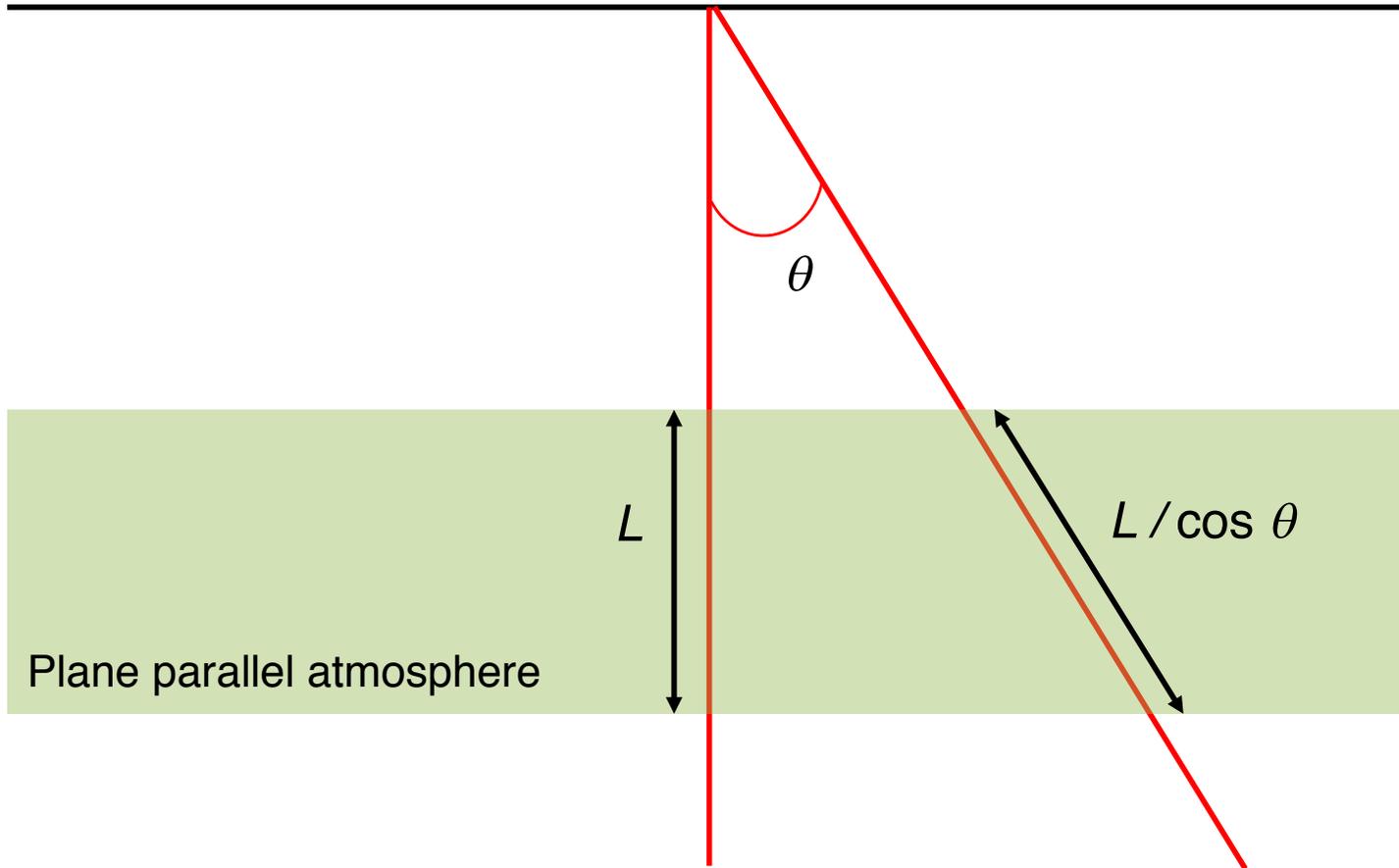


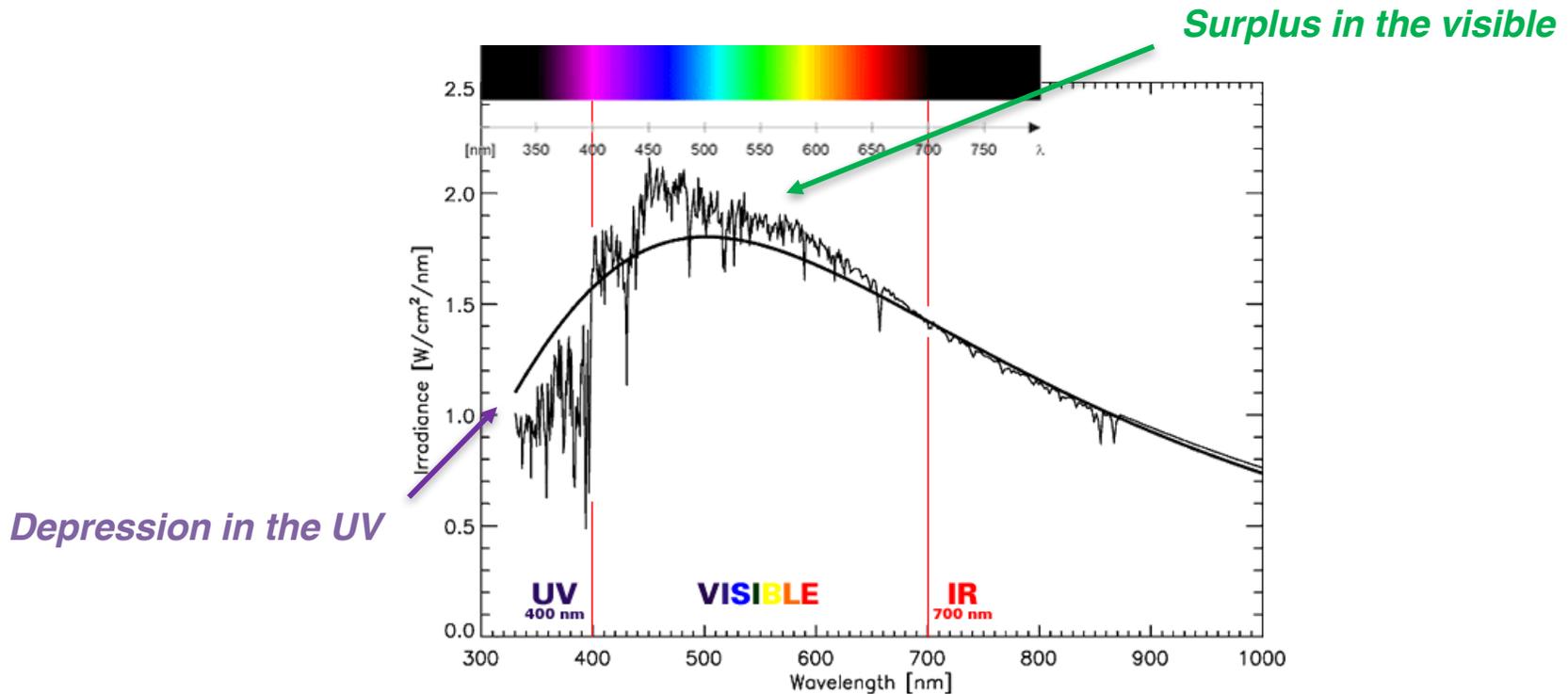
Observer



# Radiative Equilibrium

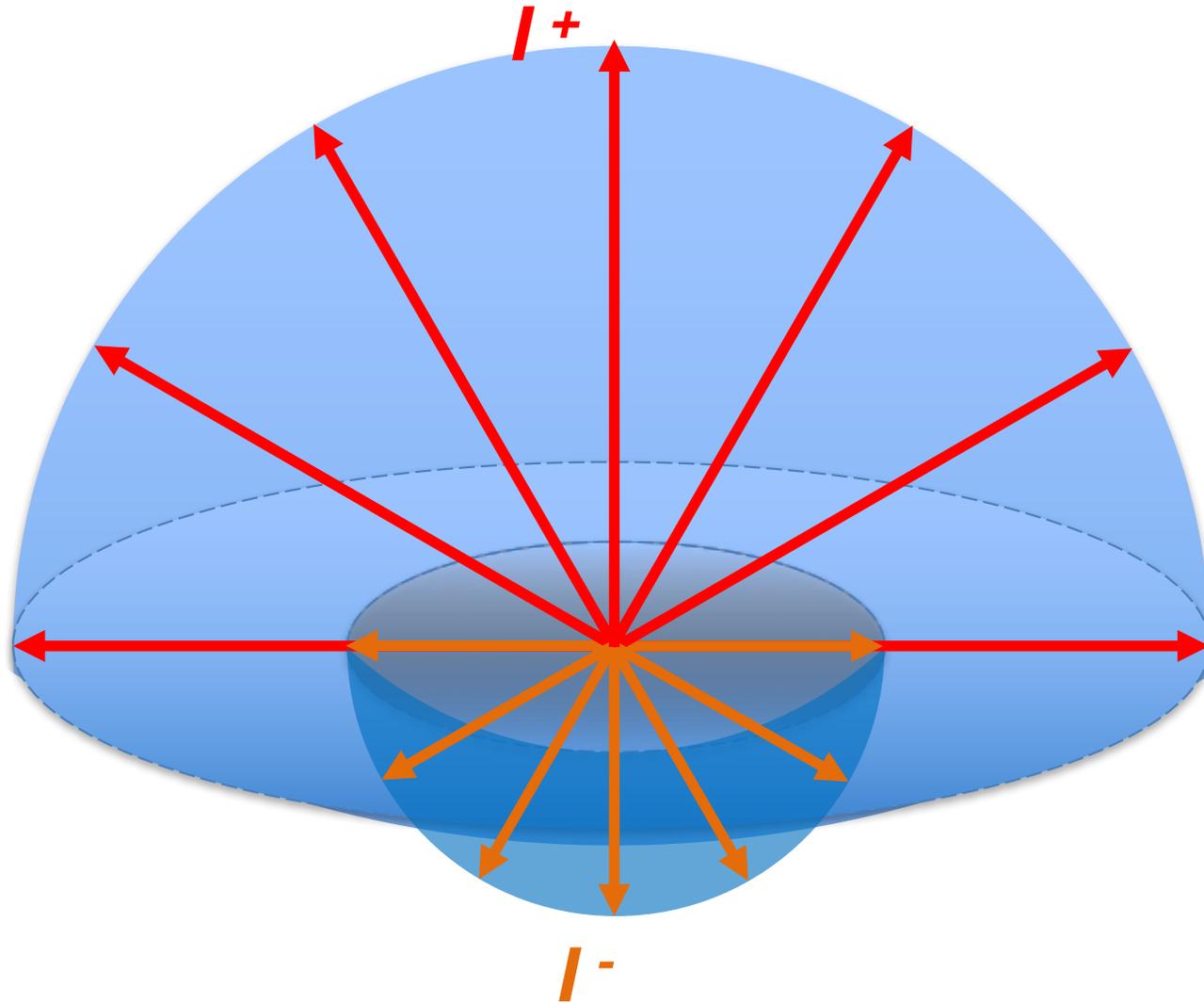
$$\int \kappa_{\nu} S_{\nu} d\nu = \int \kappa_{\nu} J_{\nu} d\nu$$

*Bolometric emission = Bolometric absorption*

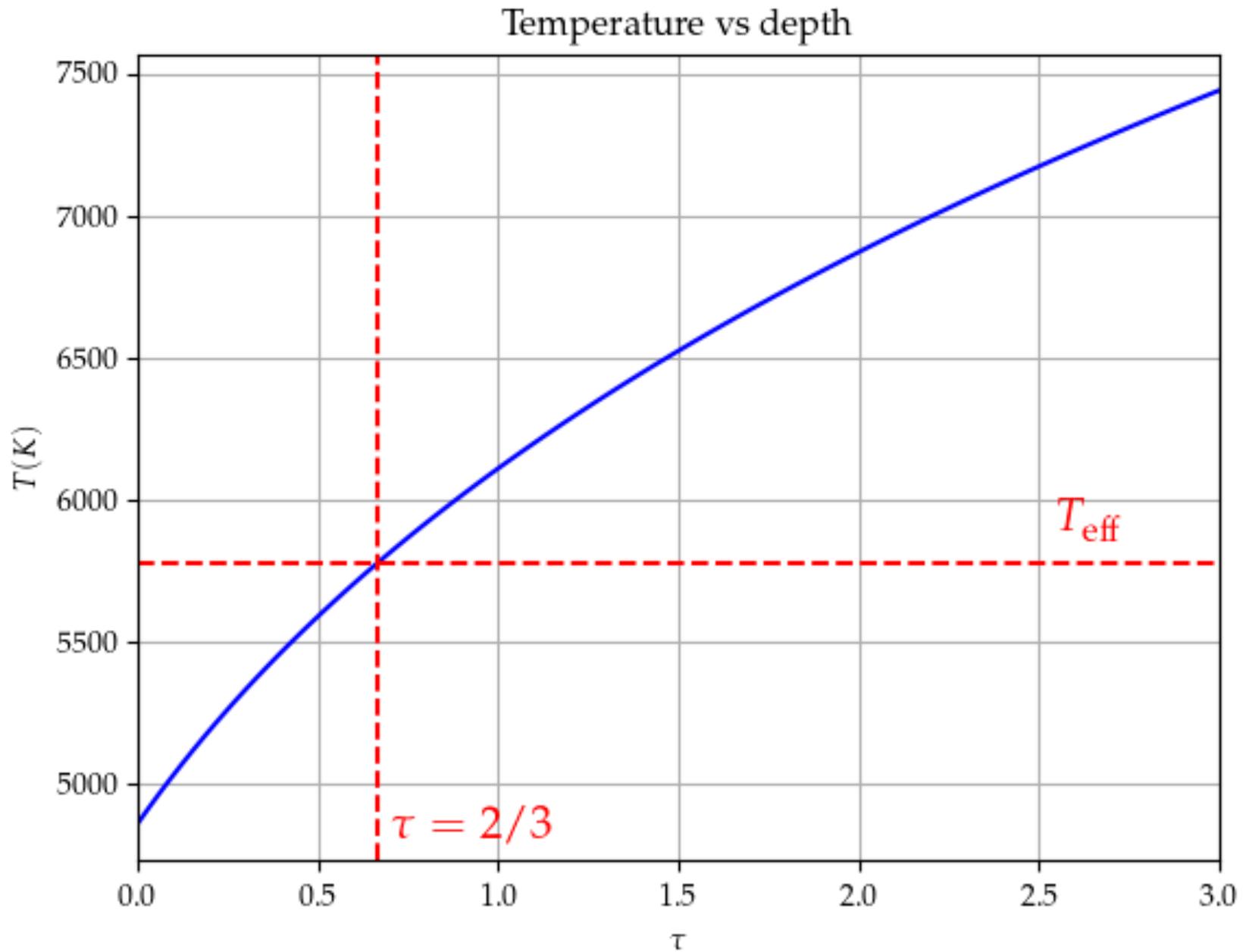


The solar irradiance spectrum from 300 to 1000nm as measured by Labs and Neckel (1962) and corrected by Neckel and Labs (1984). It roughly follows a black body radiation curve of 5777 K (thick line), particularly in the IR.

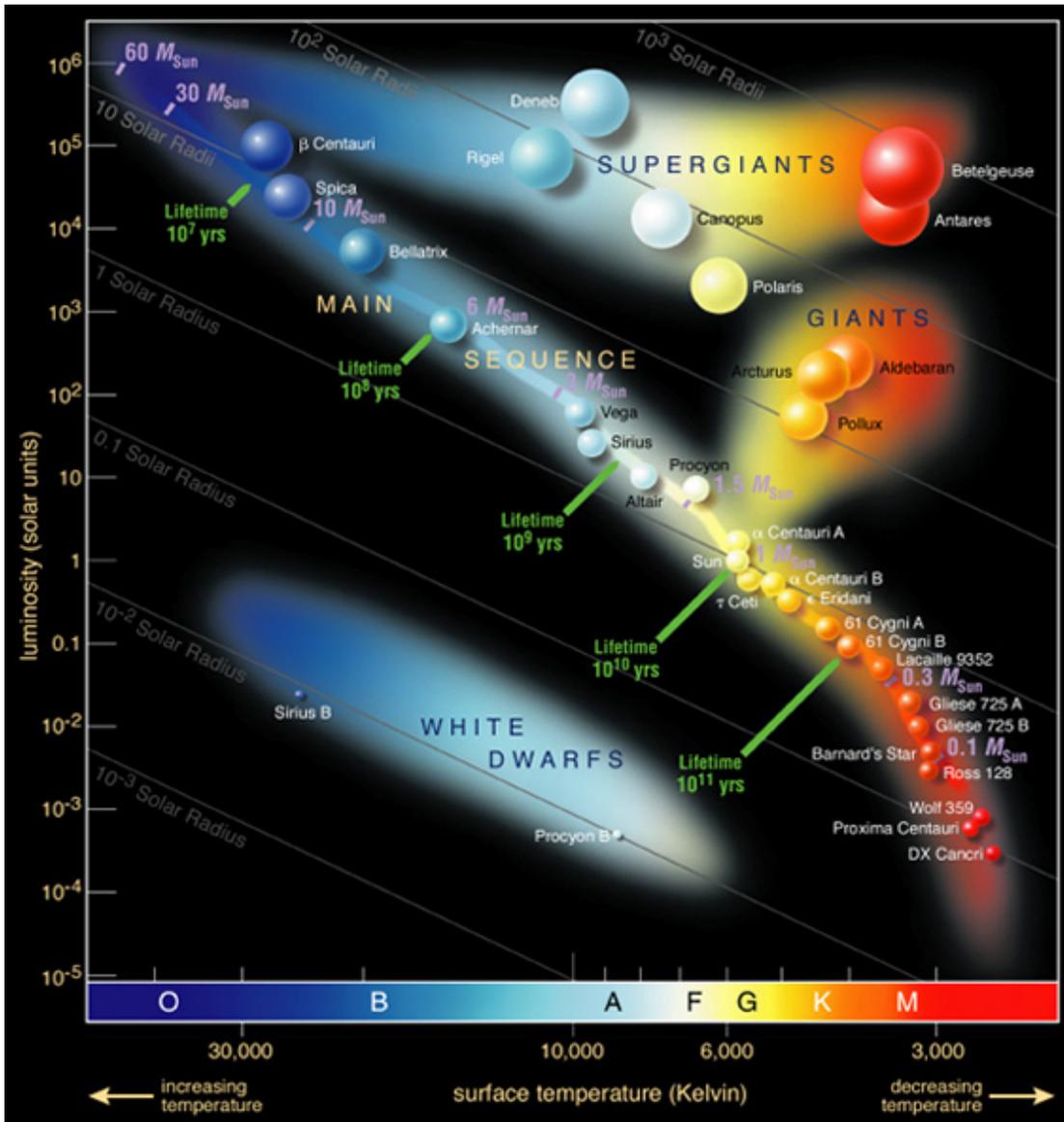
# Eddington Approximation



# Eddington Solution



# Types of stars – The HR diagram



HR stands for “Hertzsprung-Russel”

**Temperature x Luminosity**

Spectral Types

OBAFGKM

Each type is subdivided into ten numbered subtypes (eg, K8, B6)

## Spectral Types

O B A F G K M

# Types of stars – The HR diagram

## Spectral Types

OBAFGKM

Each type is subdivided into ten numbered subtypes (eg, K8, B6)

## Luminosity classes

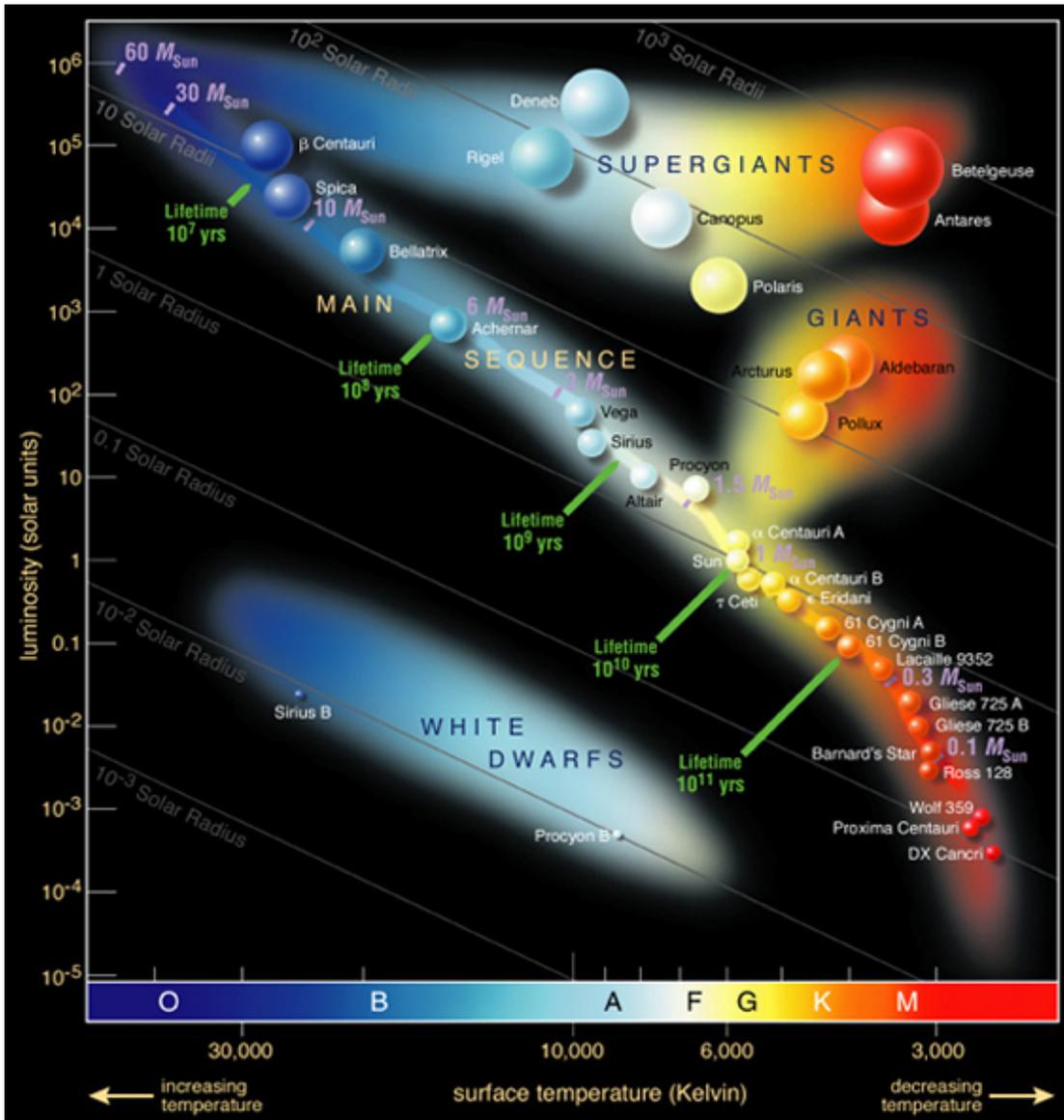
- I - Supergiants
- II - Bright giants
- III - Giants
- IV - Subgiants
- V - Dwarfs
- VI - Subdwarfs
- VII - White Dwarfs

