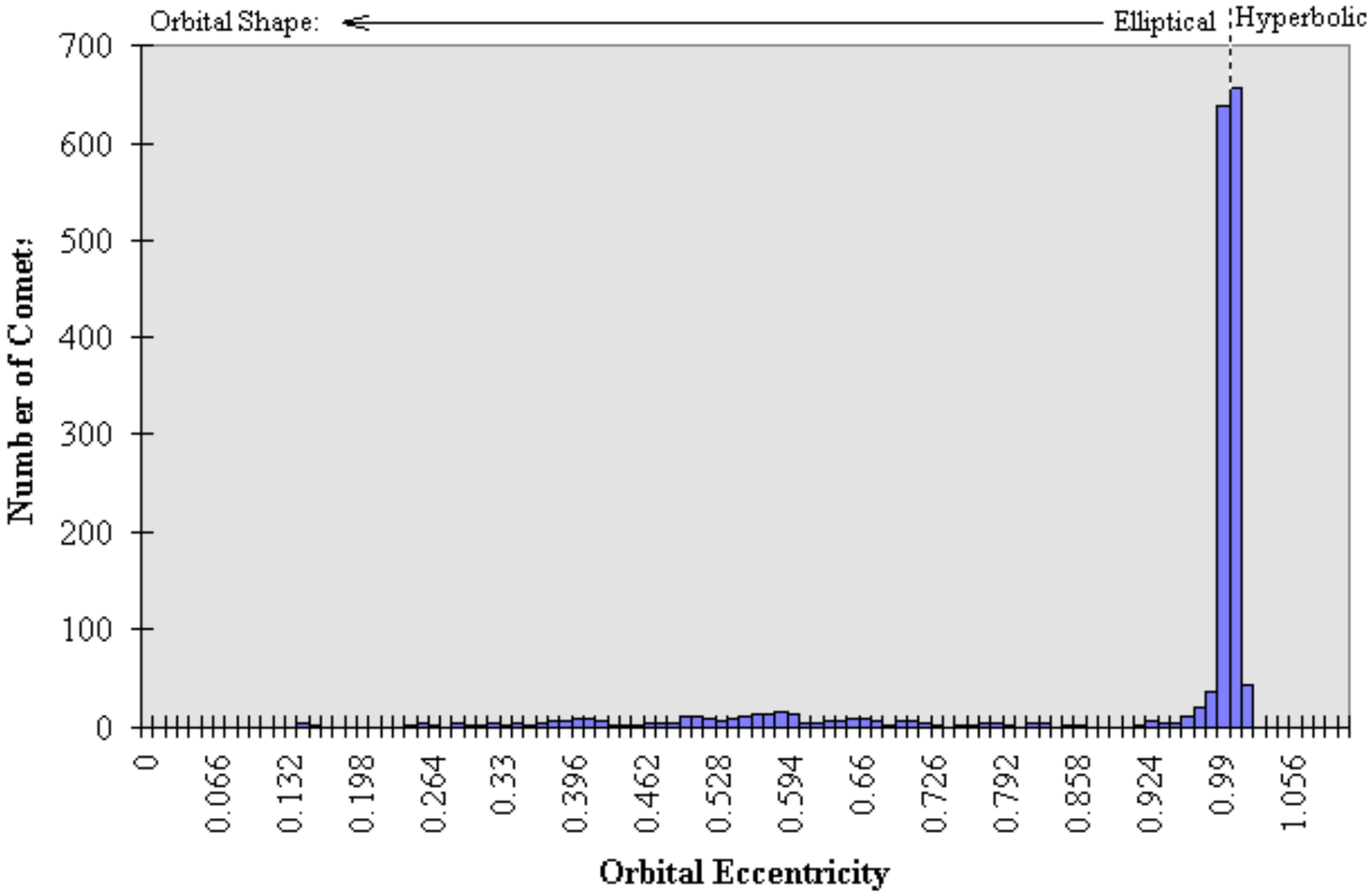


Figure 1b: Comet Eccentricity Distribution



A majority of comets have parabolic orbits