The **Sun** is a **G2V** star

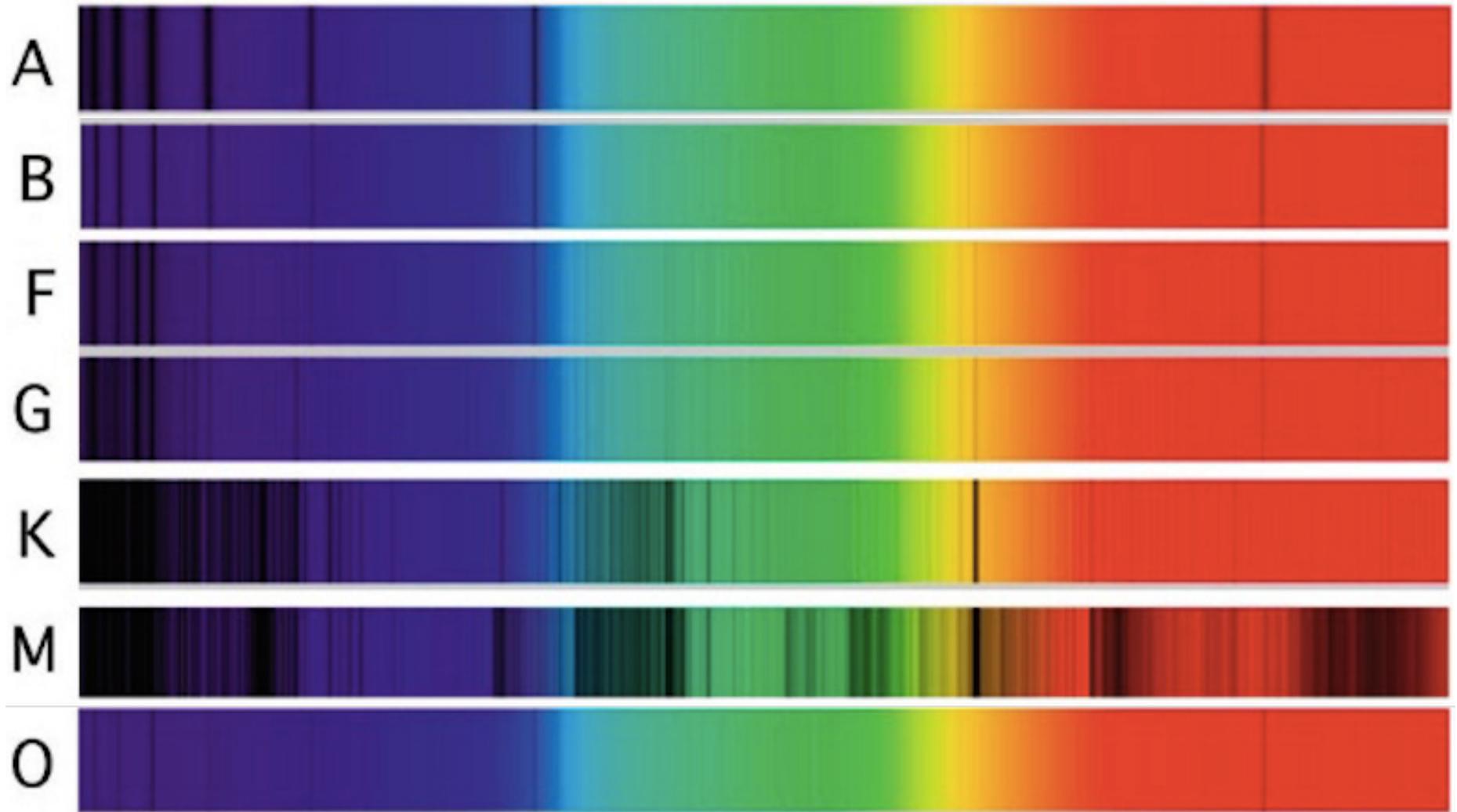
**Sirius** is a **A1V** star

**Betelgeuse** is a **M2I** star

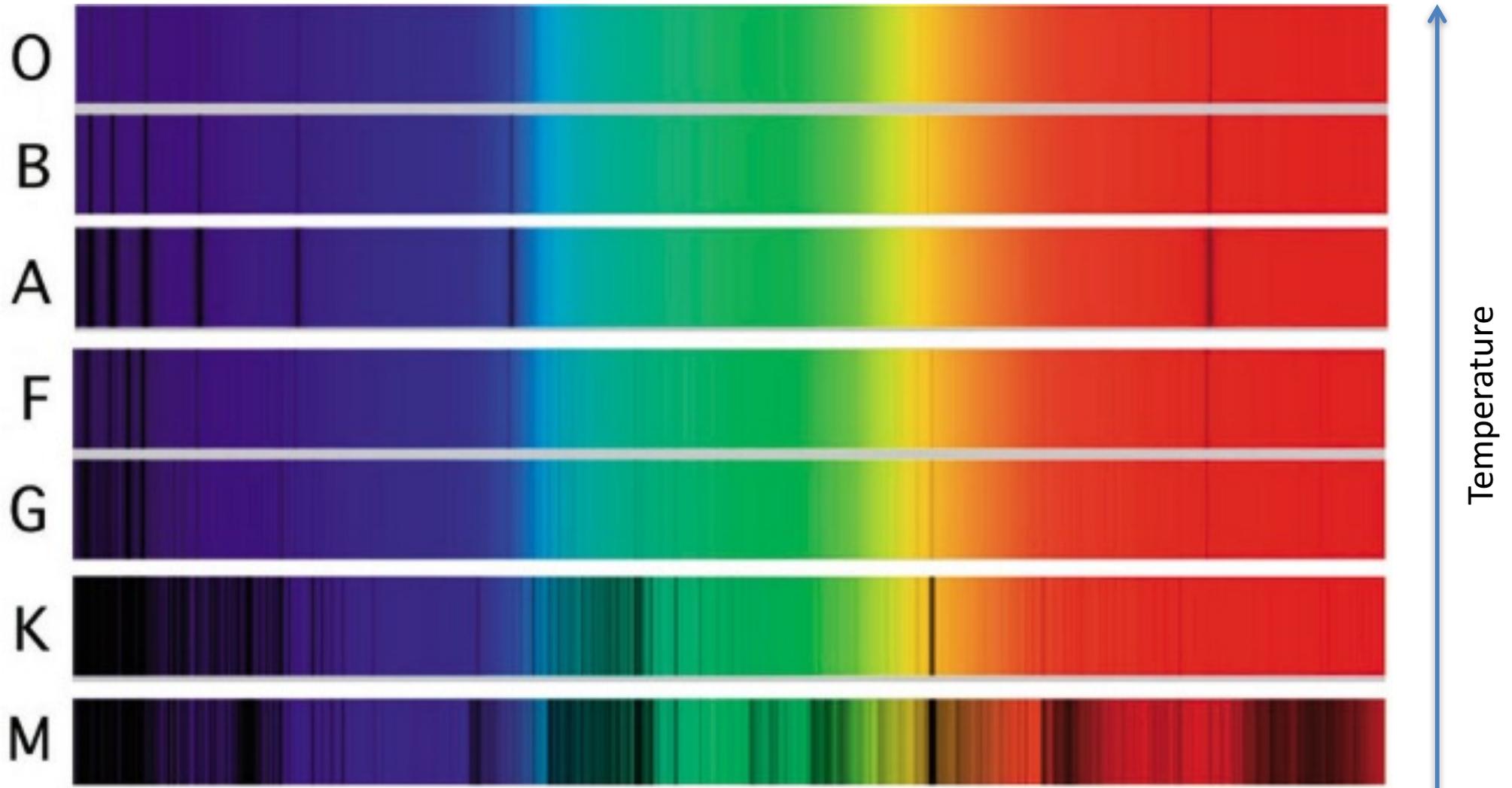
**Pollux** is a **K2III** star

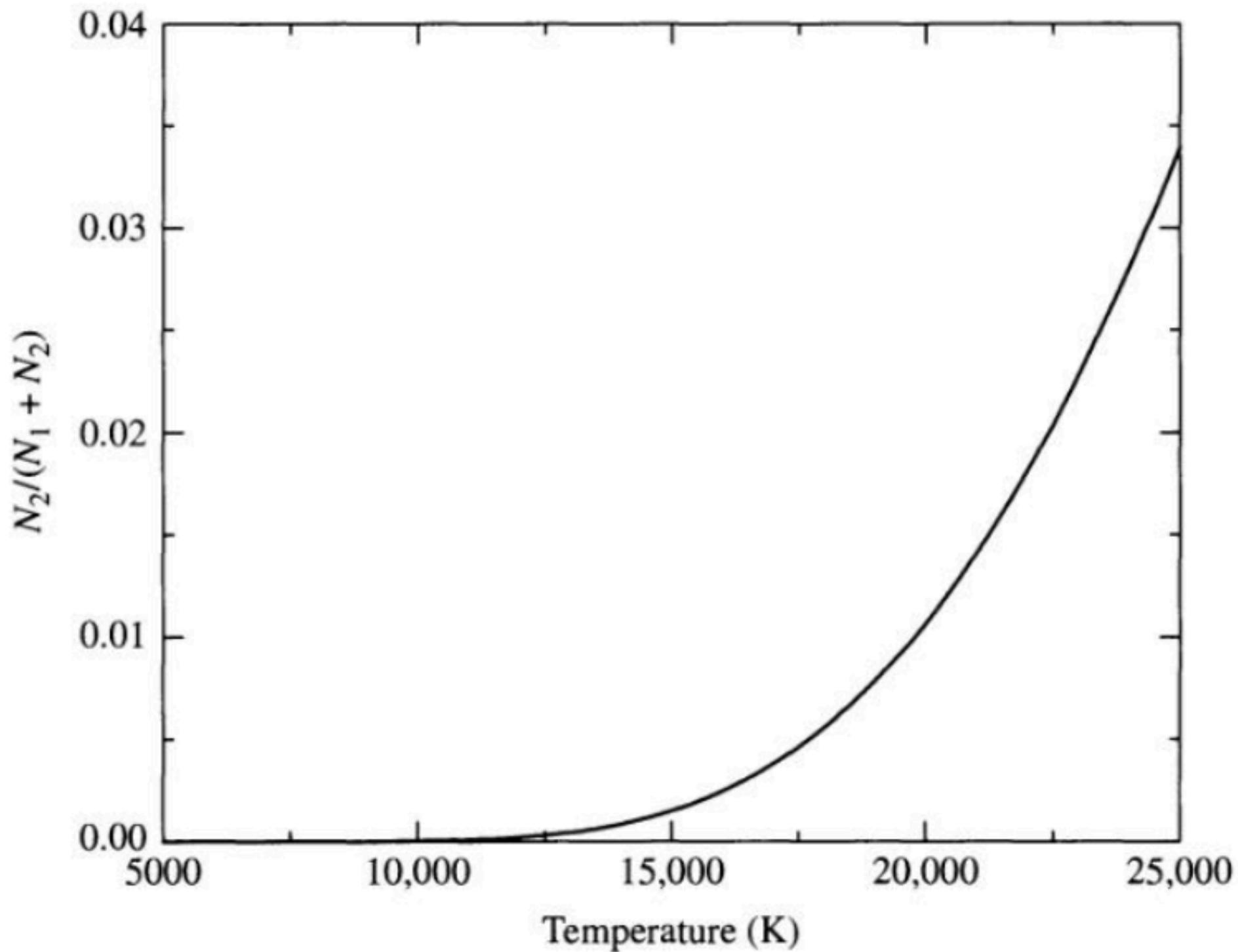


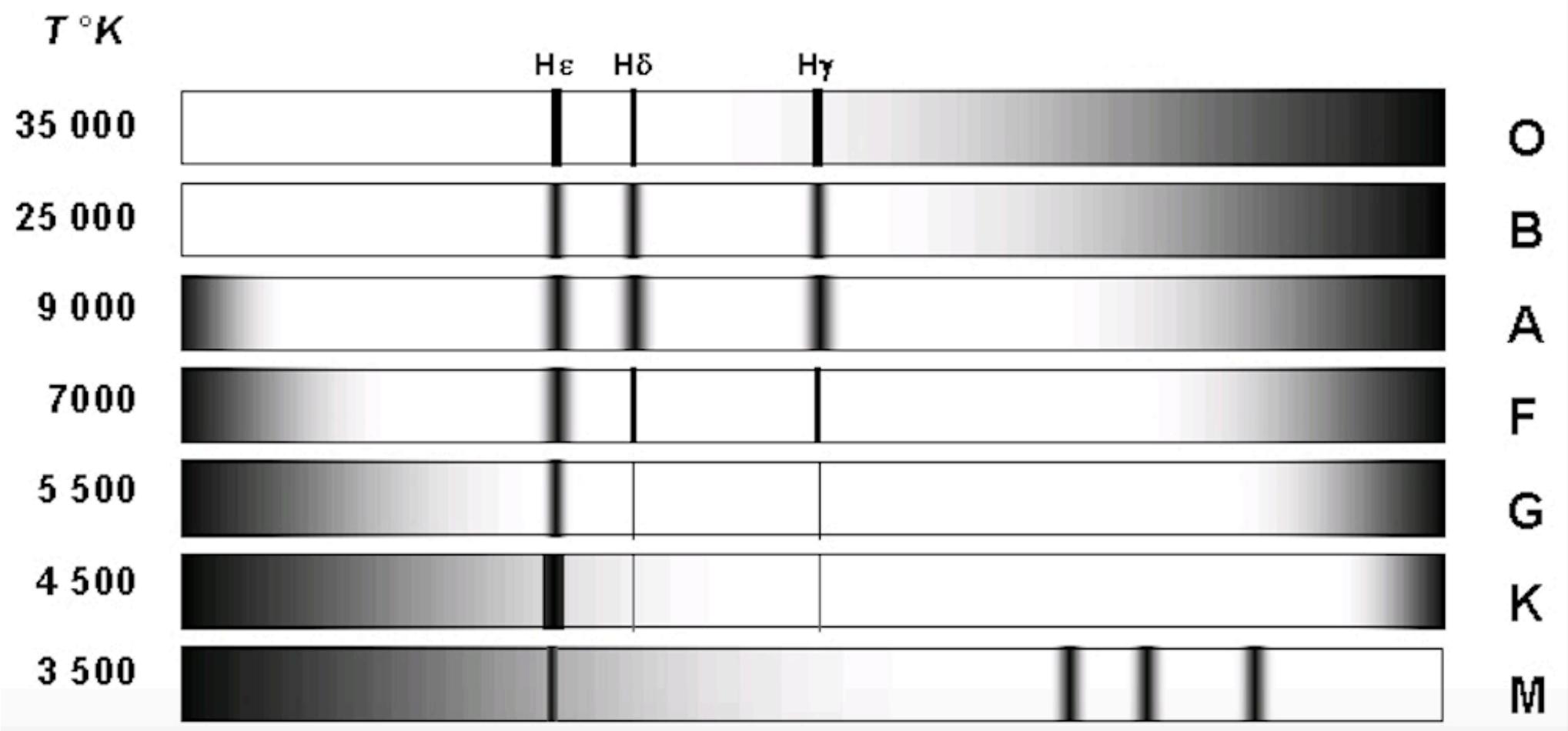
# Origin of spectral class nomenclature – Strength of Hydrogen lines