Figure 1a: Comet Semimajor Axis Distribution

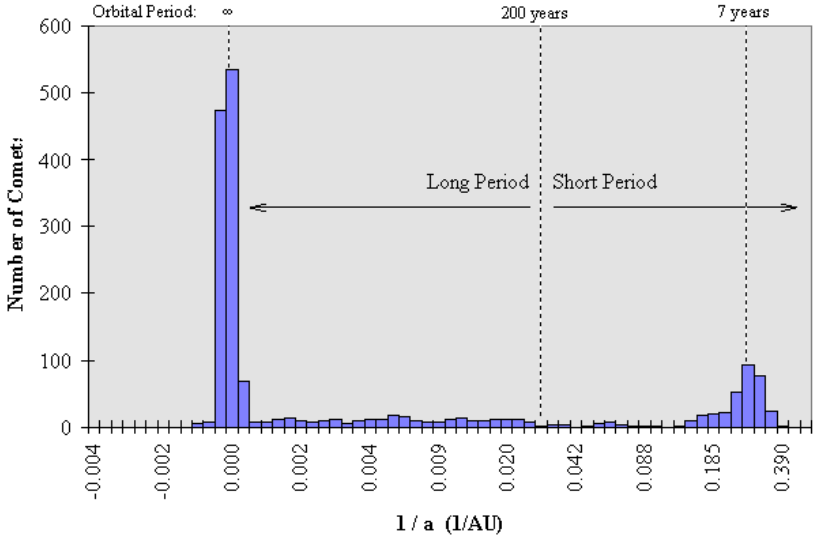
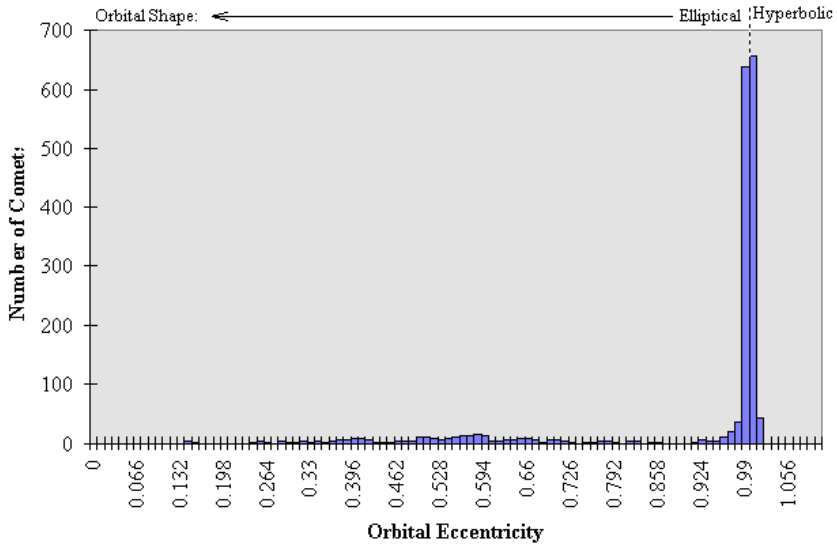
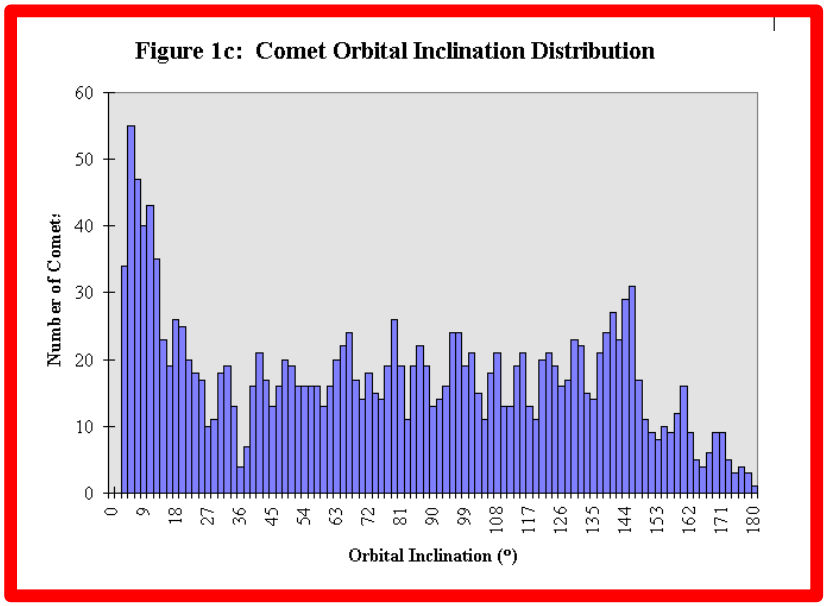