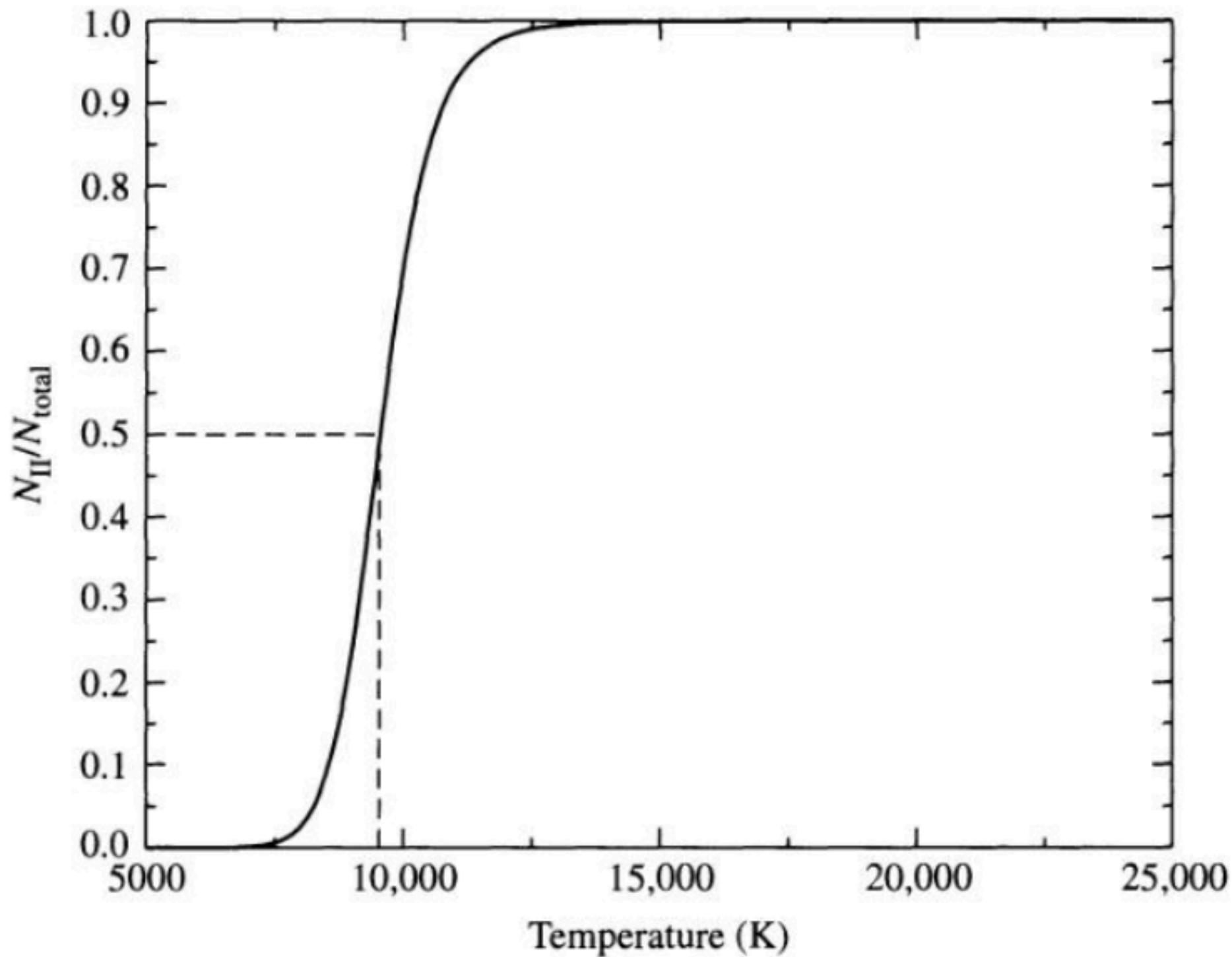


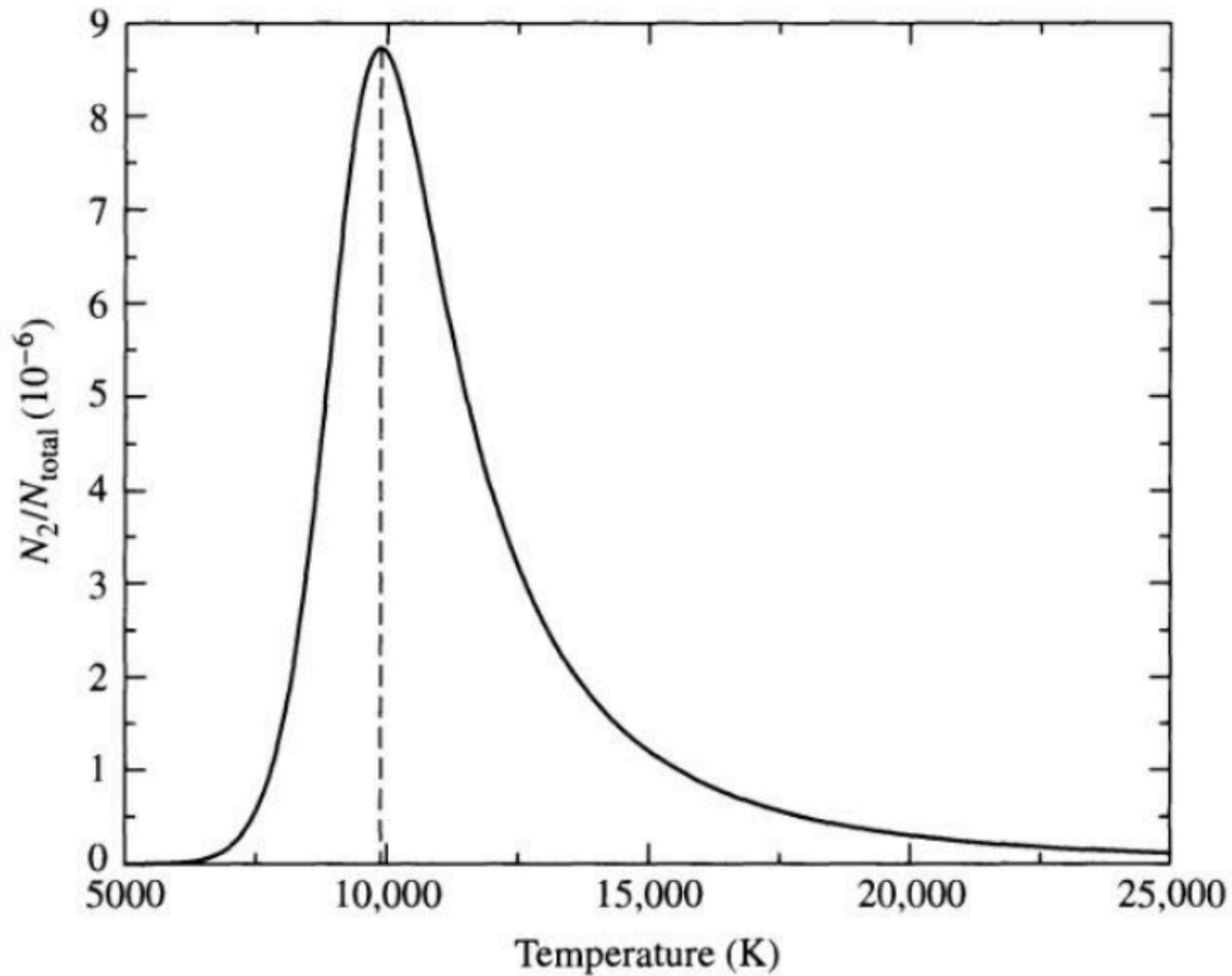
# Ordering in Temperature





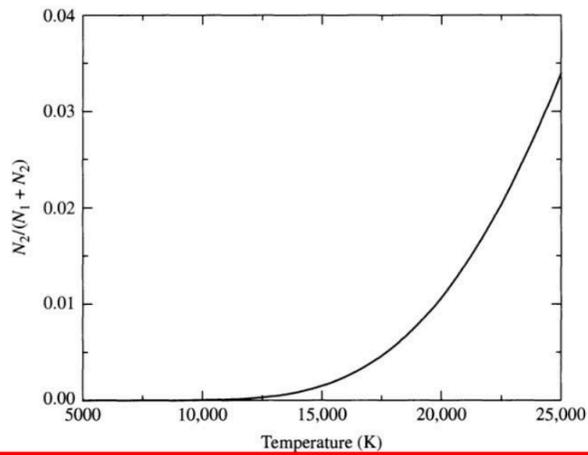






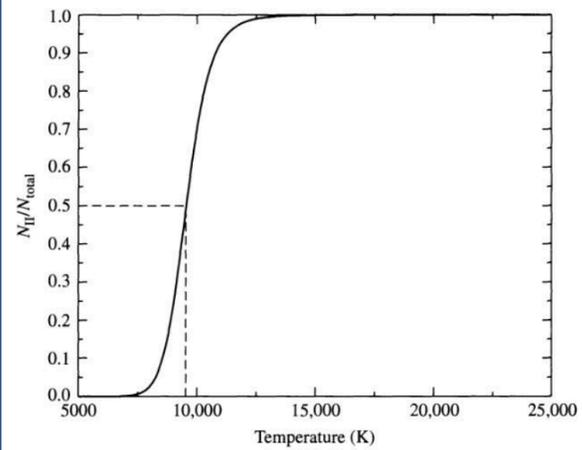
# Saha Equation

## Excitation

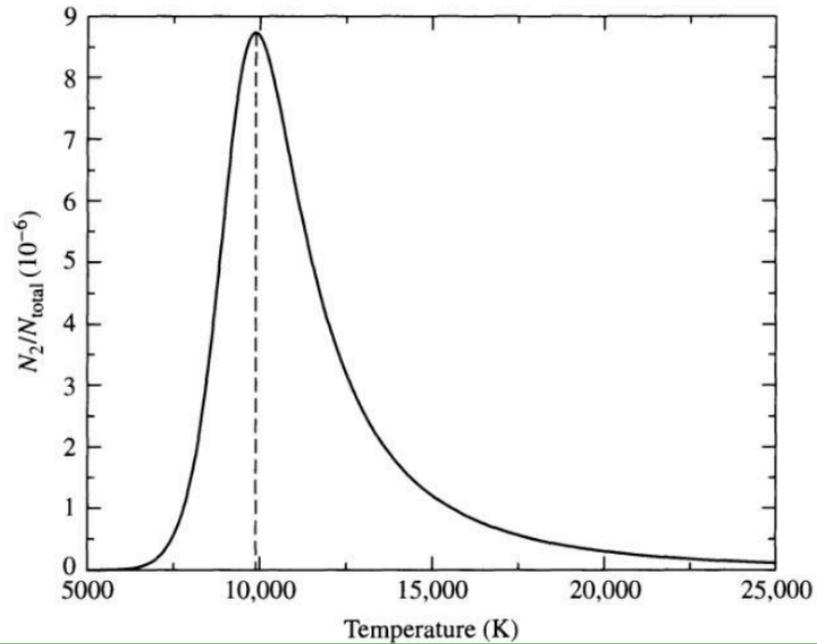


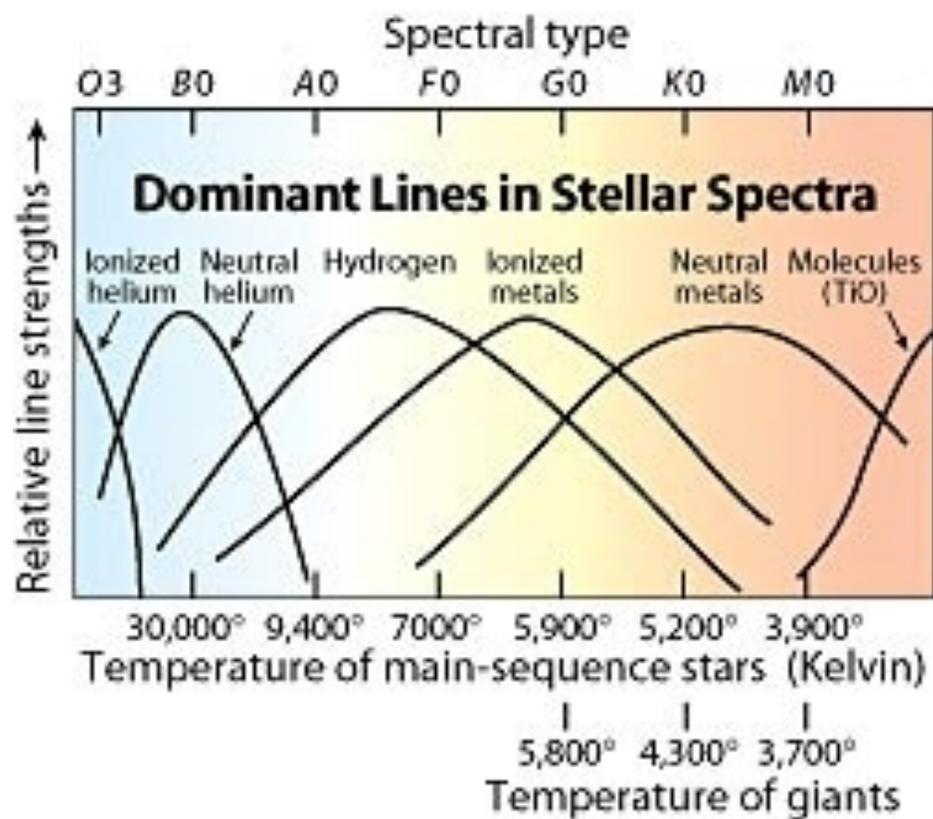
**x**

## Ionization

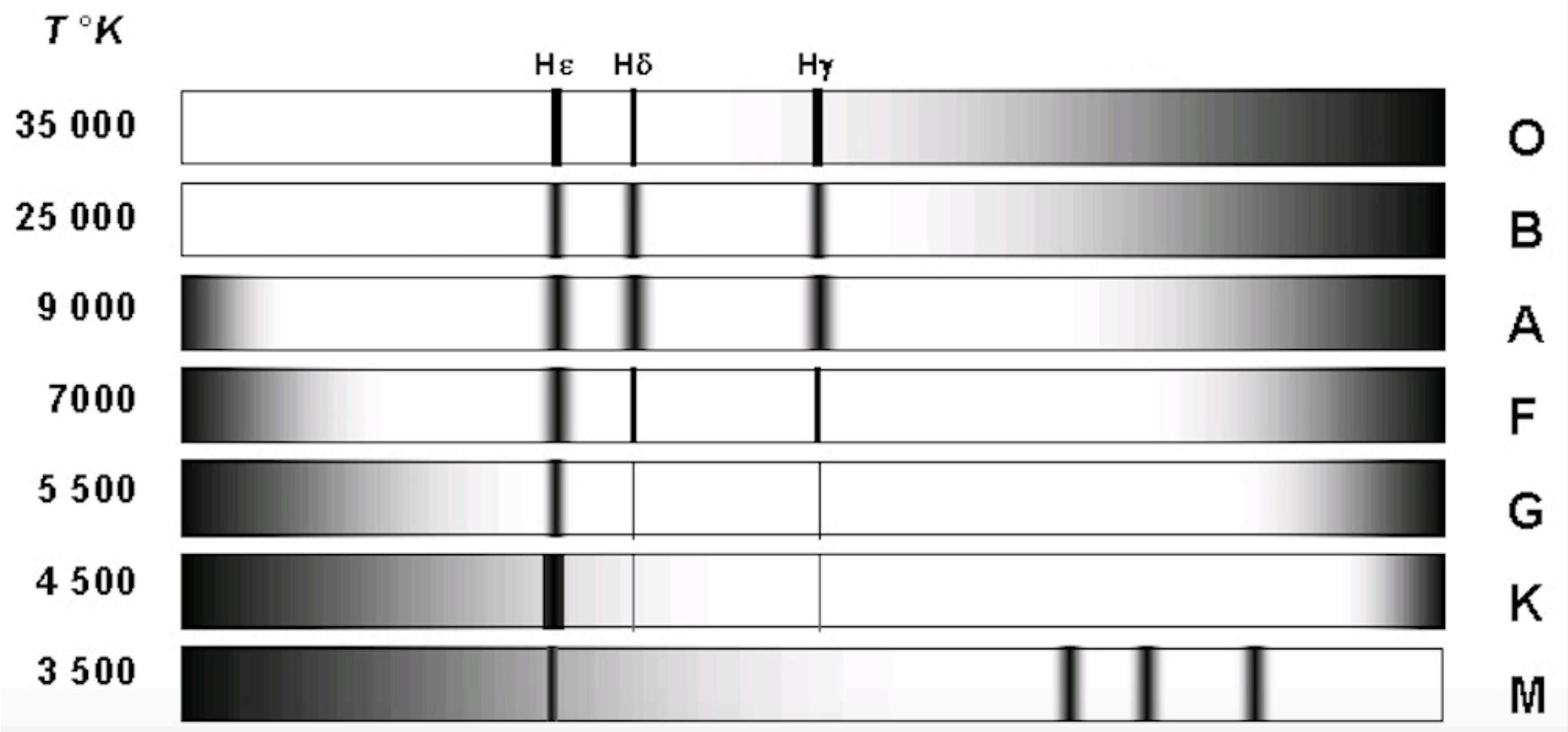


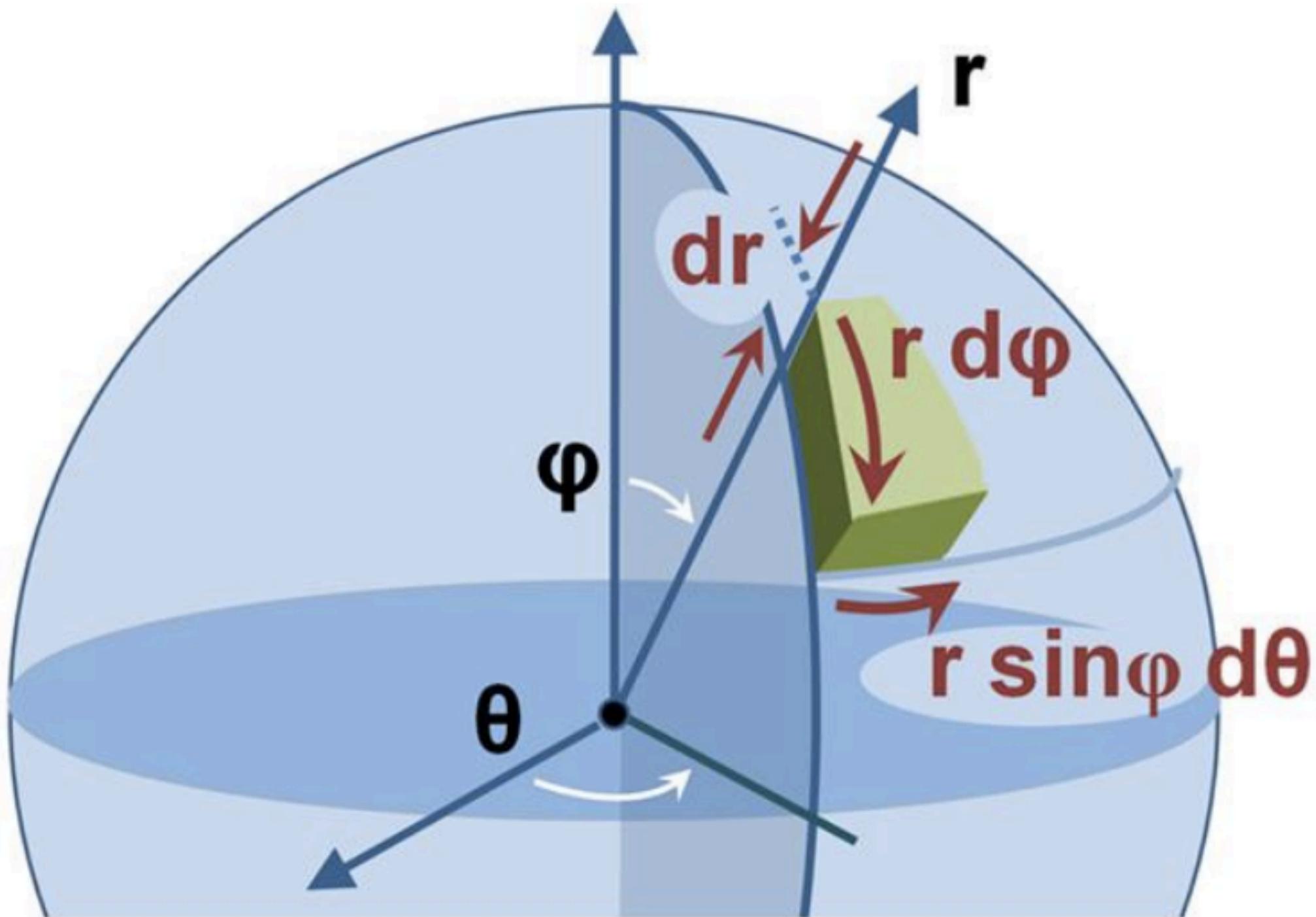
**=**

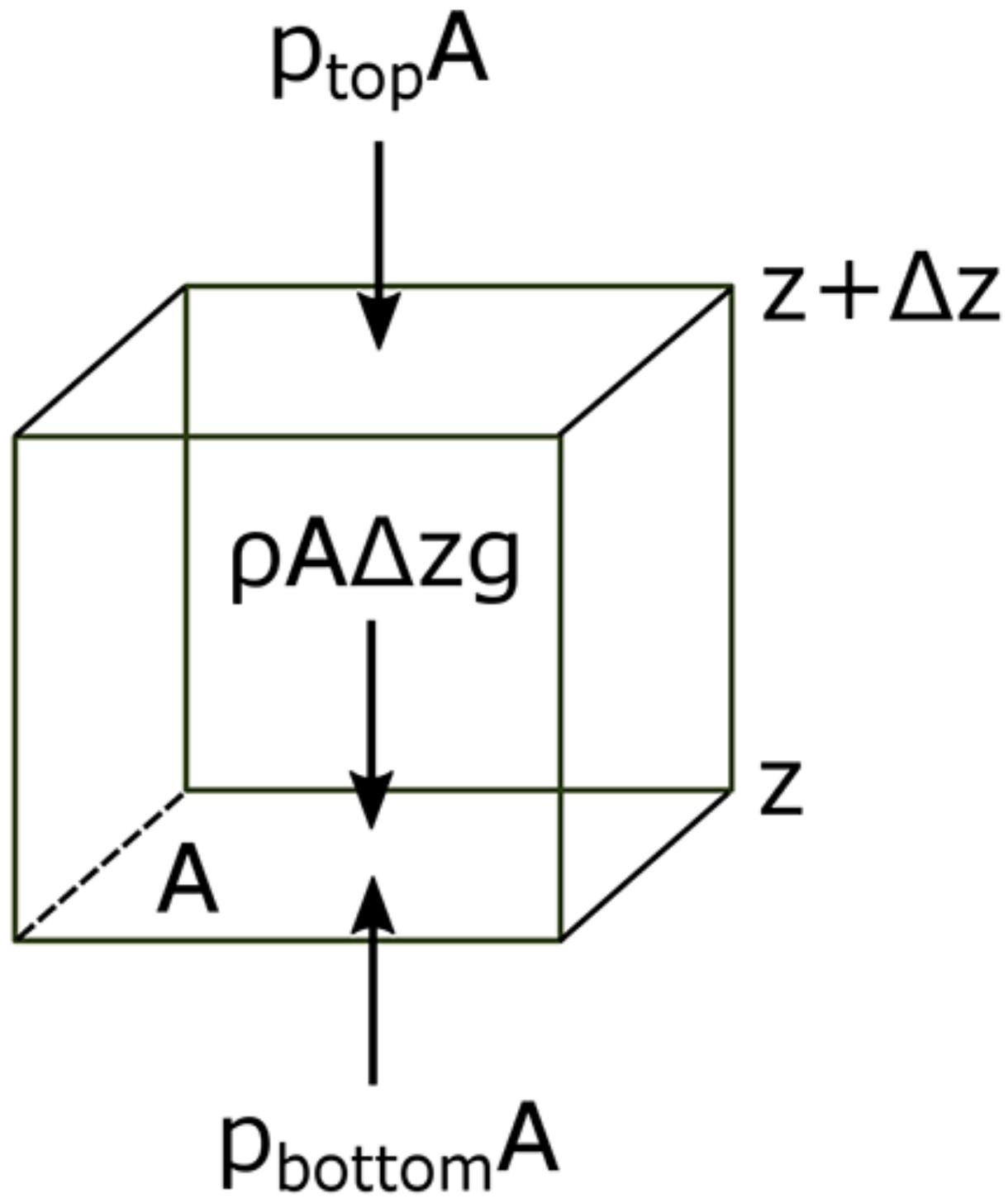




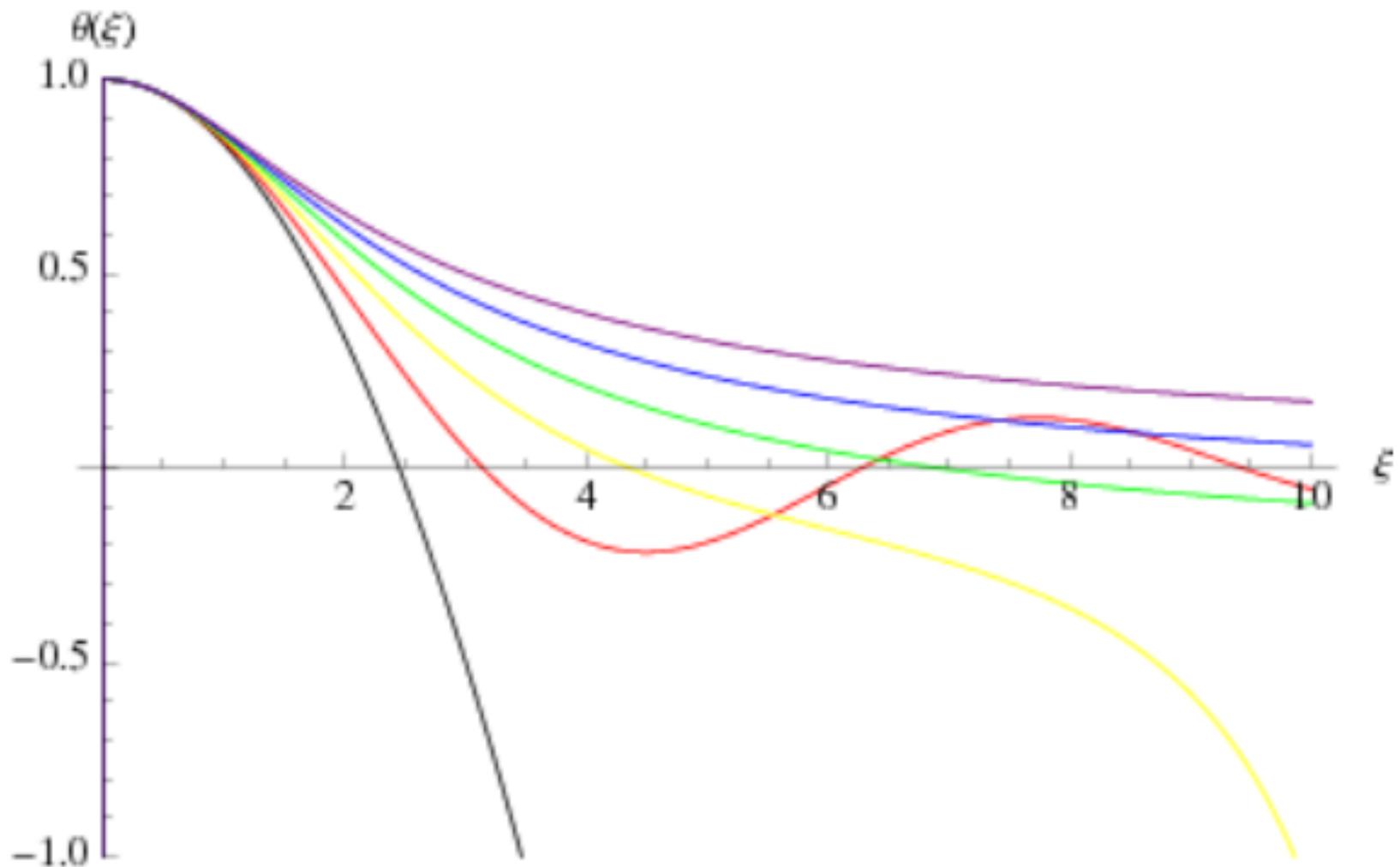






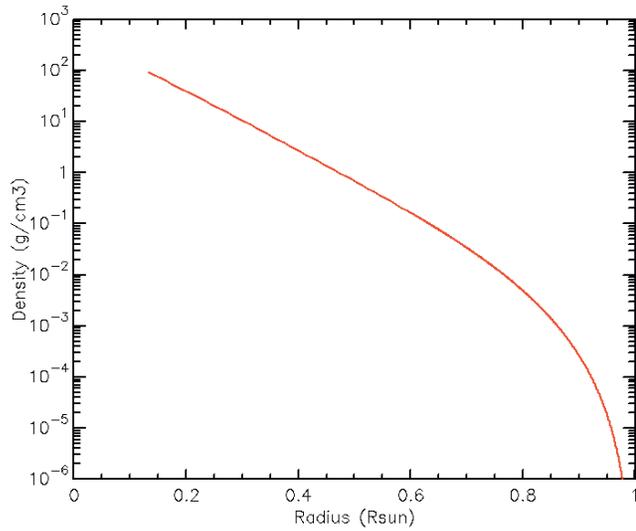


# Solutions of the Lane-Emden equation

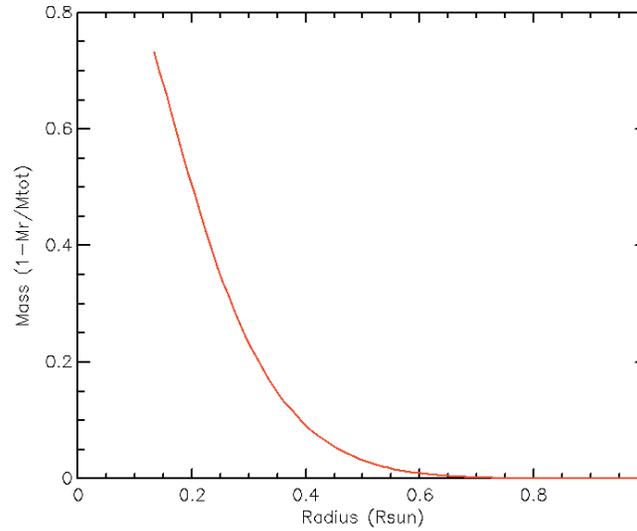


# Solar Model

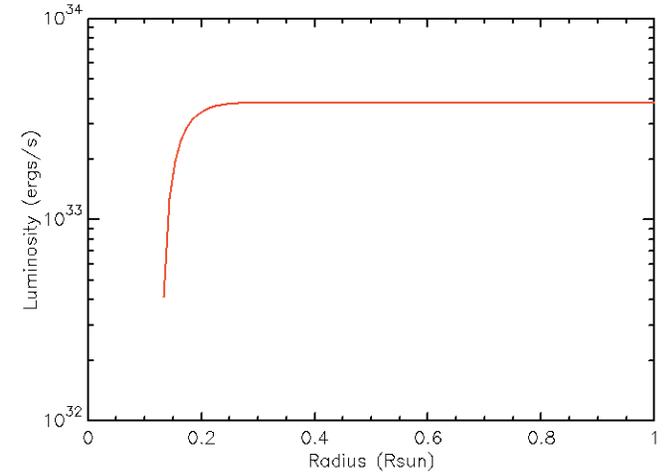
## Density



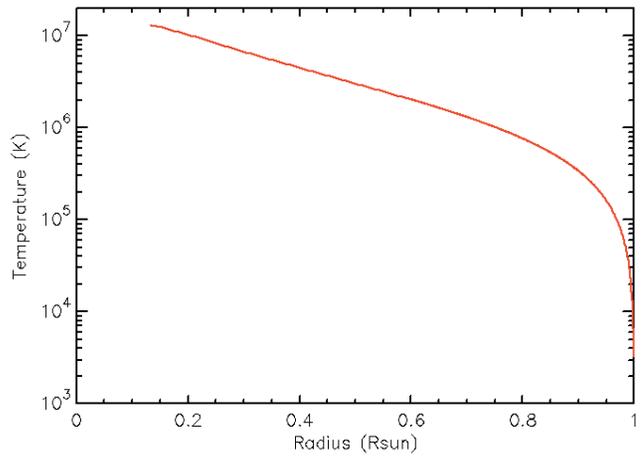
## Mass



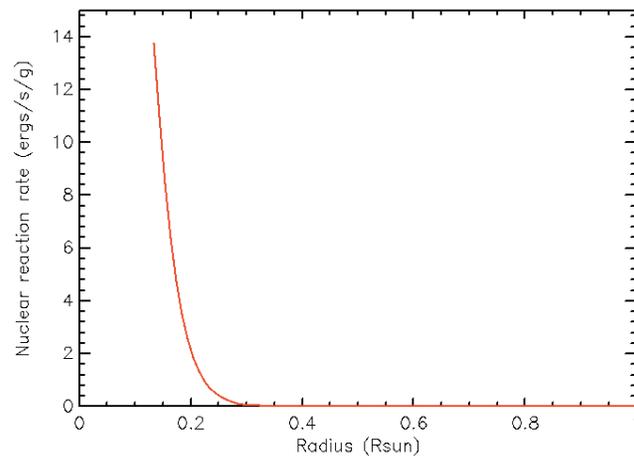
## Luminosity



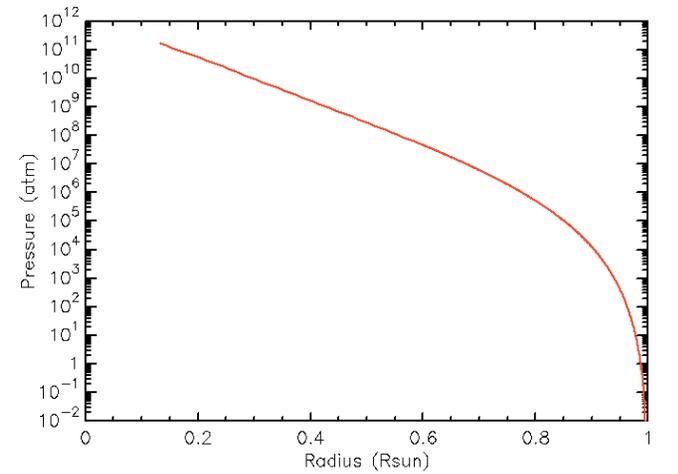
## Temperature



## Nuclear Reaction Rate



## Pressure



# Stable and Unstable Equilibria

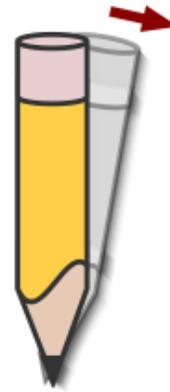
## Stable Equilibrium

Returns to equilibrium position when disturbed

stable



unstable



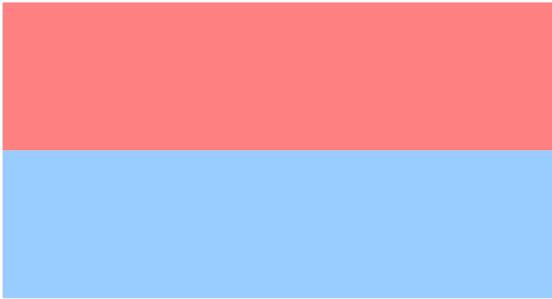
## Unstable Equilibrium

Does not return to equilibrium position when disturbed

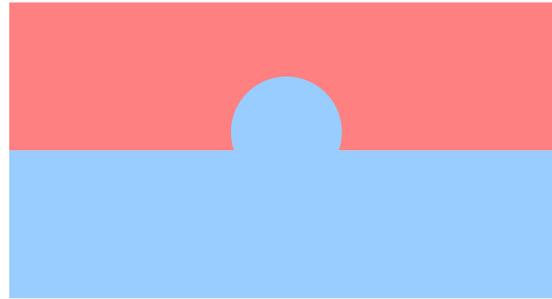
# Thermal stability

↑ *to surface*

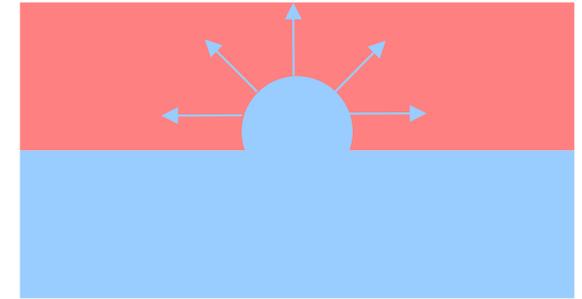
↓ *to core*



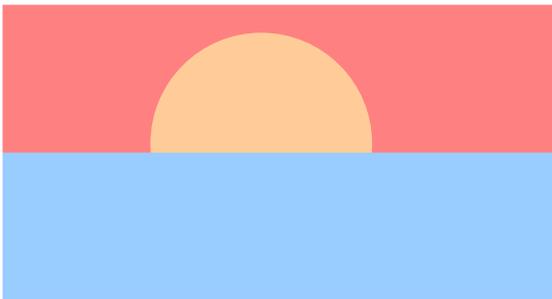
Equilibrium



a perturbation



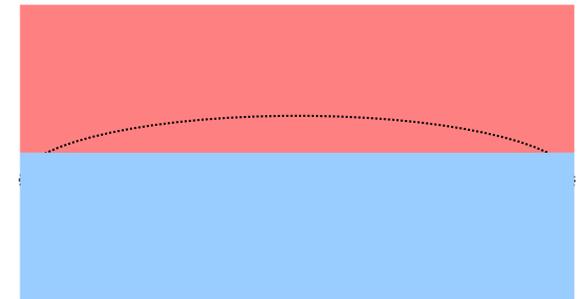
Hotter than the surroundings, it expands and rises