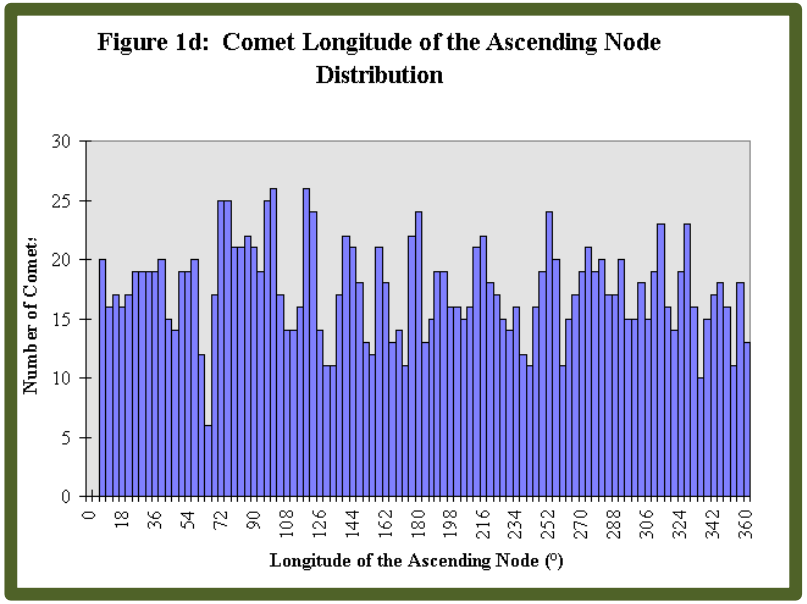
Figure 1b: Comet Eccentricity Distribution