Expansion cools the blob

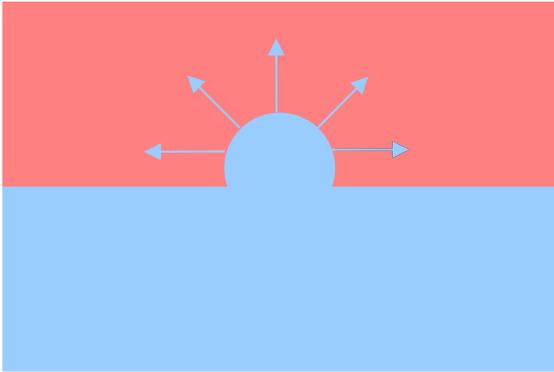


It cools further and sinks

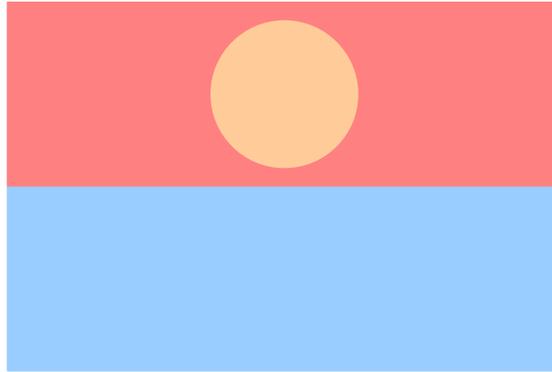


Equilibrium

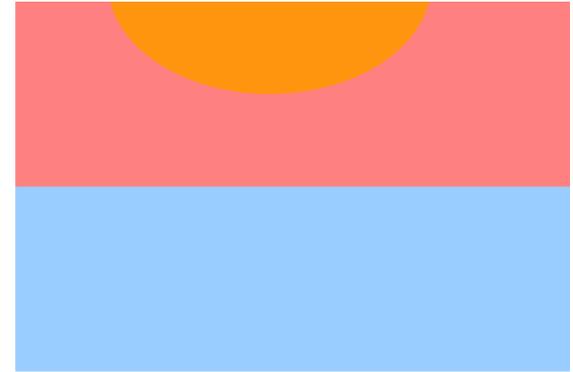
# Thermal instability



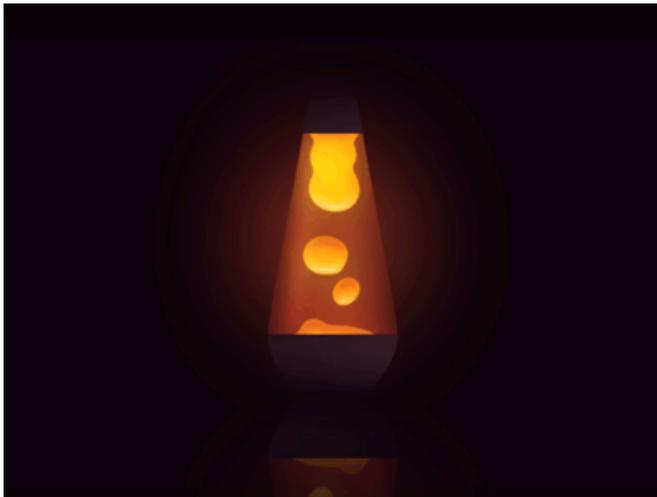
Hotter than the surroundings  
The blob expands and rises



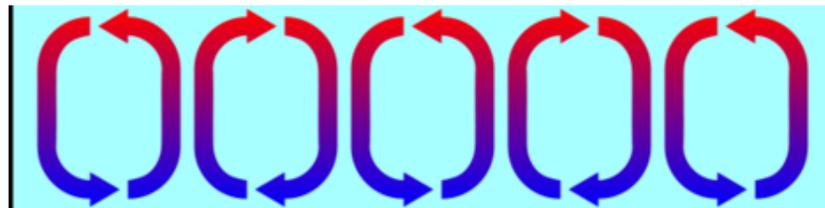
**It rises faster than it cools**



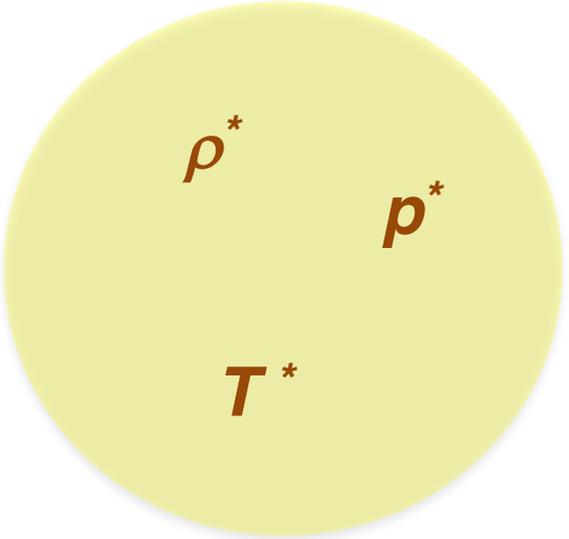
It keeps rising



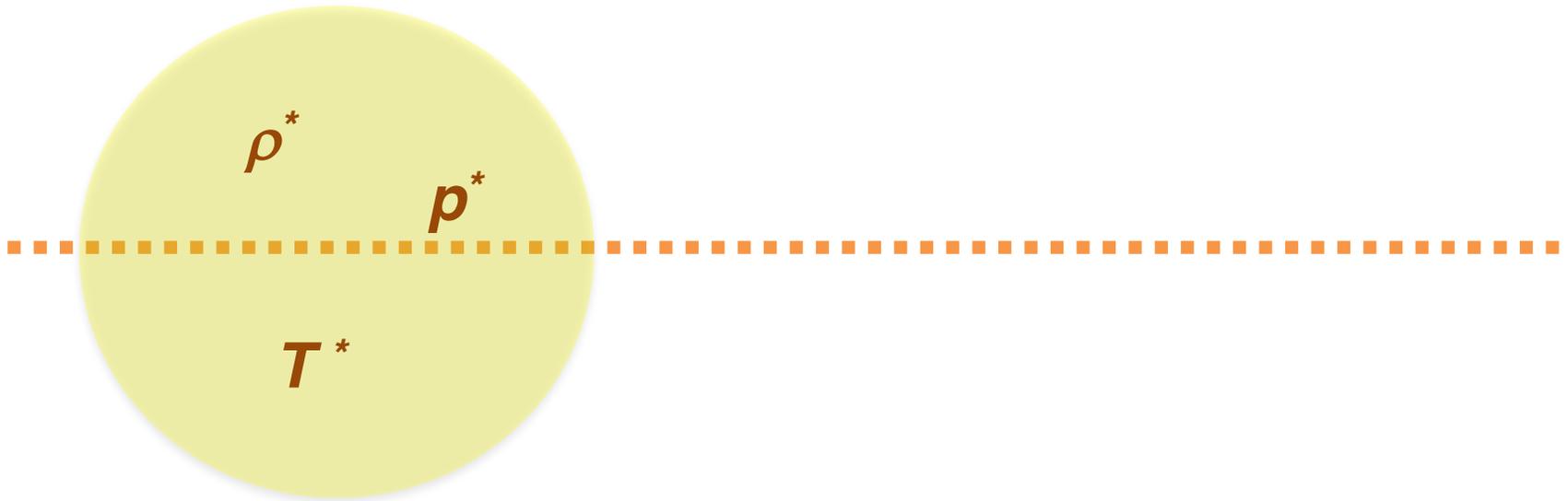
It travels a great distance  
before cooling and sinking



**CONVECTION**



**Cold**

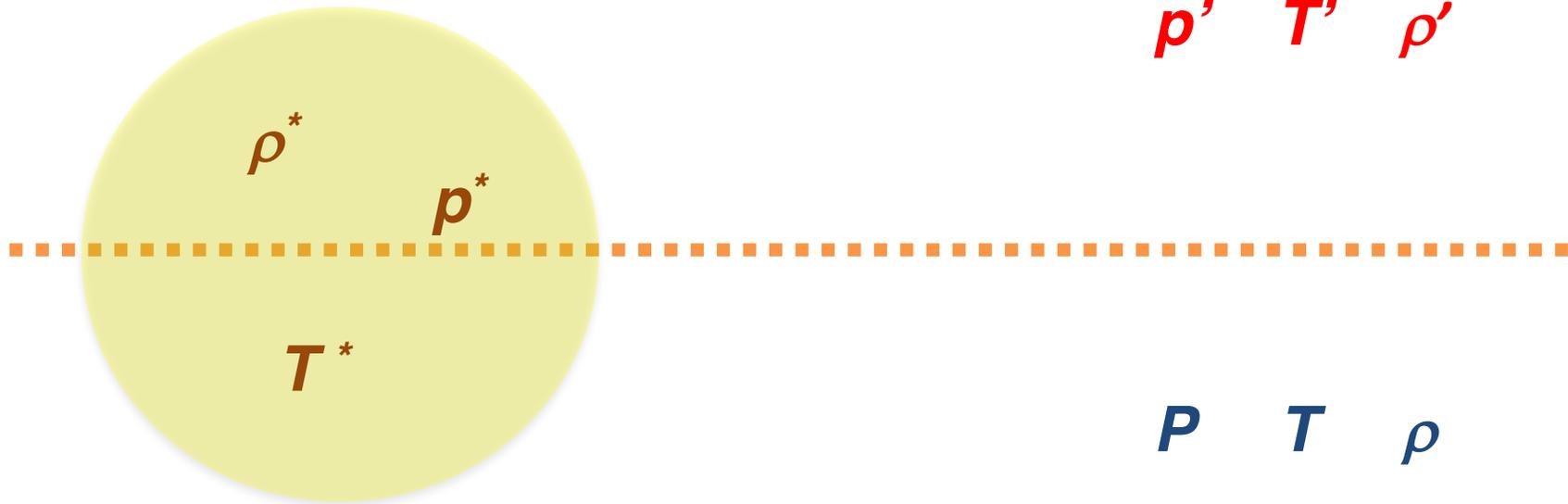


**Hot**



Cold

$\rho'$   $T'$   $\rho'$

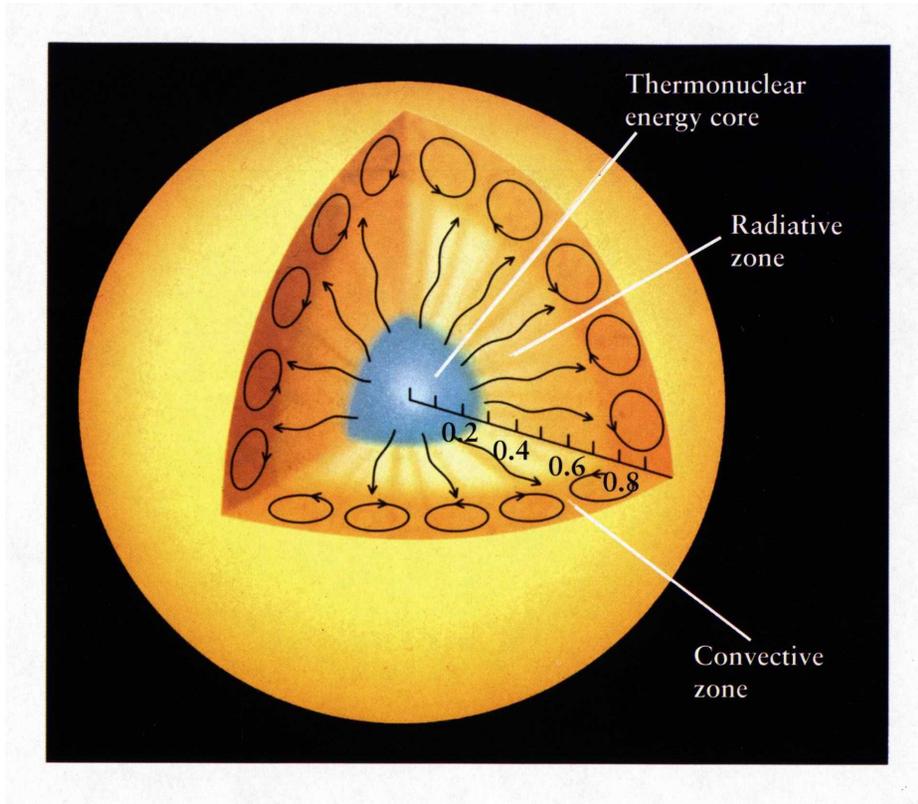


Hot

# Solar Structure

In **stable equilibrium**,  
heat is transported by **radiation**  
(without transport of mass).

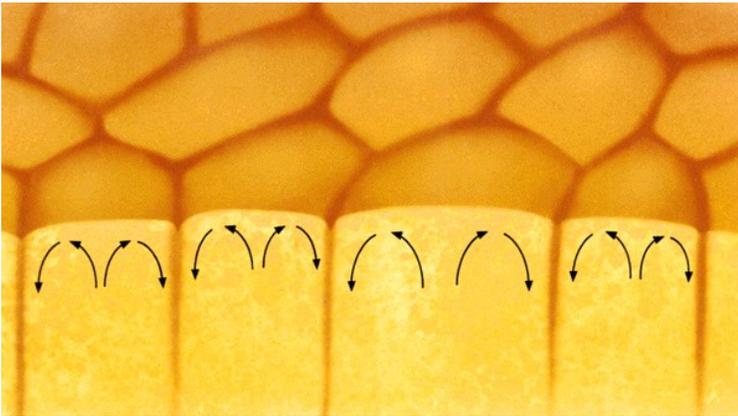
In **unstable equilibrium**,  
heat is transported by **convection**  
(with transport of mass).



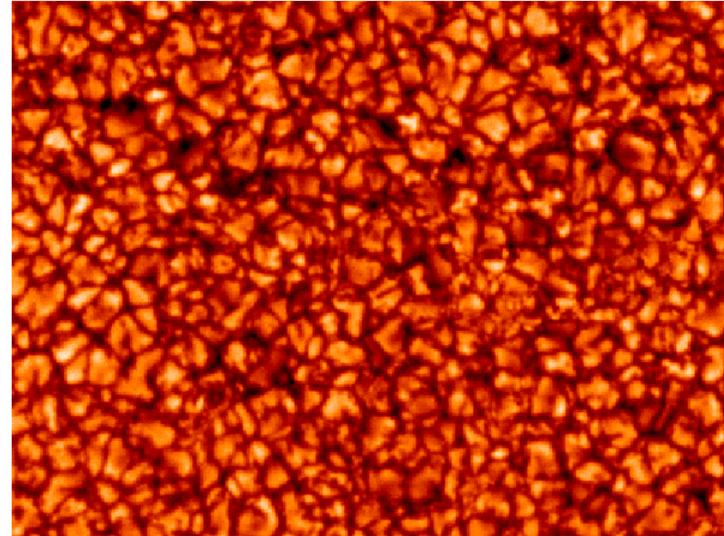
The Sun has a  
**radiative zone**  
**in the interior**

And a  
**convective zone**  
**near the surface**

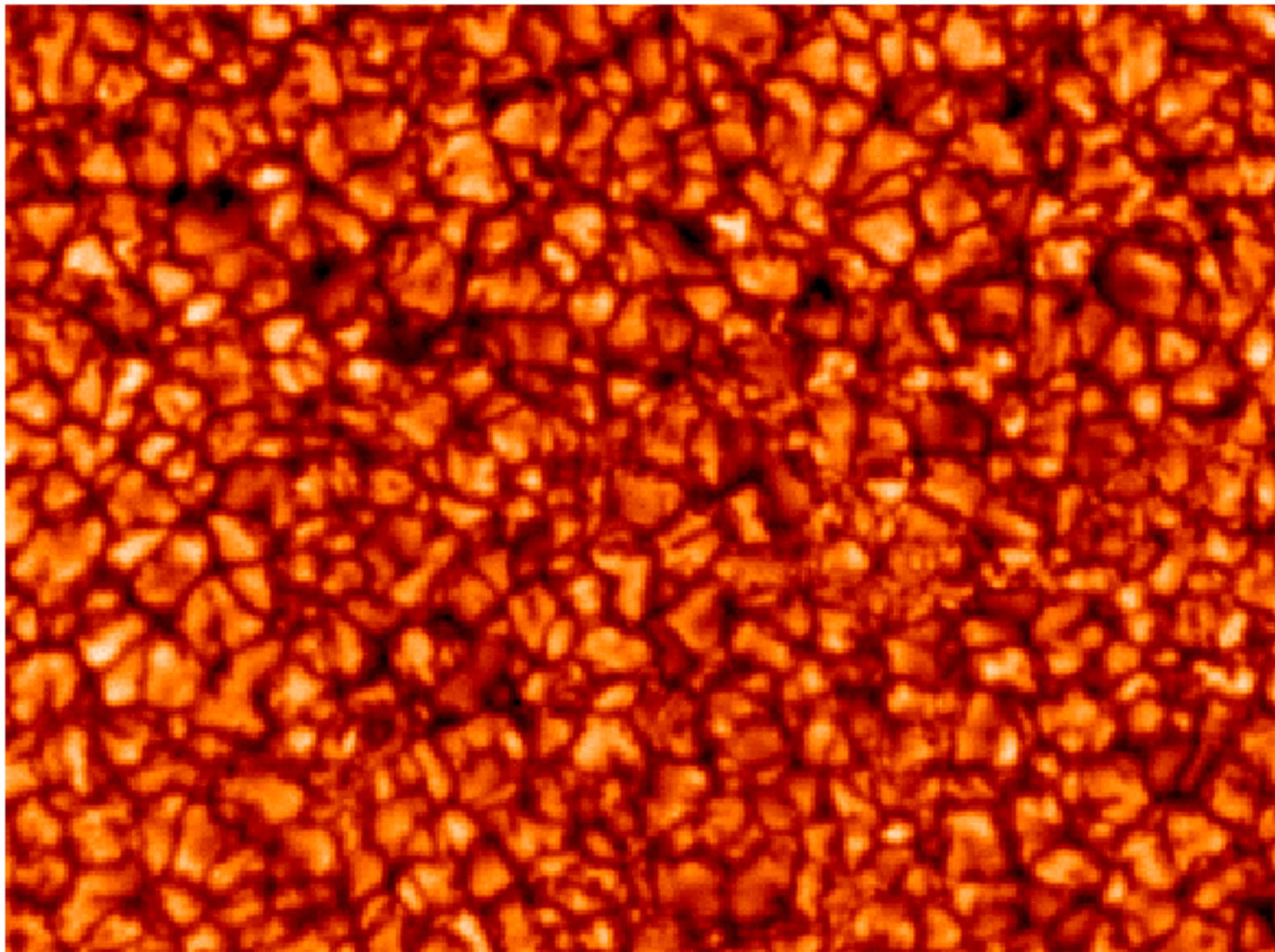
# Granulation

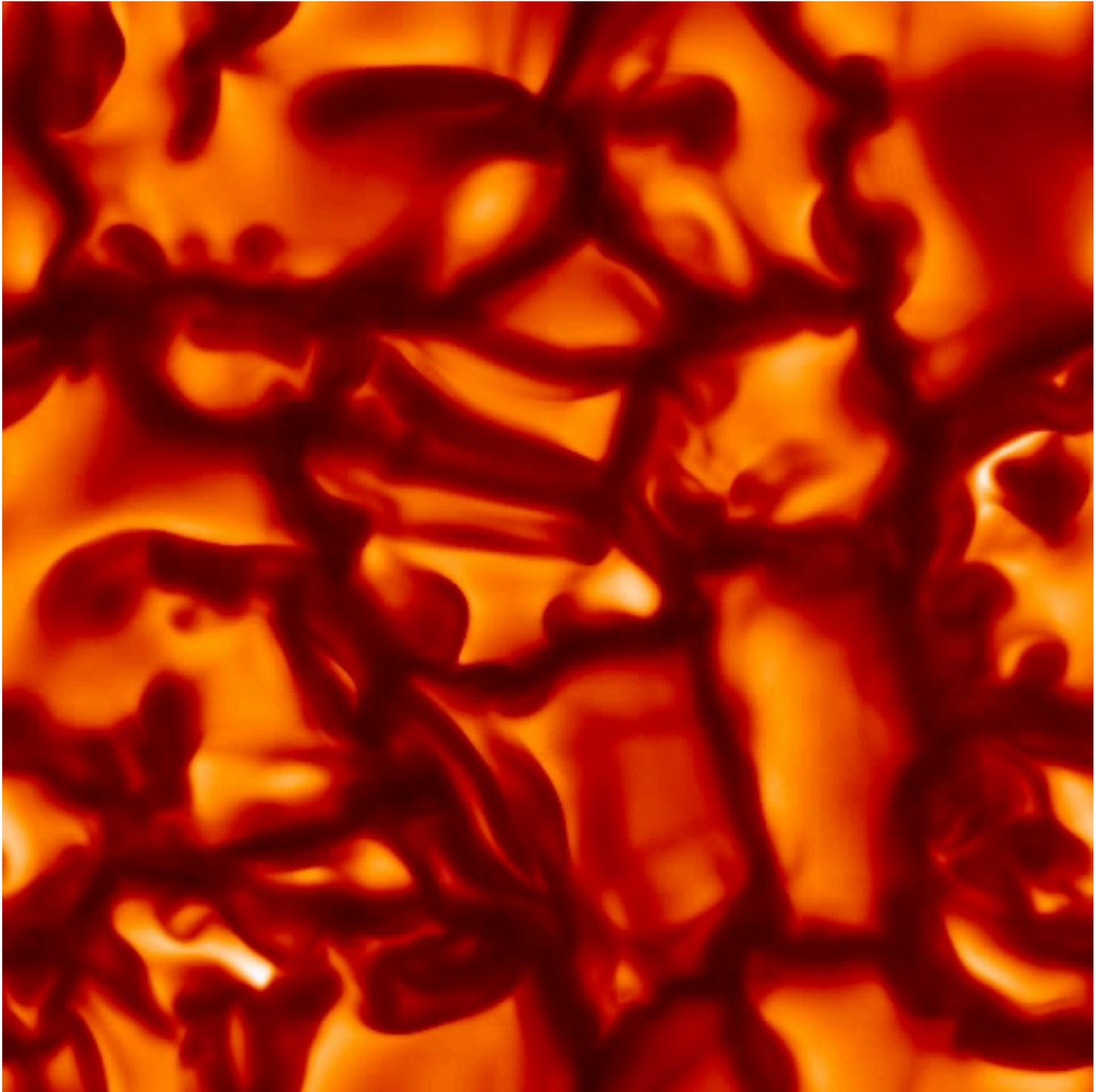


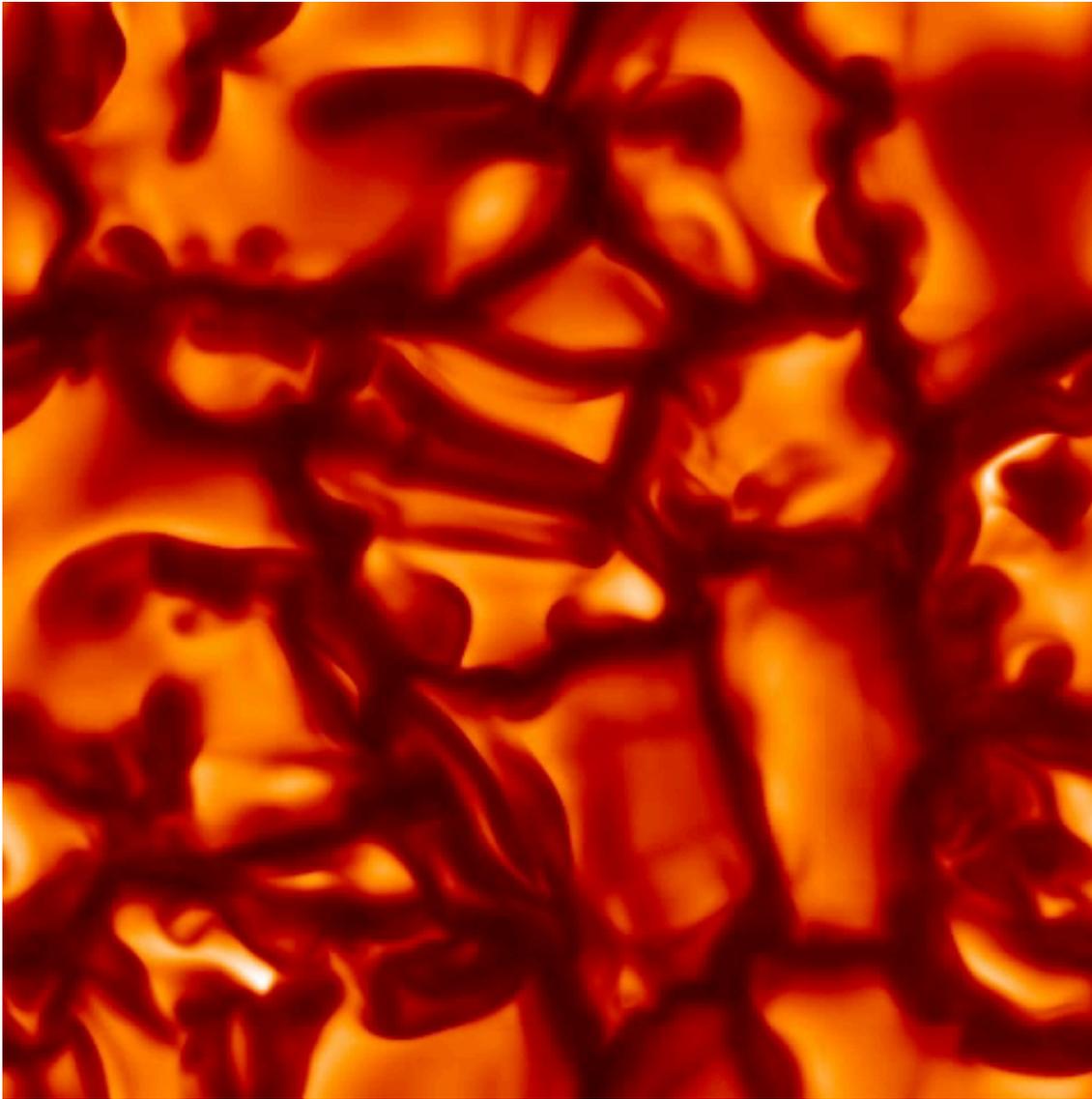
Convective cells



Close-up of the surface of the Sun







Simulation by Remo Collet.

Surface intensity

$$T_{\text{eff}} = 4400 \text{ K}$$

$$L = 4.4 \times 10^6 \text{ km}$$



Atmosphere

Photosphere

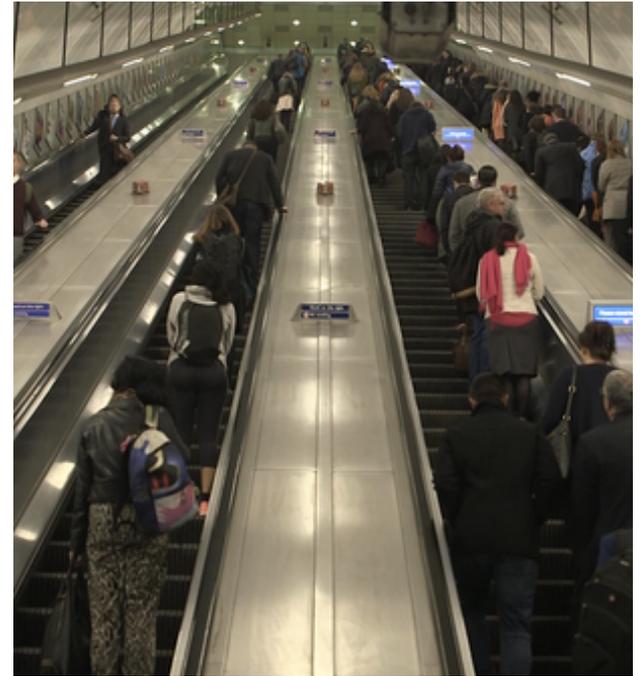
Interior

$$L_z = 2 \times 10^6 \text{ km}$$

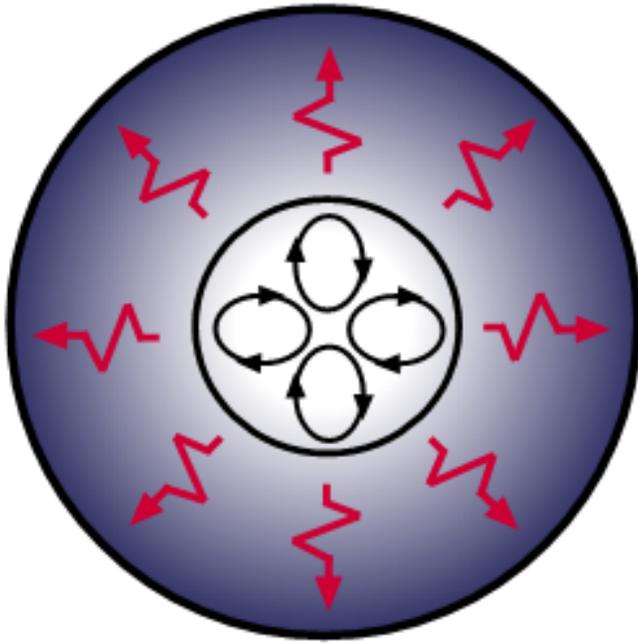
Radiative Zone



Convective Zone

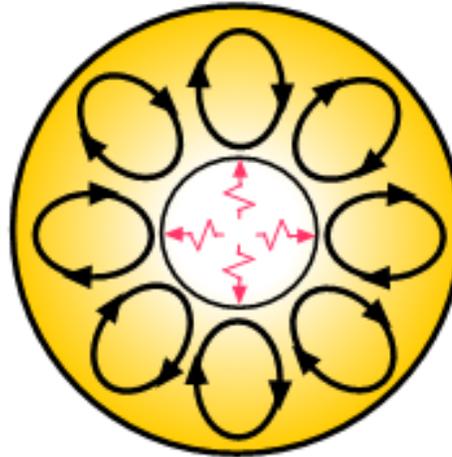


# Stellar Structure



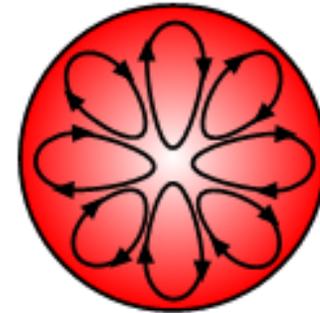
$$M > 1.5$$

Convective Core  
Radiative Envelope



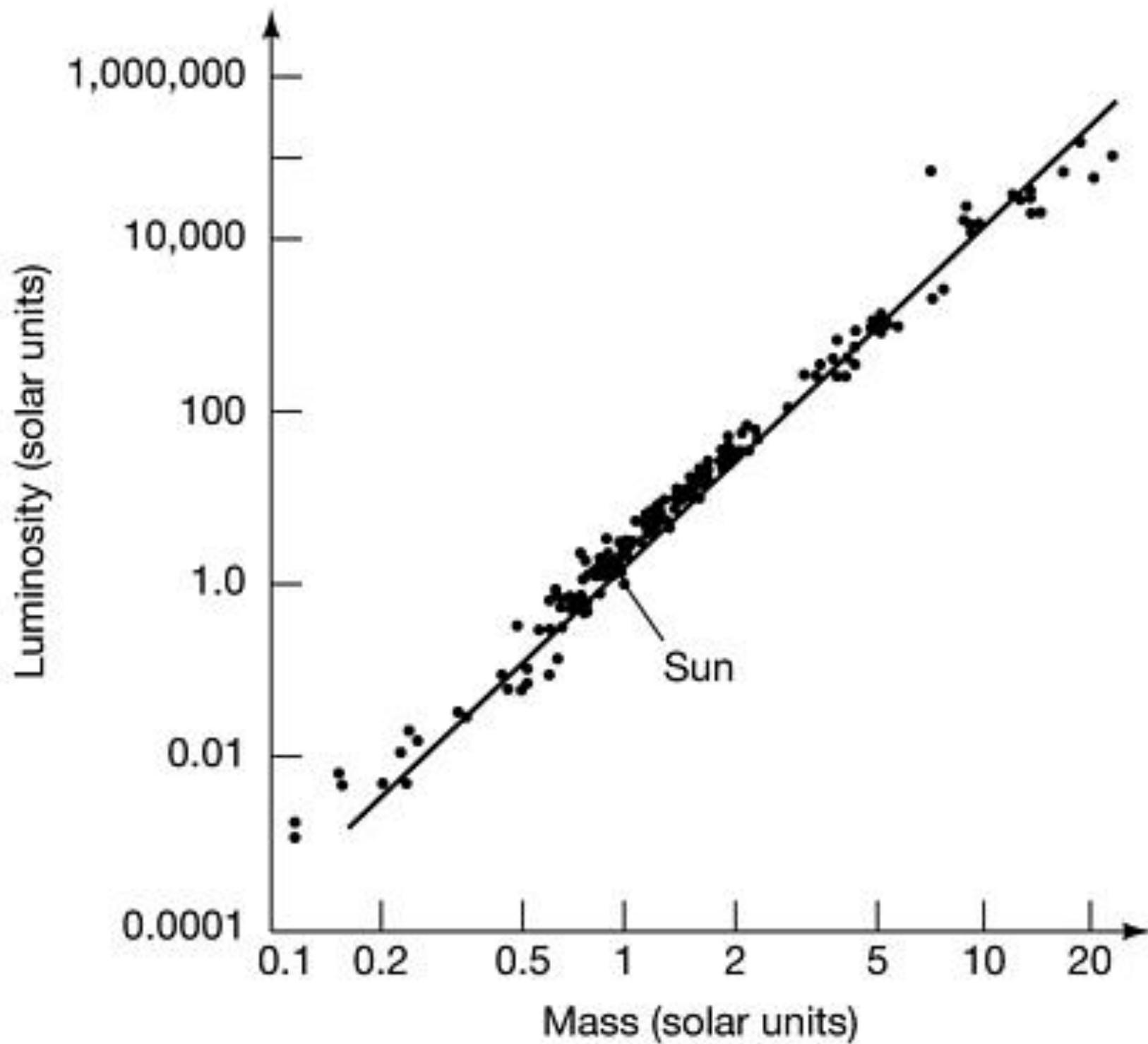
$$0.5 < M < 1.5$$

Radiative Core  
Convective Envelope



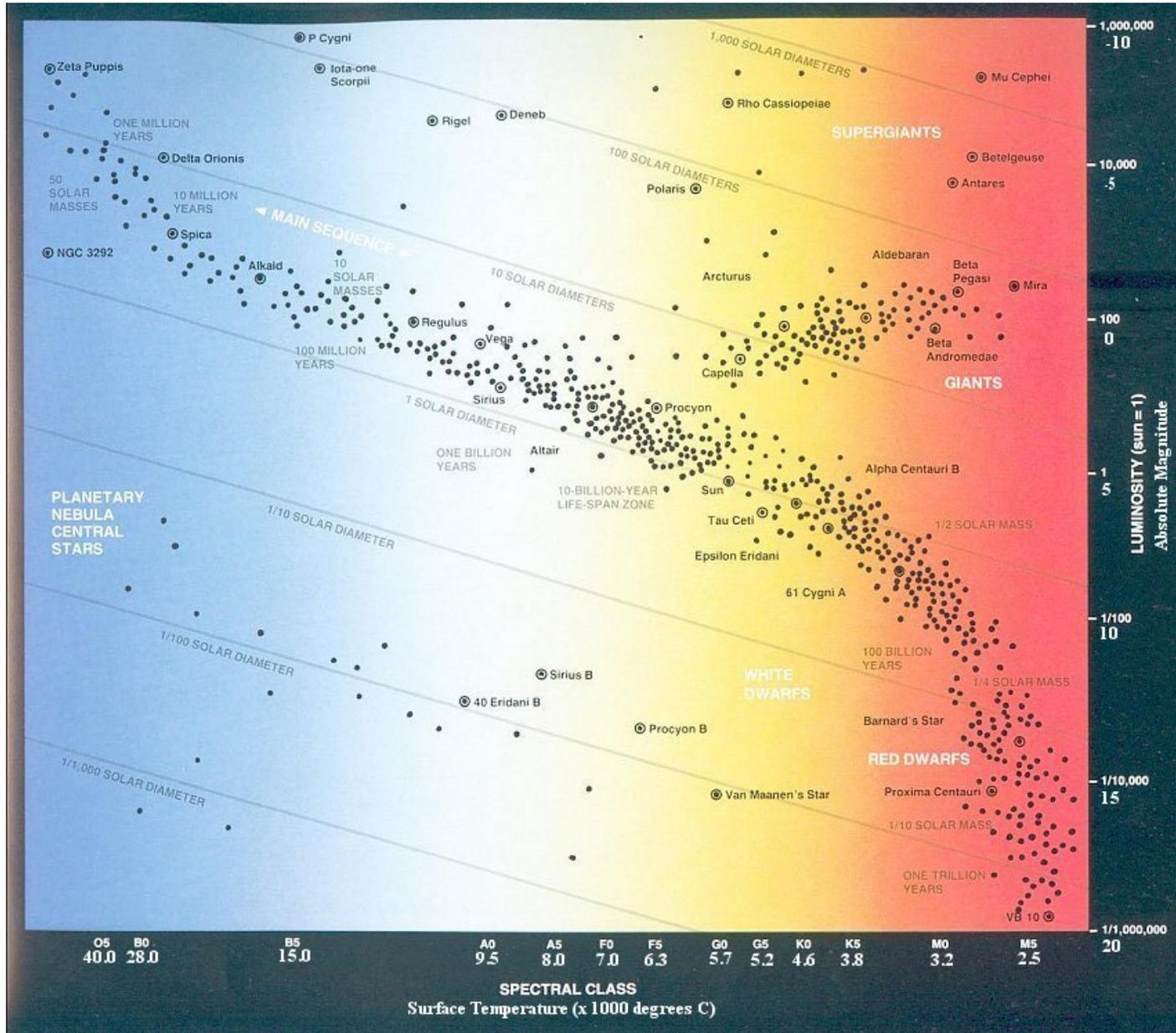
$$M < 0.5$$

Fully Convective



# Main Sequence: Temperature-Luminosity relation

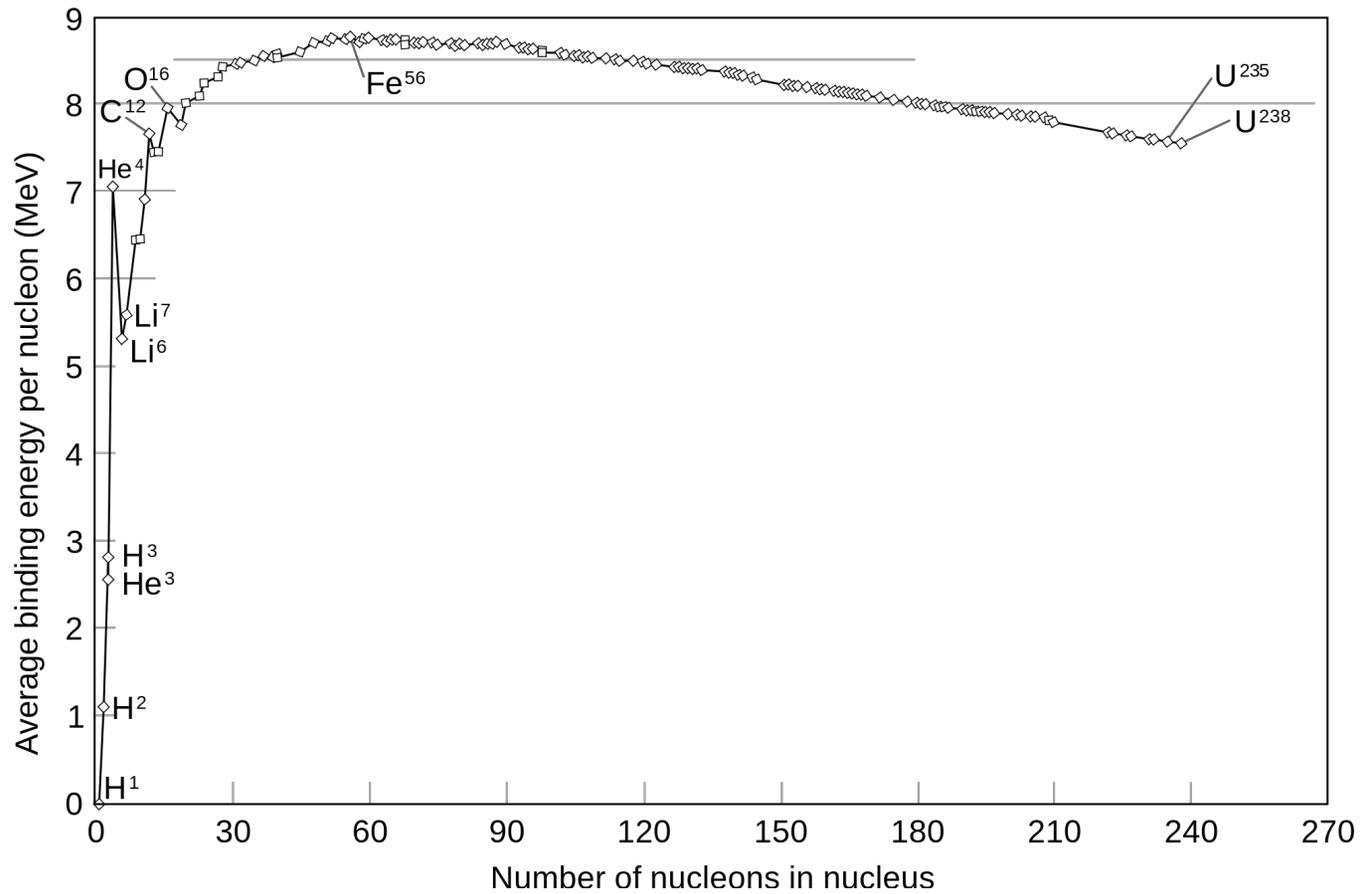
$L \propto T^6$

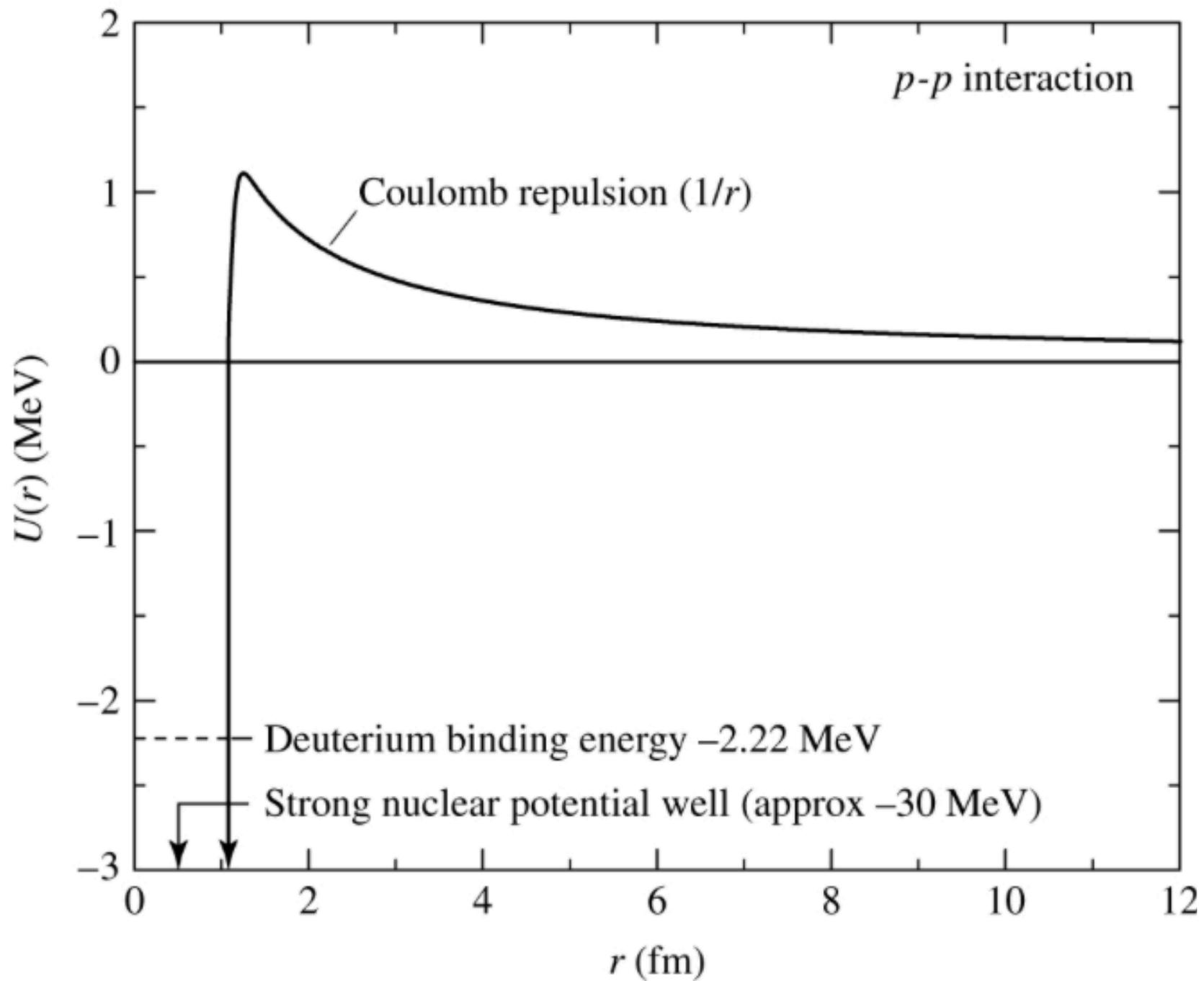


# Nucleosynthesis

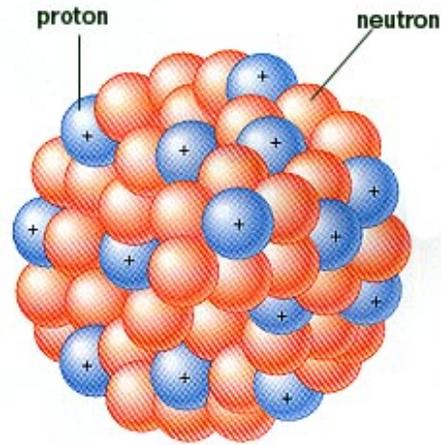
A nucleus is always found to be less massive than the combined mass of nucleons.  
The difference is the **binding energy**.

When a nucleus is formed, this binding energy is released.

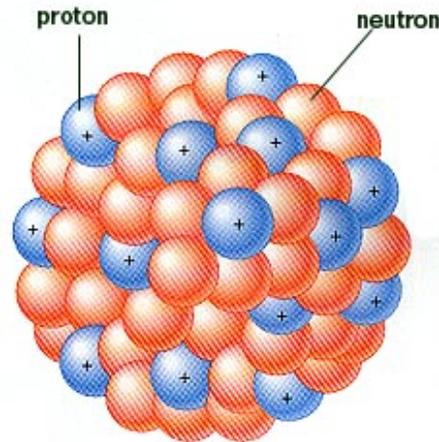




**How is an atomic nucleus bound together  
if the protons are like-charged?**



**How is an atomic nucleus bound together  
if the protons are like-charged?**



**Another force of nature exists at nuclear distances  
Not Gravity. Not Electromagnetism.**

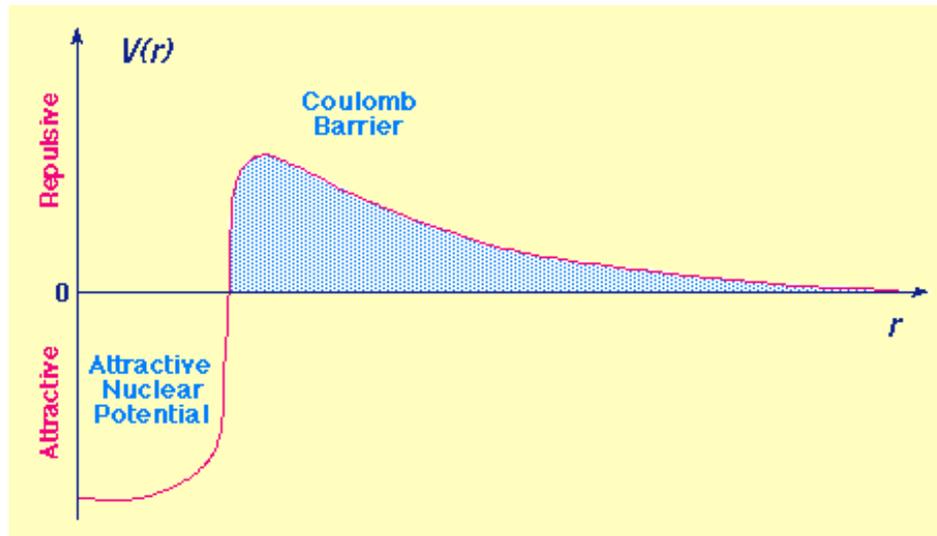
***Strong Force***

**The Coulomb force (EM) between protons is repulsive,  
but the strong force between protons is attractive!**

# The Coulomb Barrier

Protons are like-charged and thus repel each other

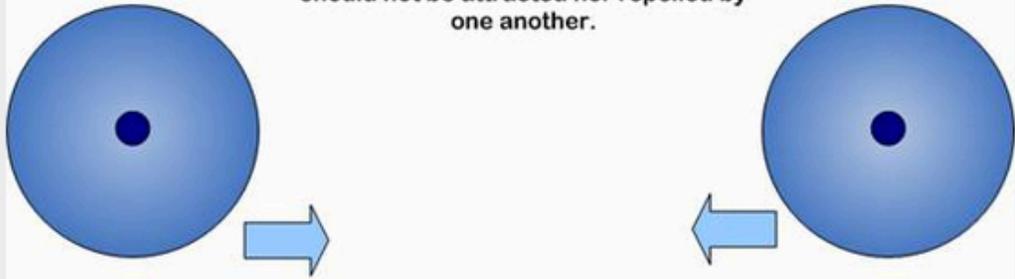
You need to get them really close so that nuclear forces start to operate



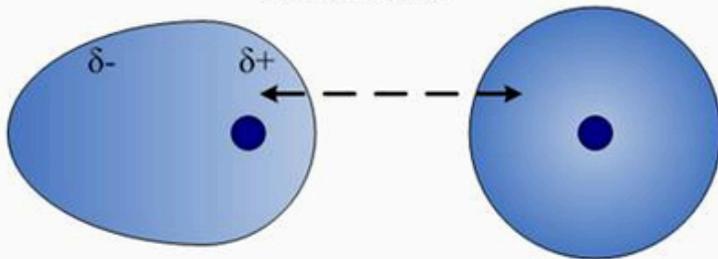
Really close means: packed together  
fast speeds

- HIGH DENSITY
- HIGH TEMPERATURE

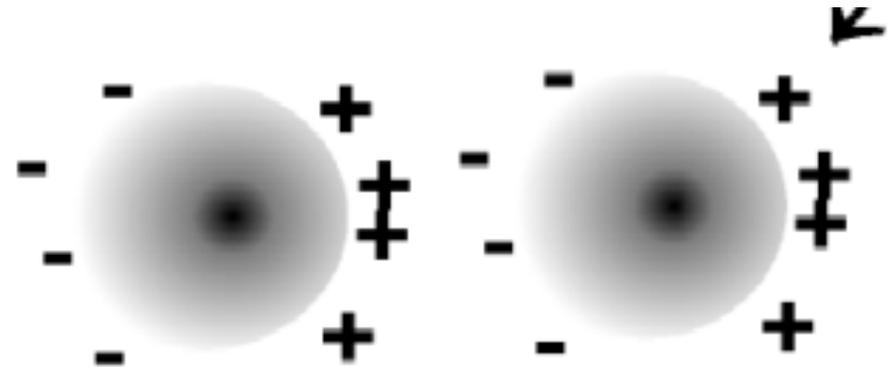
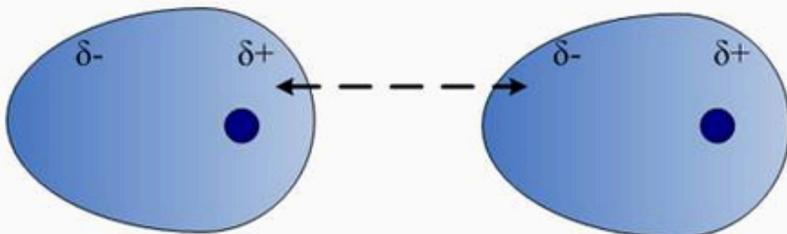
Roughly spherical atoms of an ideal gas should not be attracted nor repelled by one another.