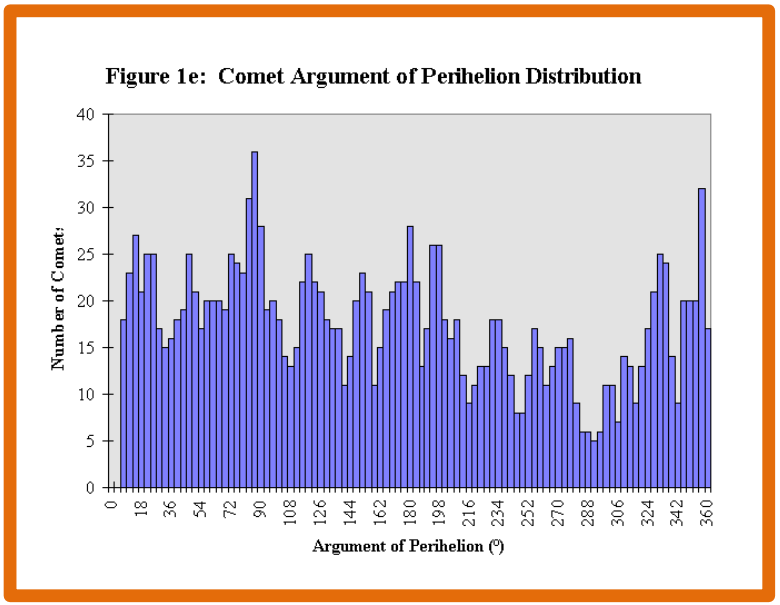




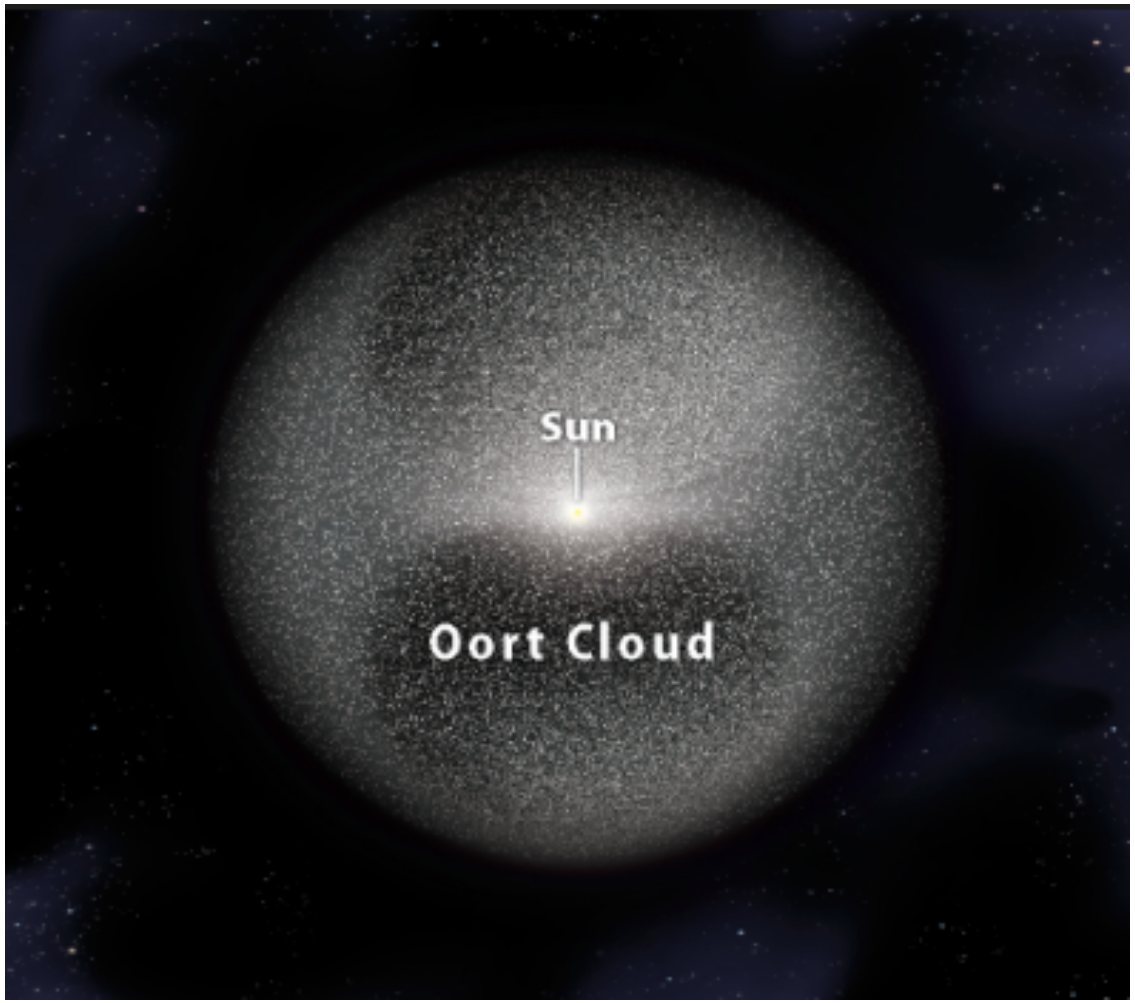
Inclination

Longitude of Ascending Node

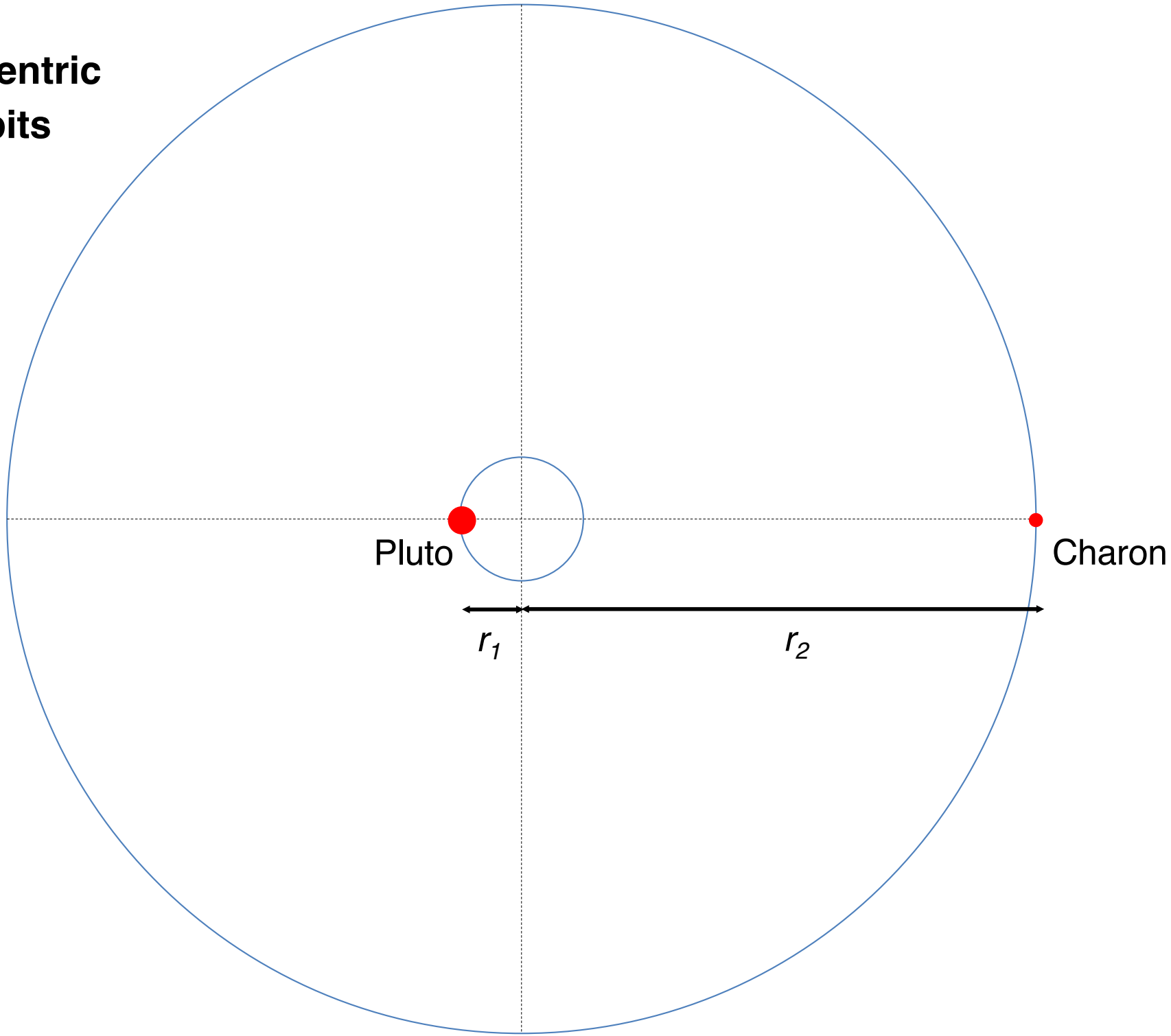




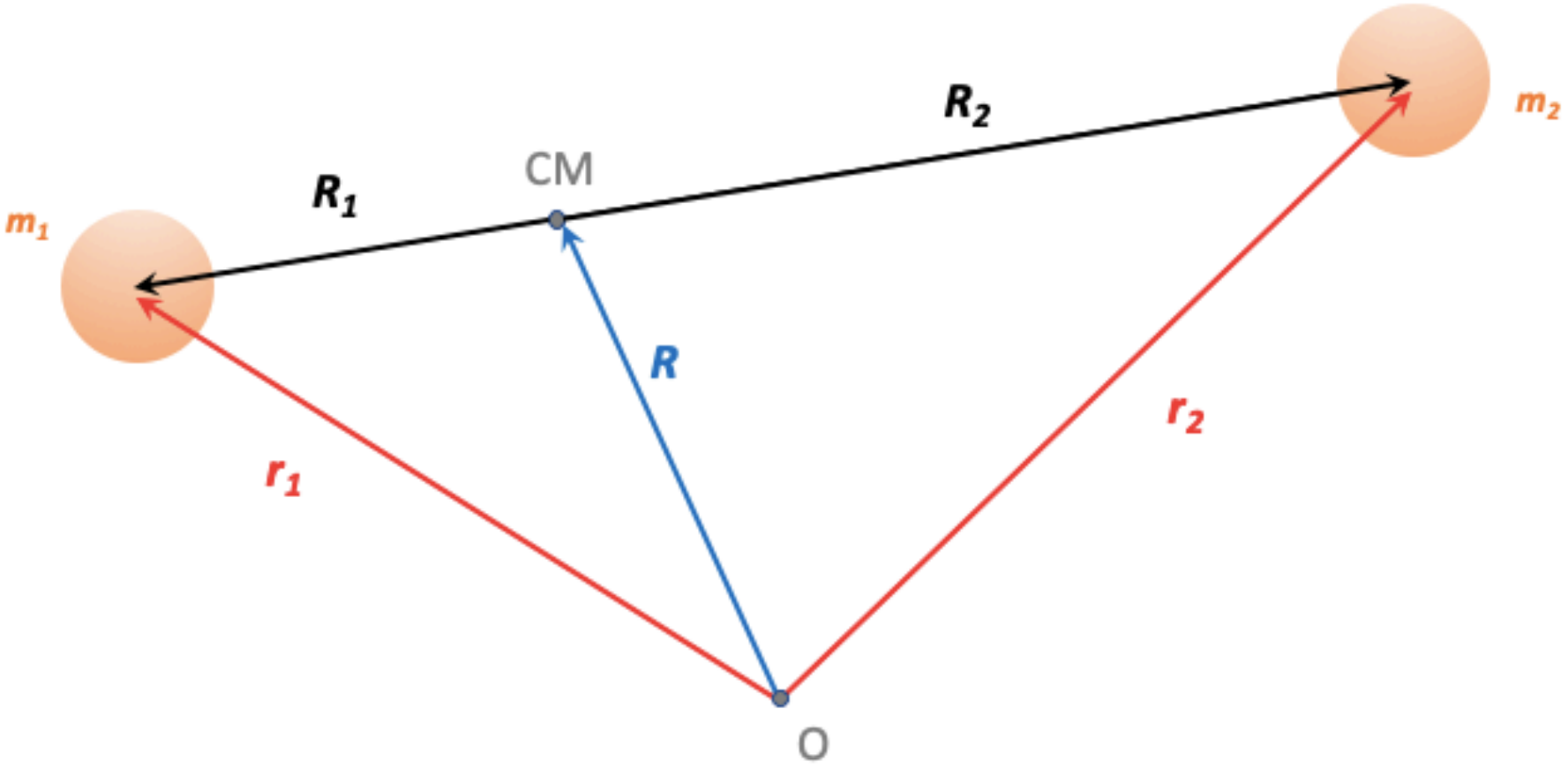
Argument of Perihelion


Sun
Oort Cloud

Barycentric Orbits



Barycentric Orbits