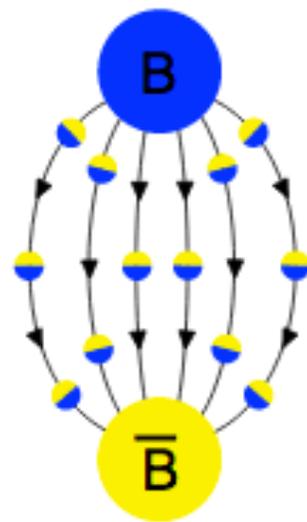
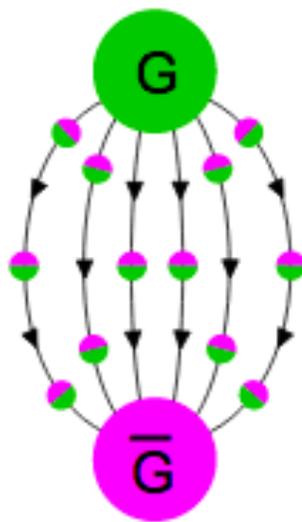
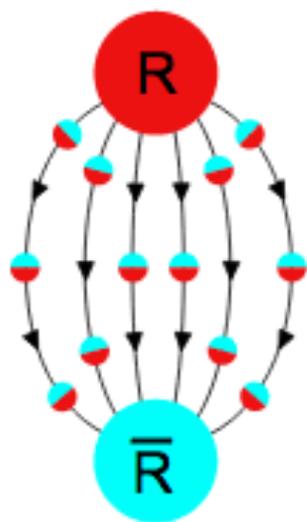
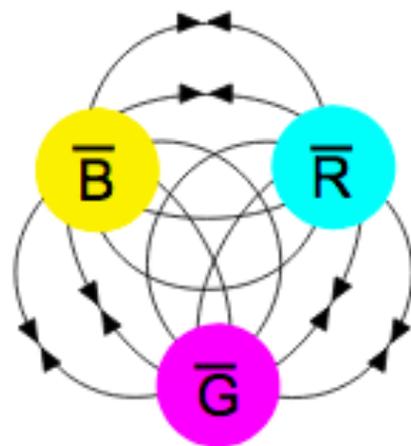
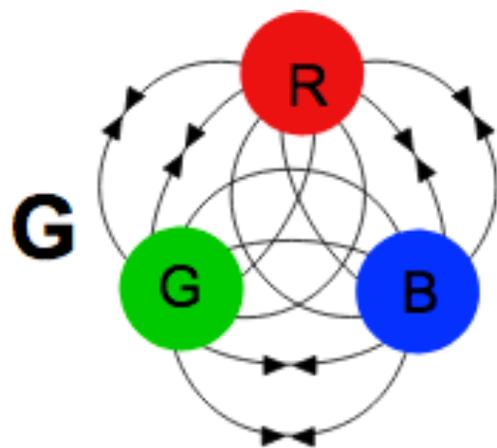
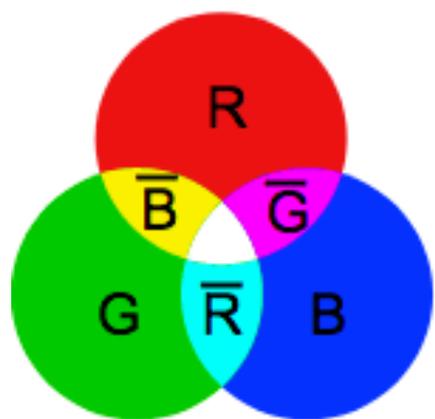
A real gas atom can have an instantaneous dipole. Partial charges on one atom cause a neighboring atom to distort due to the electrostatic attractions/repulsions of their electron clouds.



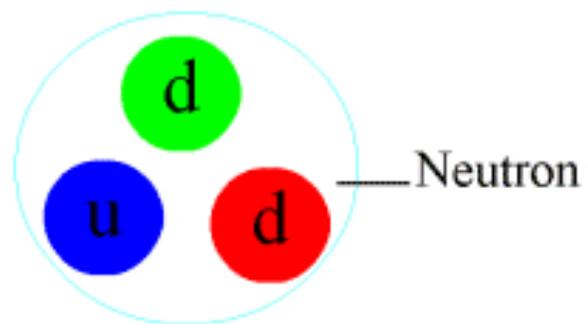
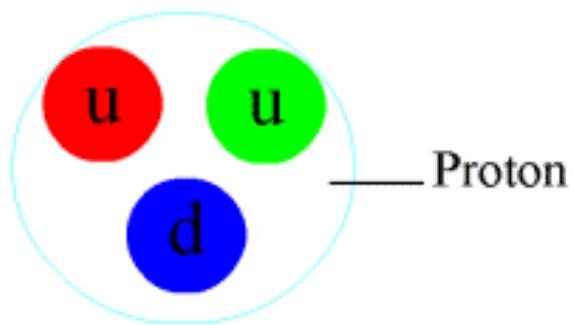
Attractions between opposite partial charges of neighboring induced dipoles cause atoms to "stick together" for a very short time.



**Van der Waals  
interaction**

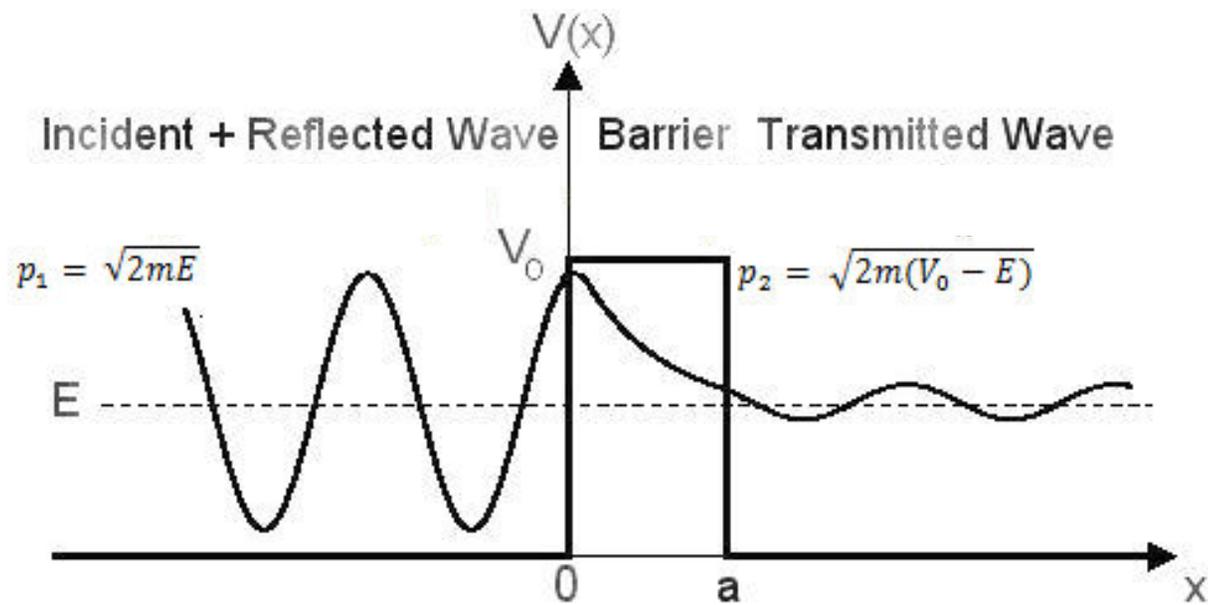
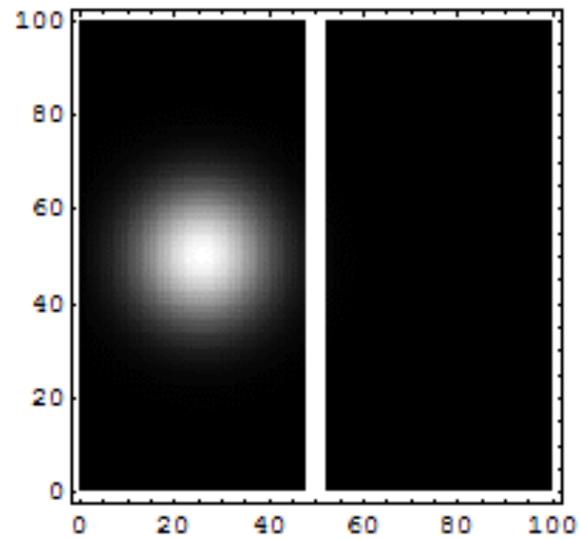


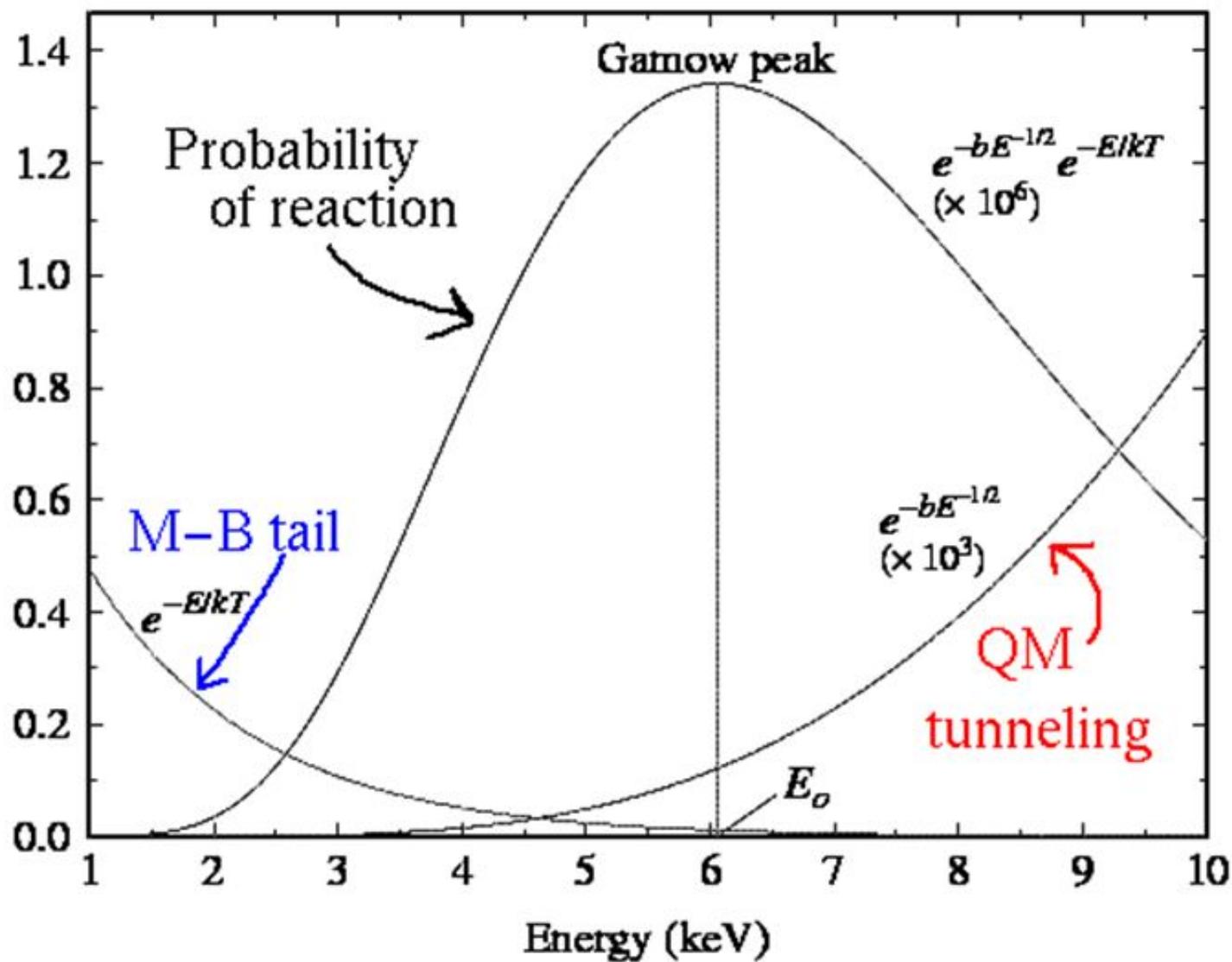




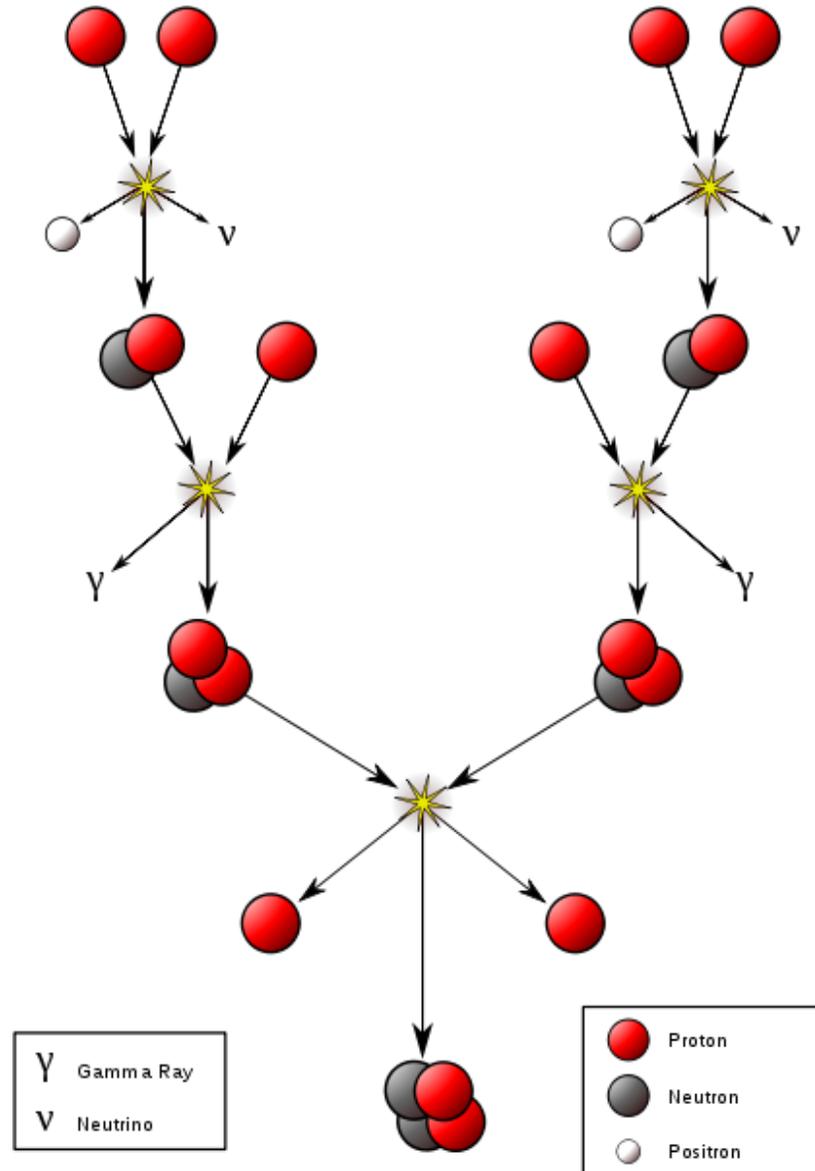
$$V_{\text{Yukawa}}(r) = -g^2 \frac{e^{-\mu r}}{r},$$

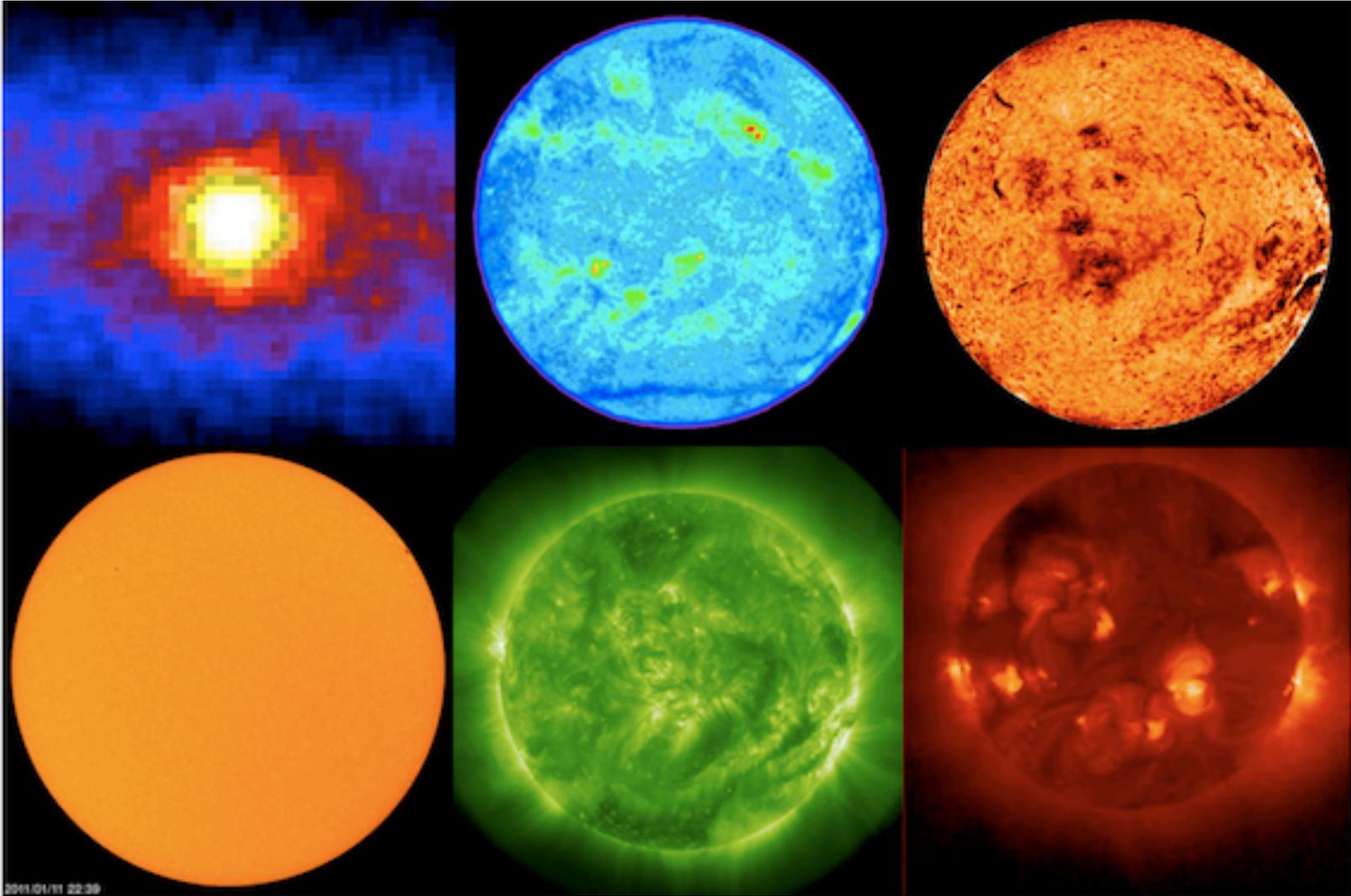
# Quantum Tunneling



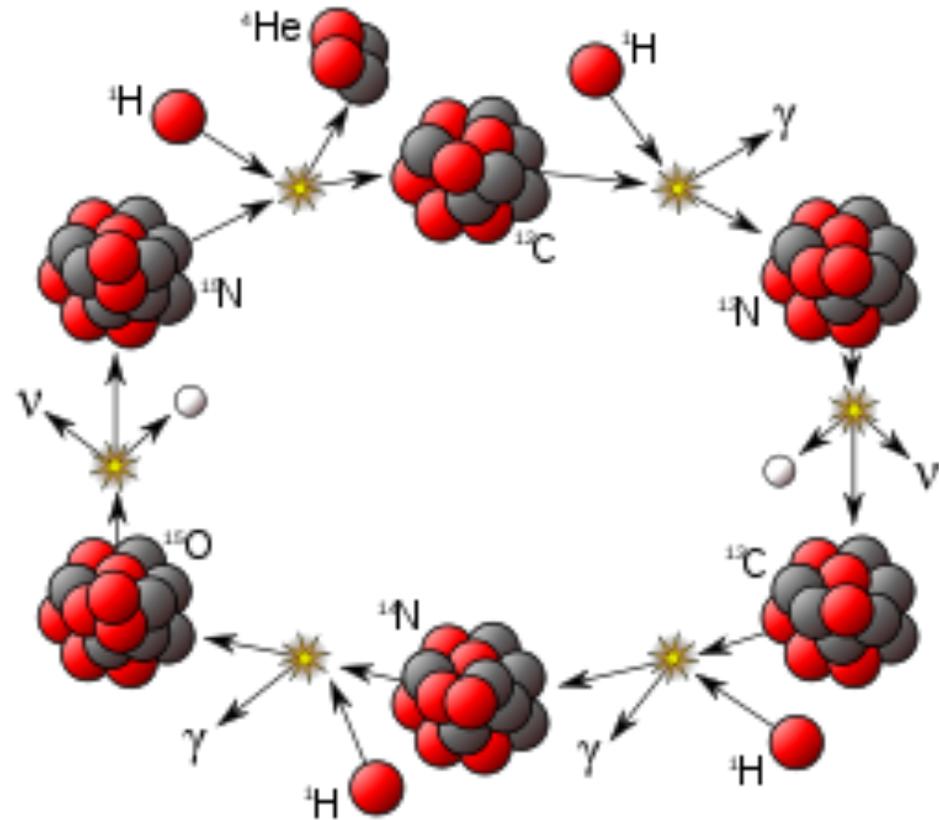
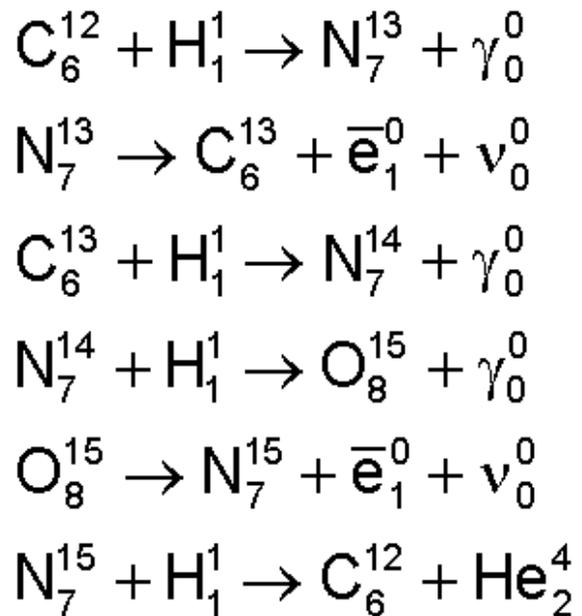