m_1
m_2
CM
R_1
R_2
R
r_1
r_2
O

Potential of oblate bodies

Newton's second theorem

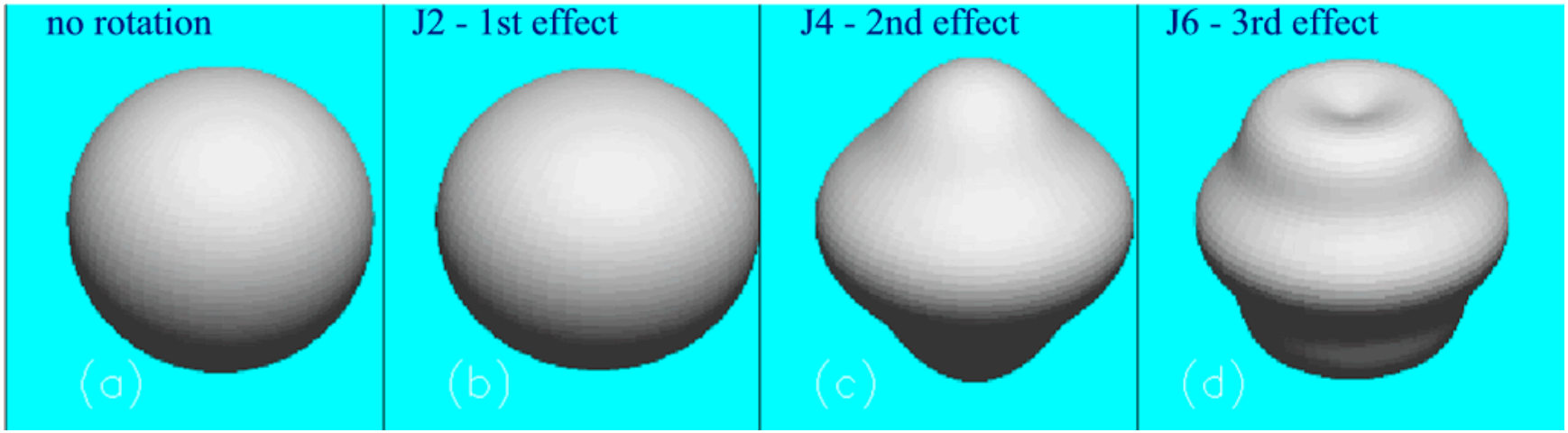
"A spherically symmetric body affects external objects as if all its mass was concentrated in its center"

But planets are not spherically symmetric

Oblateness caused by rotation

Gravitational Potential

$$\Phi_g(r,\phi,\theta) = -\frac{GM}{r}\left[1 - \sum J_n P_n(\cos\theta)\left(\frac{R}{r}\right)^n\right]$$



	J_2 (x10^{-6})	J_4 (x10^{-6})	J_6 (x10^{-6})
Jupiter	14696.4+/-0.2	587+/-2	34+/-5
Saturn	16290.7+/-0.3	936+/-3	86+/-9

$$n^2 > \kappa^2$$