# Proton-proton chain





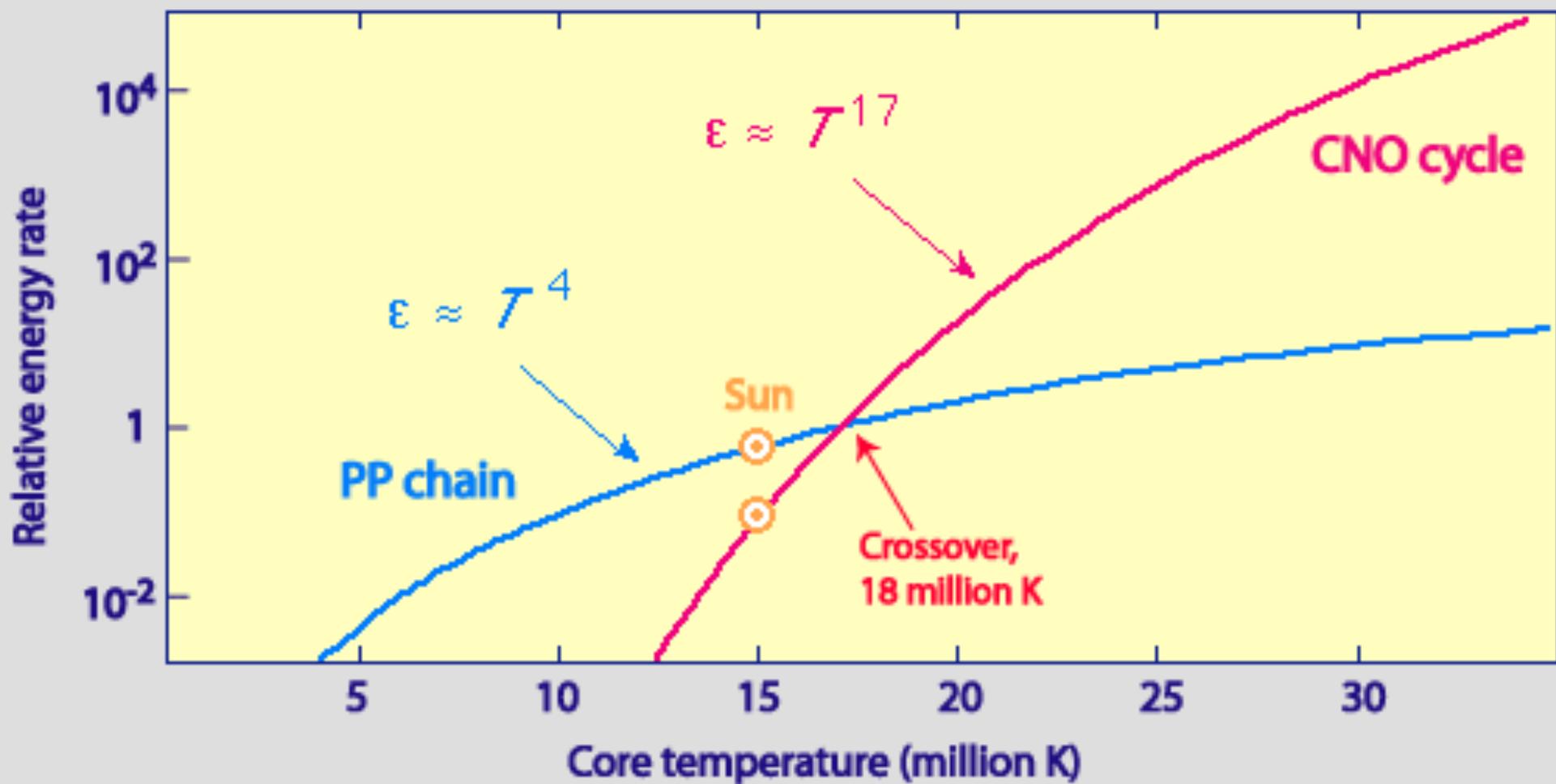
# CNO cycle



	Proton	$\gamma$	Gamma Ray
	Neutron	$\nu$	Neutrino
	Positron		

## End of cycle:

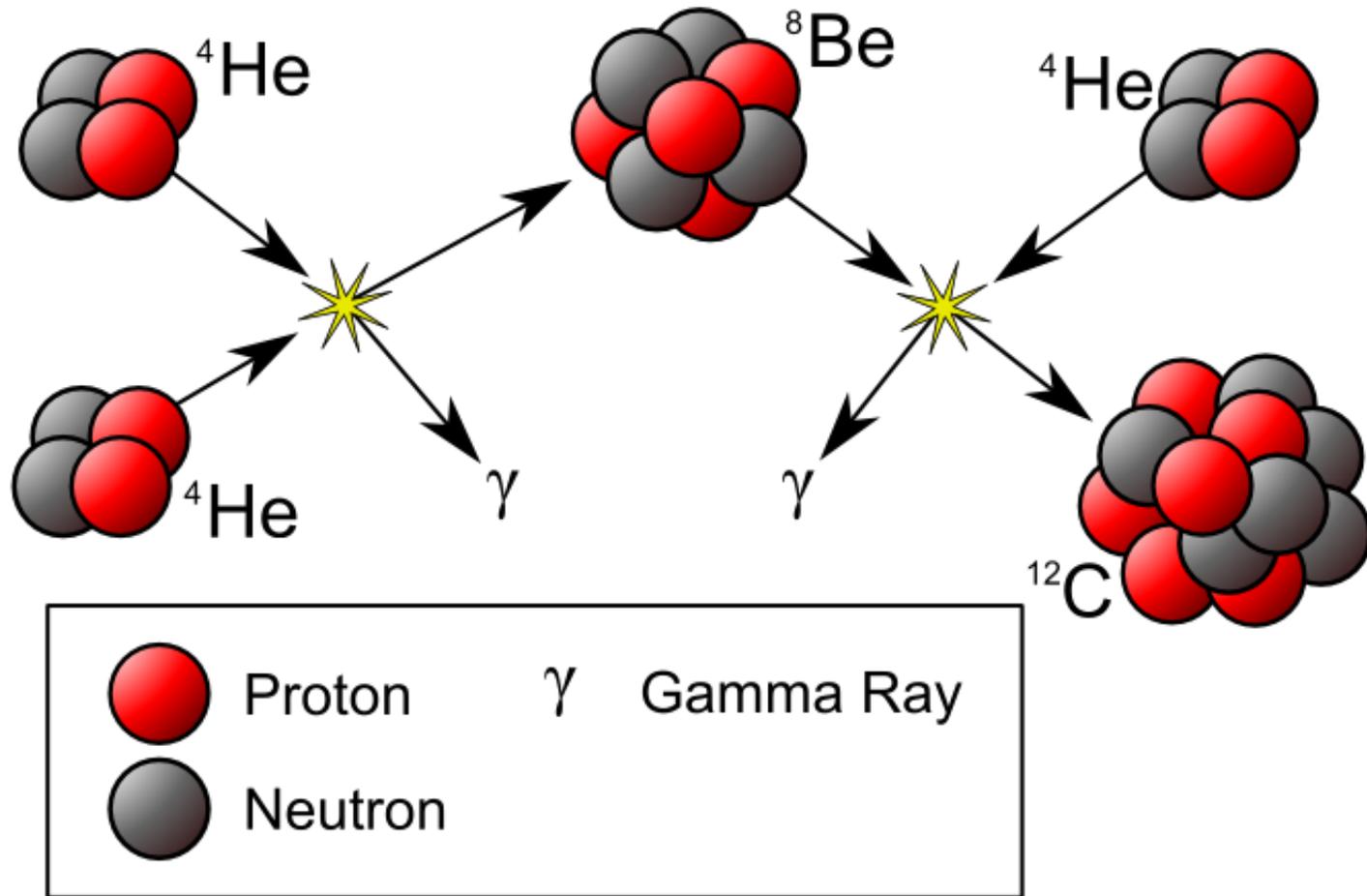
4 H burned into He  
C used as catalyst







# Triple alpha



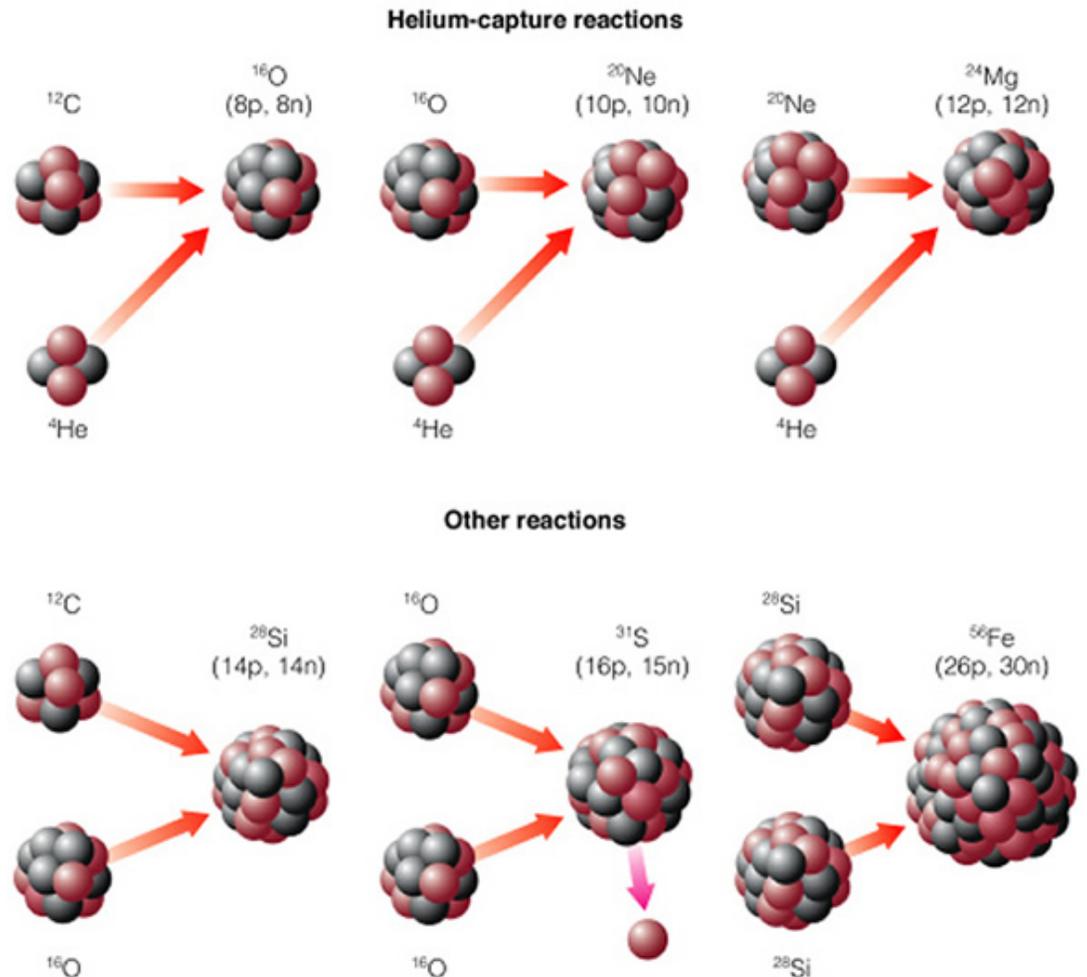
# Alpha Ladder

Carbon  $\rightarrow$  O, Ne, Mg (600 million K)

Neon  $\rightarrow$  O, Mg (1.5 Billion K)

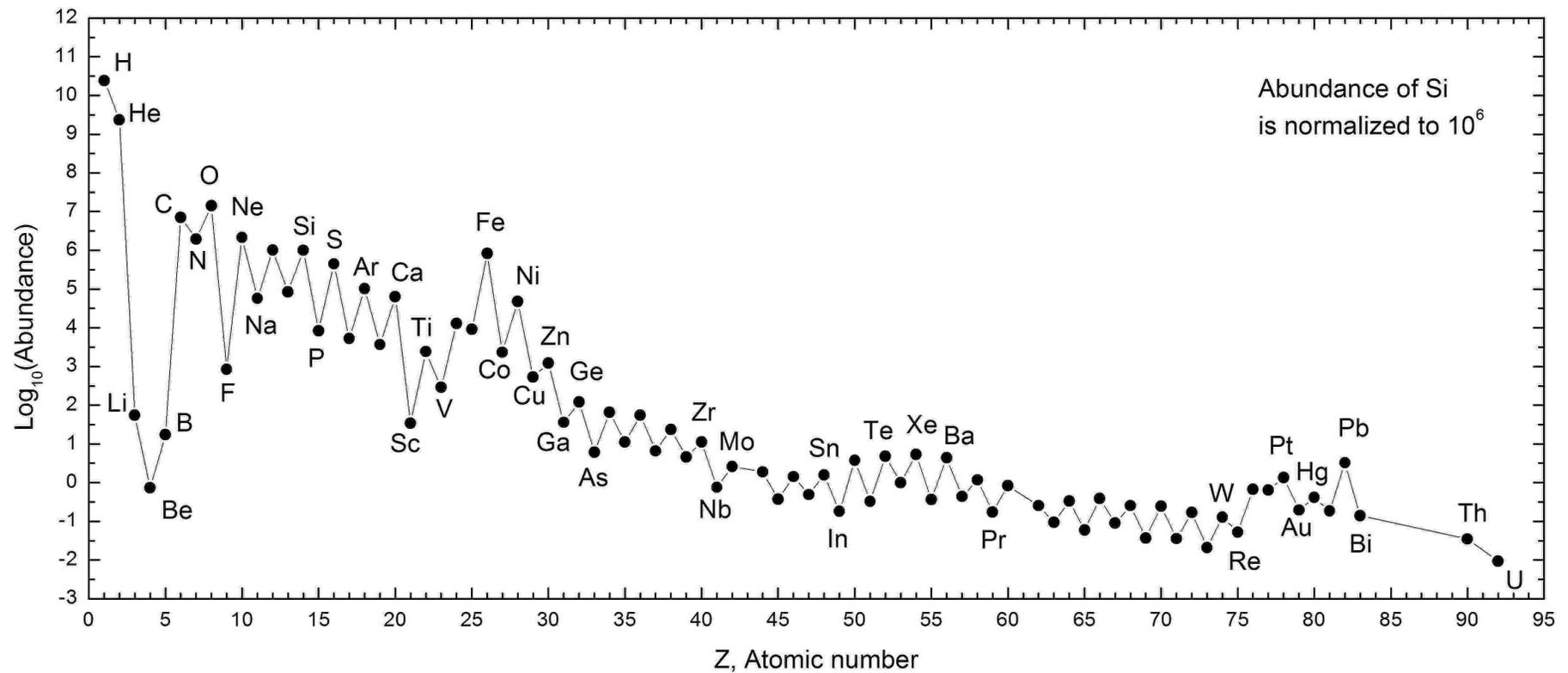
Oxygen  $\rightarrow$  Si, S, P (2.1 Billion K)

Silicon  $\rightarrow$  Fe, Ni (3.5 Billion K)

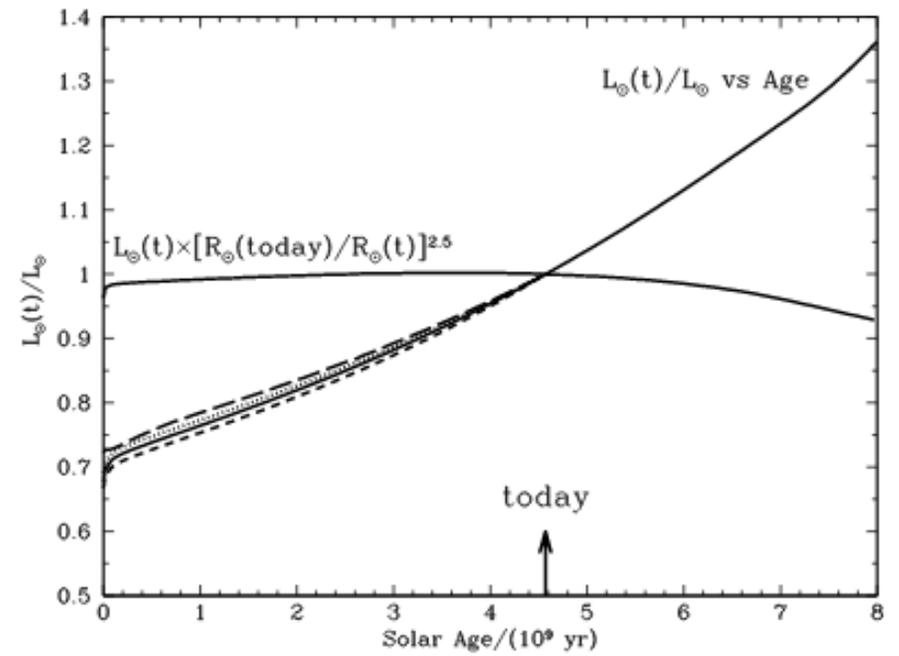
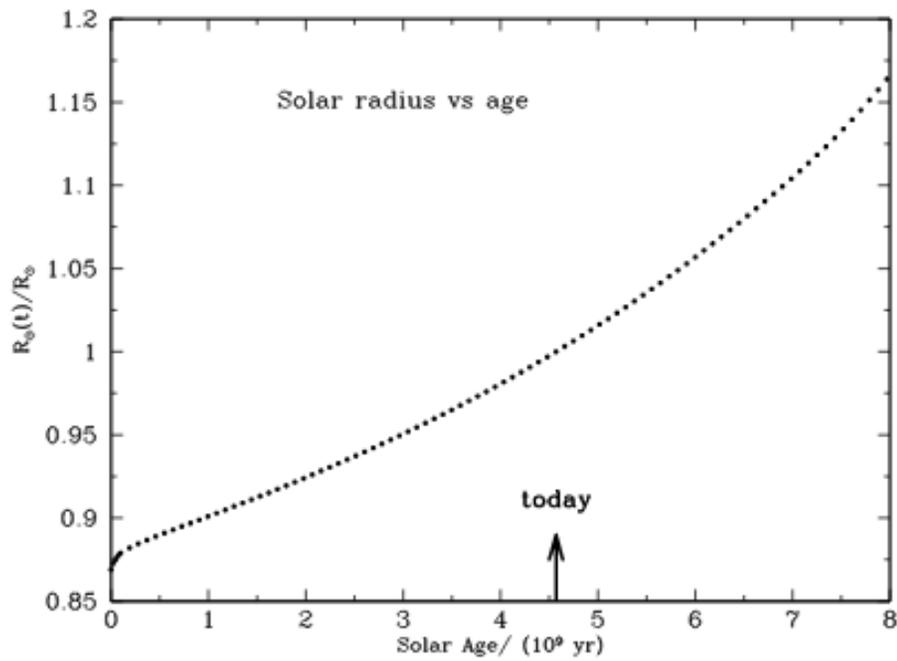


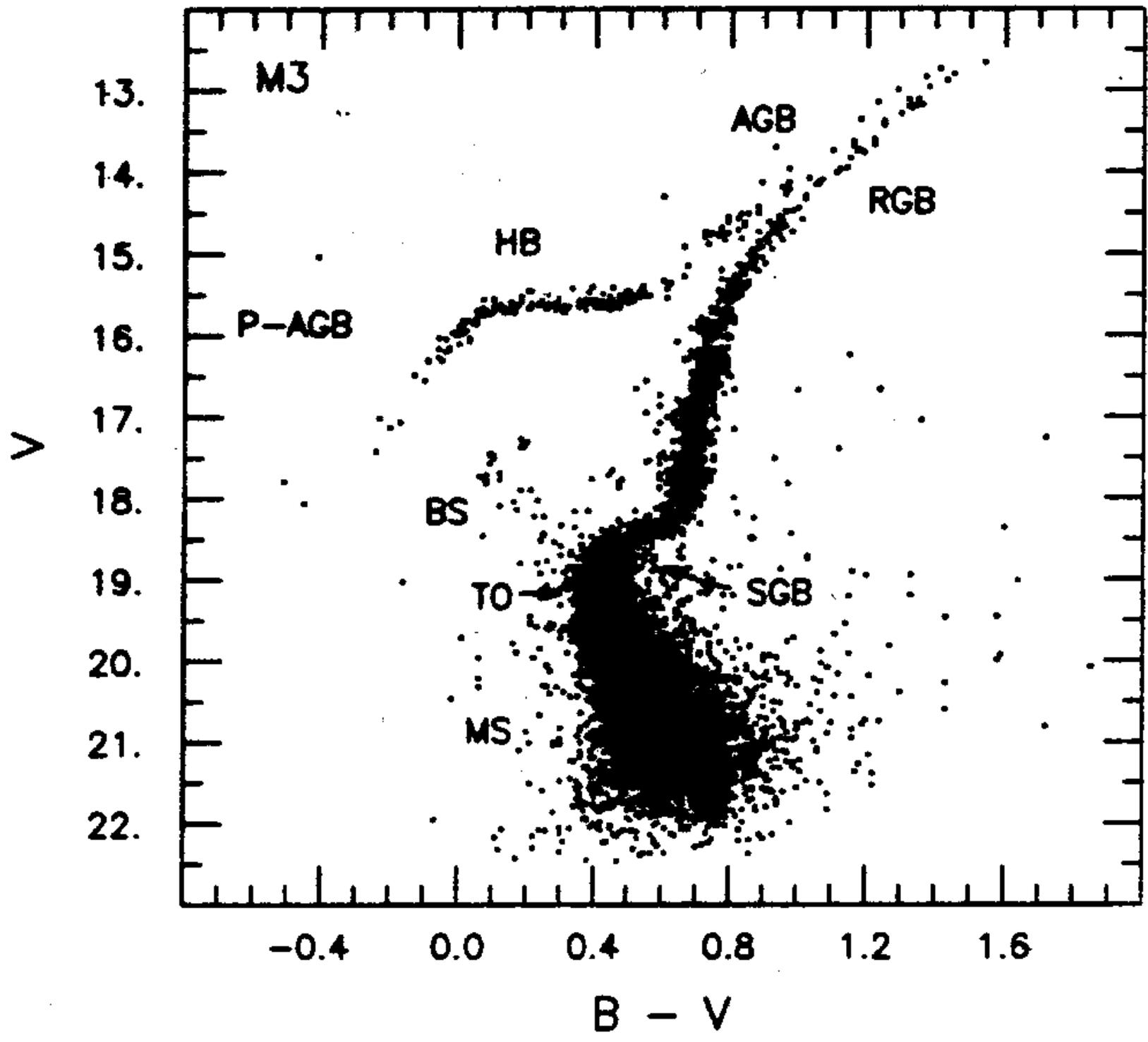
# The Sun's abundance pattern

Elements with even atomic number are more abundant than those with odd

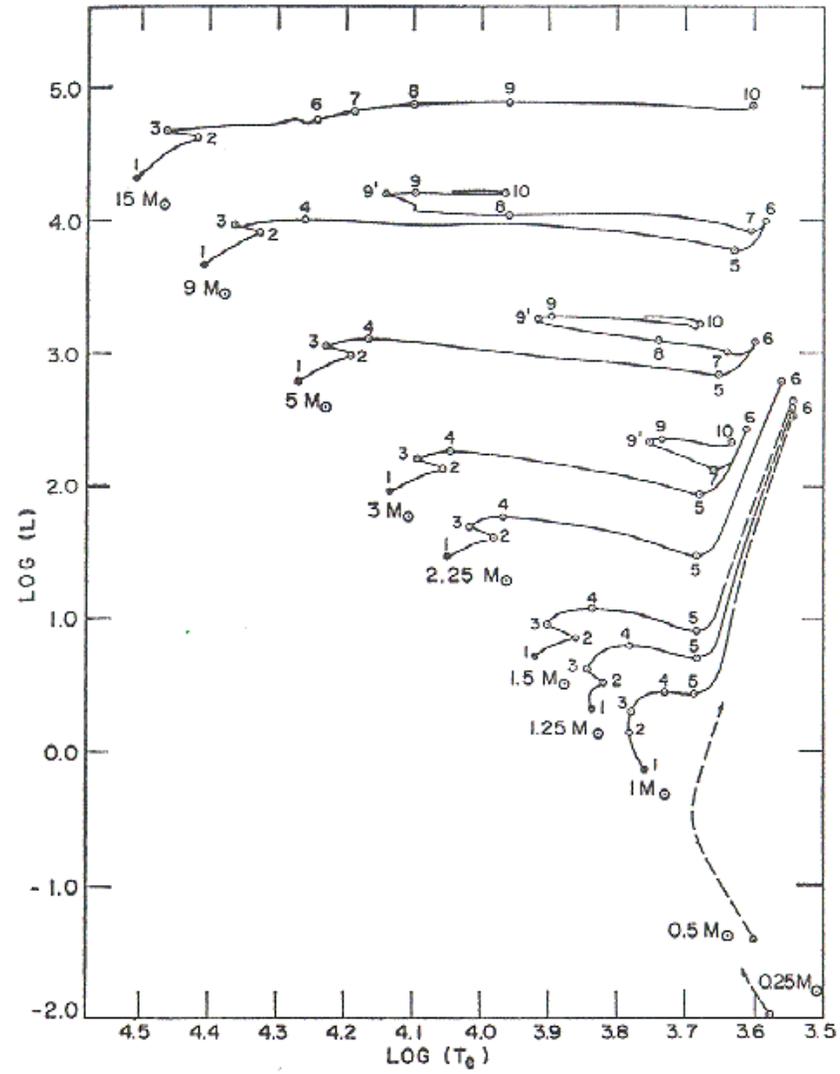


# Solar evolution in the main sequence

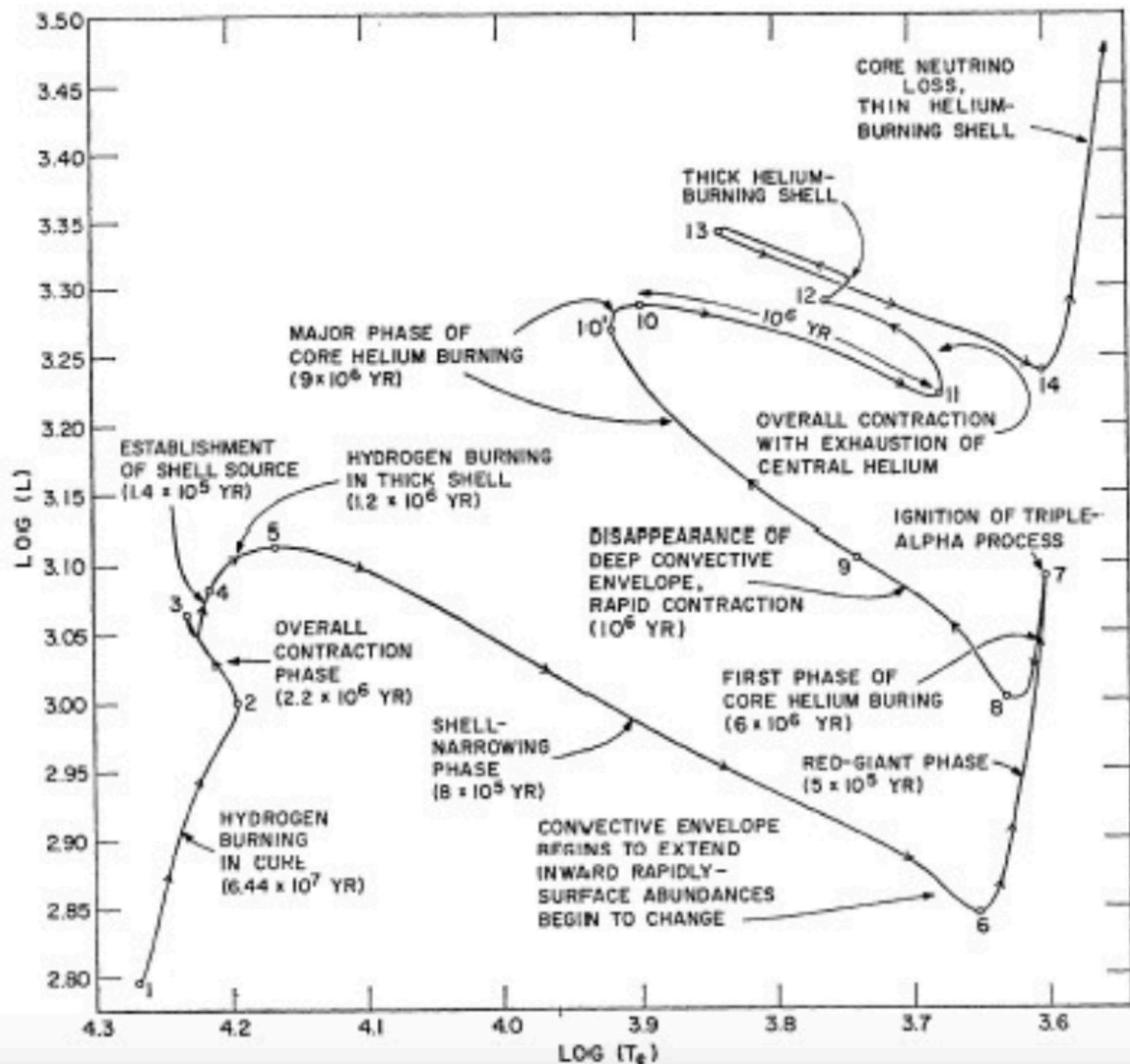




# Evolutionary tracks



Schaller et al. (1992)



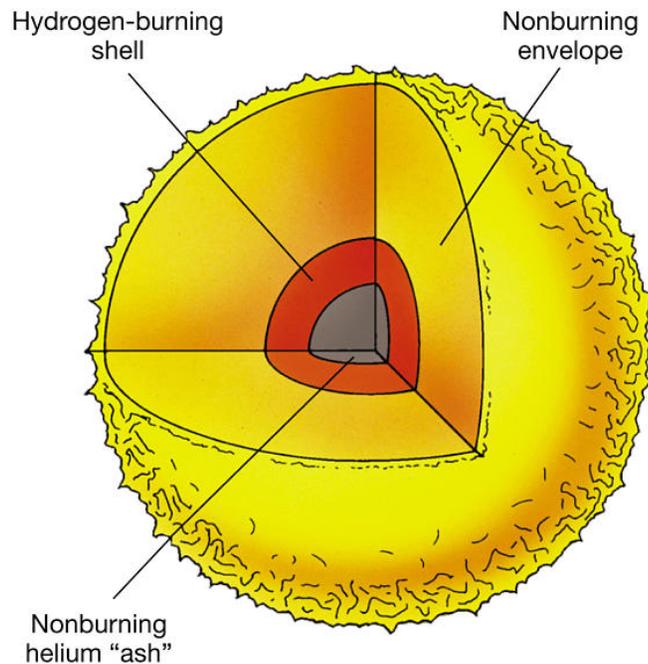
# Hydrogen gone in the core

Star stops producing energy.

The star **contracts** and **heats** up.

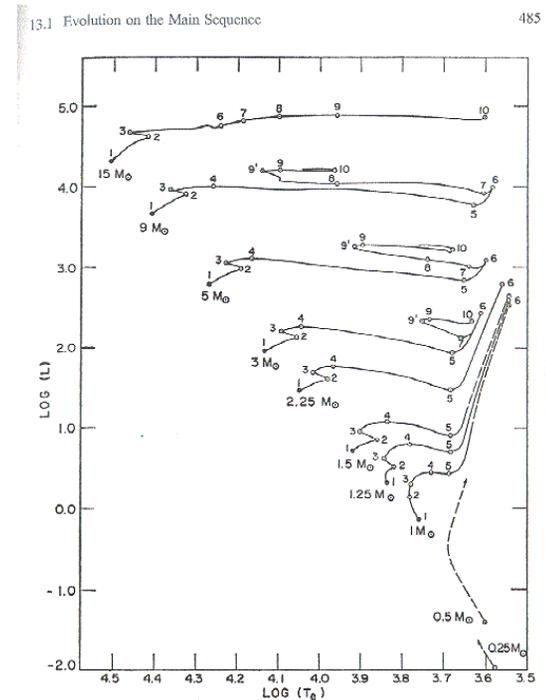
Eventually, the temperature becomes high enough to **burn hydrogen around the Helium core**

## Hydrogen shell burning



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The star reaches  
the subgiant branch



# Red giant branch

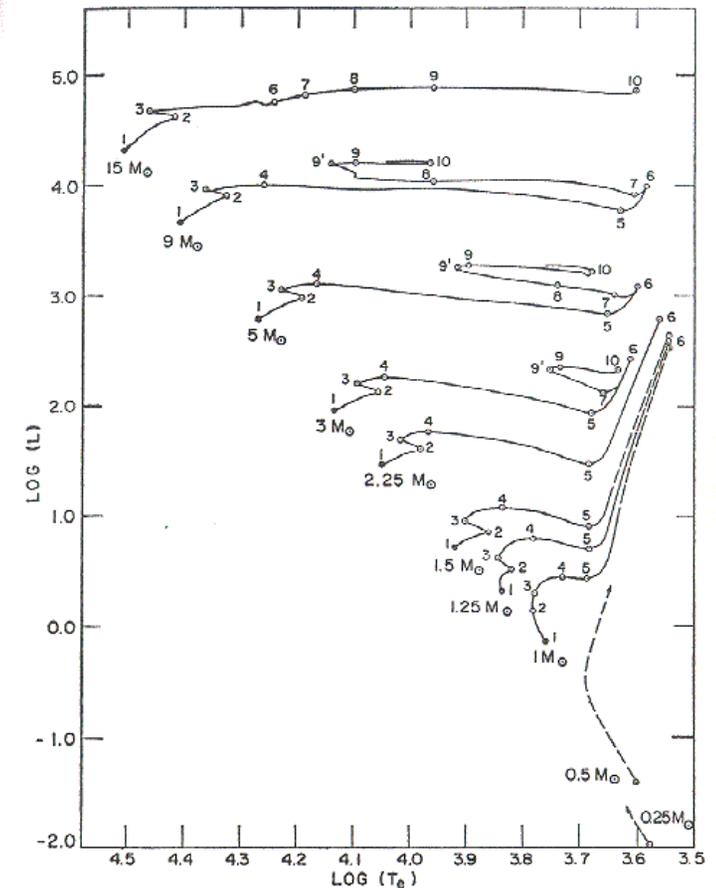
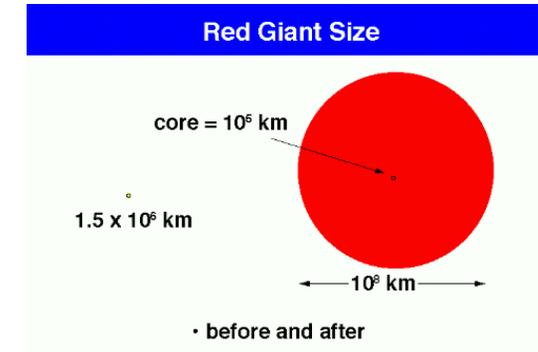
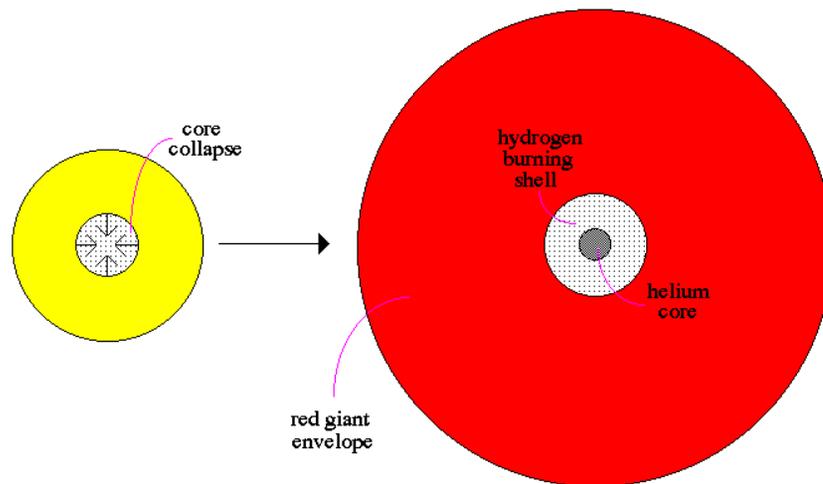
Hydrogen shell burning involves:

*More fuel than in MS-hydrogen burning  
Higher temperatures  
(thus more efficient)*

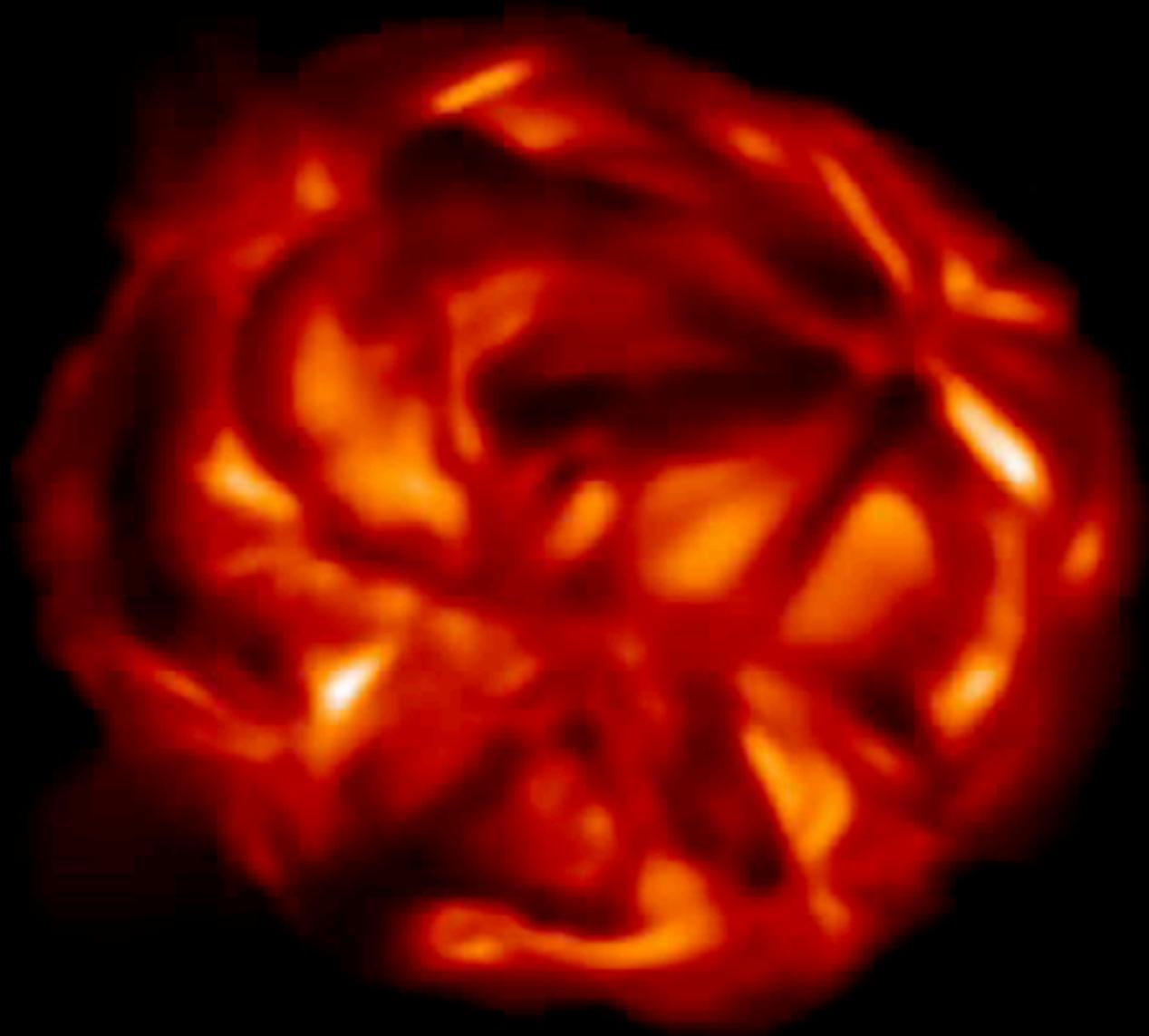
**A lot more of energy is being produced than in the MS-phase.**

The star gets very luminous and **swells**.

Hydrogen Shell Burning

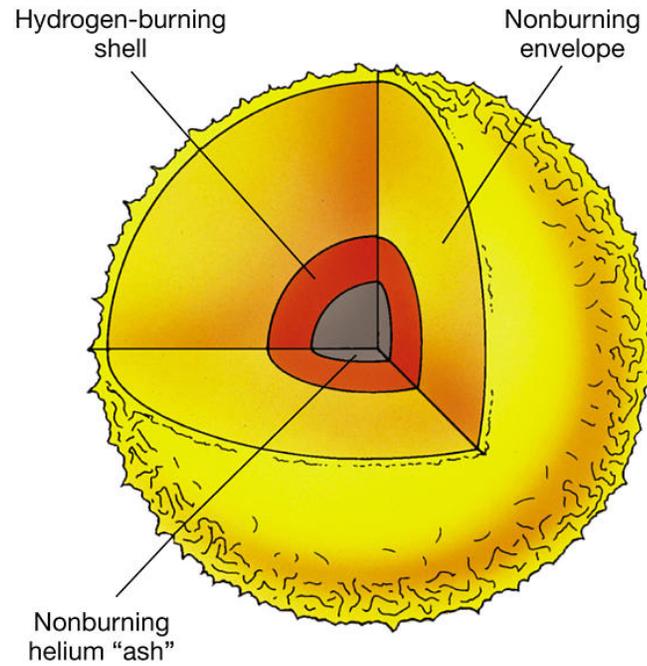


st35gm04n26: Surface Intensity(11), time( 0.0)=30.263 yrs



# What happens to the inert Helium core?

## Hydrogen shell burning



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# What happens to the inert Helium core?

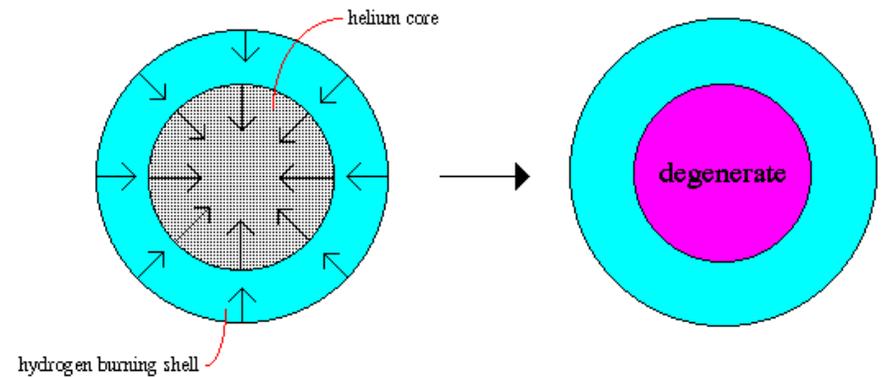
It keeps contracting and heating

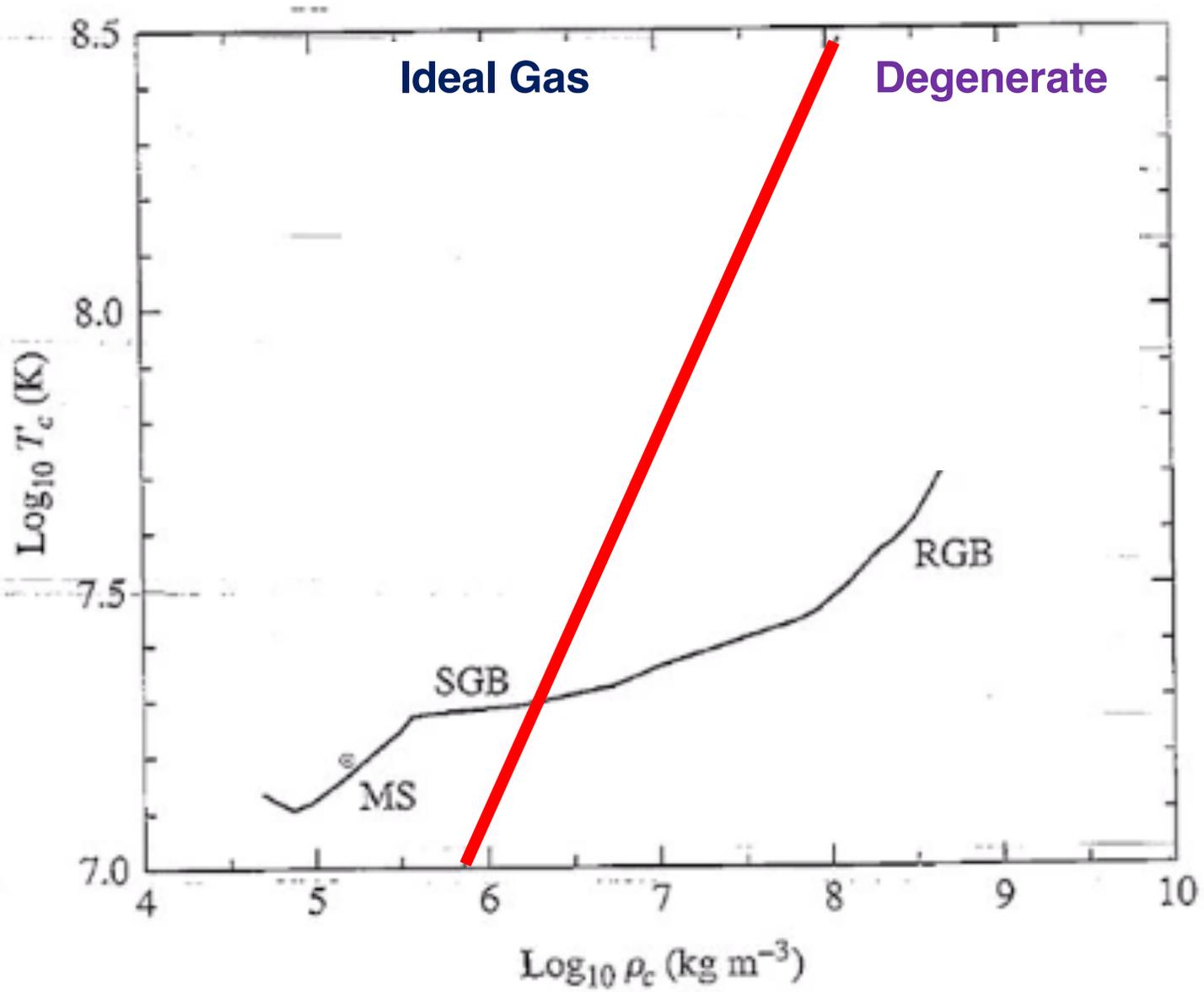
At some point the density is so high it goes **degenerate**

A **phase transition** has occurred

The core stops behaving like a gas and starts behaving more **like a solid**

Core Degeneracy





# What happens to the inert Helium core?

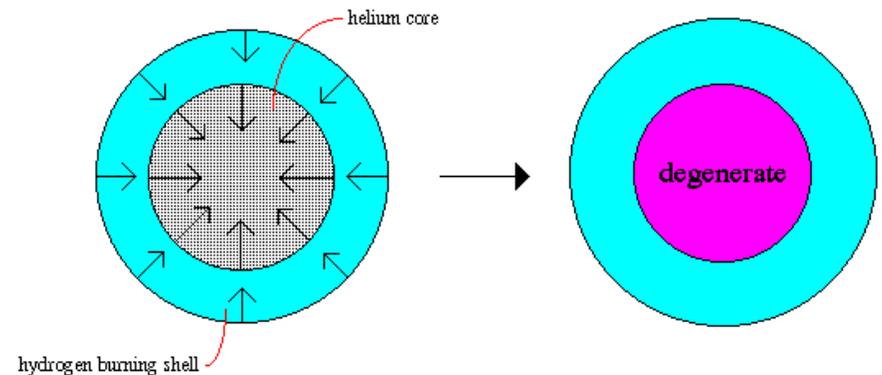
It keeps contracting and heating

At some point the density is so high it goes **degenerate**

A **phase transition** has occurred

The core stops behaving like a gas and starts behaving more **like a solid**

Core Degeneracy



## Ideal Gas

$$P \propto \rho T$$

Temperature rises, pressure rises

Temperature falls, pressure falls

**Radiative loss** → cooling →  
less support against gravity → **contraction**

## Degenerate Matter

$$P \propto \rho^{4/3}$$

If temperature rises or falls, pressure  
couldn't care less

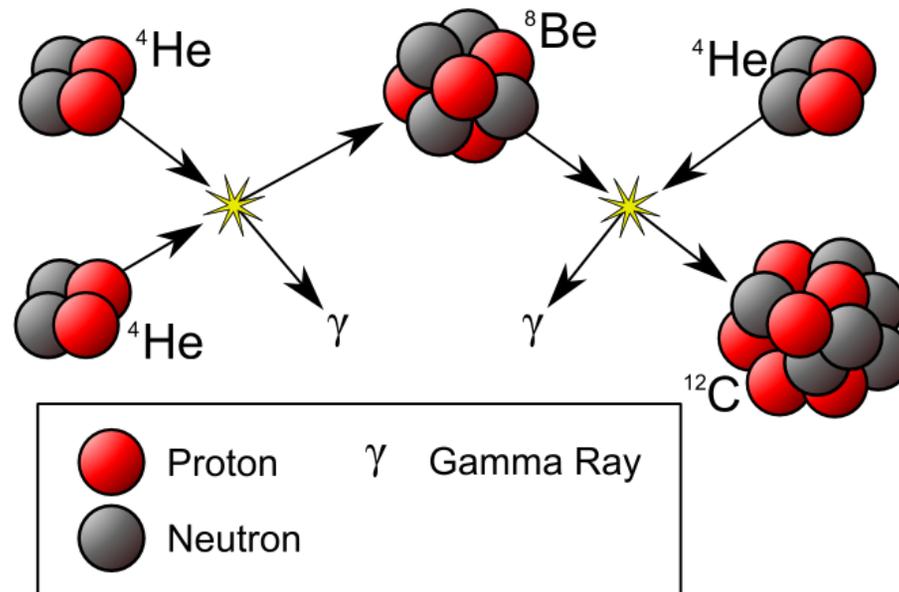
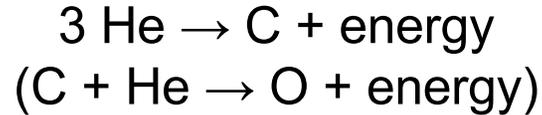
Radiative losses can continue indefinitely  
**The degenerate core is stable**

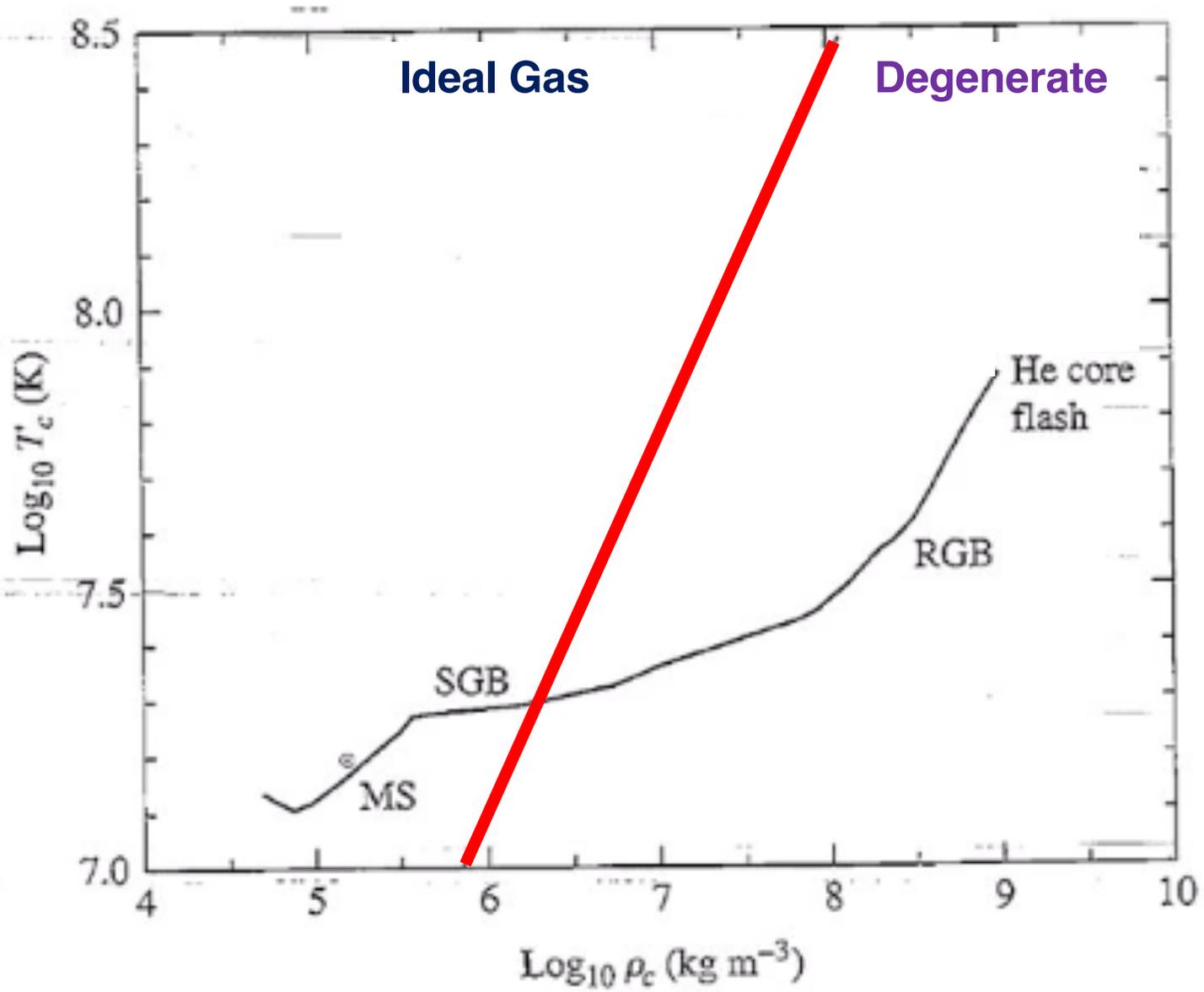
# Helium Fusion

The inner degenerate Helium core is stable  
But the outer Helium core keeps contracting and heating

At the tip of the Red Giant Branch,  
when the temperature reaches 100 million K,  
**HELIUM FUSION** begins

## Triple Alpha





# The Helium Flash

Under normal (non-degenerate) conditions ...

**Ideal Gas**

$$P \propto \rho T$$

Nuclear reactions start

**Heating → Expansion → Cooling**

Cooling = Less nuclear reactions

**Cooling → Contraction → Heating**

Thermostat keeps nuclear reactions “tuned”

**Controlled fusion**

# The Helium Flash

Non-degenerate vs degenerate

## Ideal Gas

$$P \propto \rho T$$

Nuclear reactions start

**Heating → Expansion → Cooling**

Cooling = Less nuclear reactions

**Cooling → Contraction → Heating**

Thermostat keeps nuclear reactions “tuned”

**Controlled fusion**

## Degenerate Matter

$$P \propto \rho^{4/3}$$

Nuclear reactions start

Heating

Star does not expand

Nuclear burning increases

More heating

# The Helium Flash

Fusion ignition in degenerate matter is a bomb ready to explode

## Ideal Gas

$$P \propto \rho T$$

Nuclear reactions start

**Heating → Expansion → Cooling**

Cooling = Less nuclear reactions

**Cooling → Contraction → Heating**

Thermostat keeps nuclear reactions “tuned”

**Controlled fusion**

## Degenerate Matter

$$P \propto \rho^{4/3}$$

Nuclear reactions start

Heating

Star does not expand

**Nuclear burning increases  
More heating**

**No thermostat**

Runaway temperature rise

**Runaway fusion**

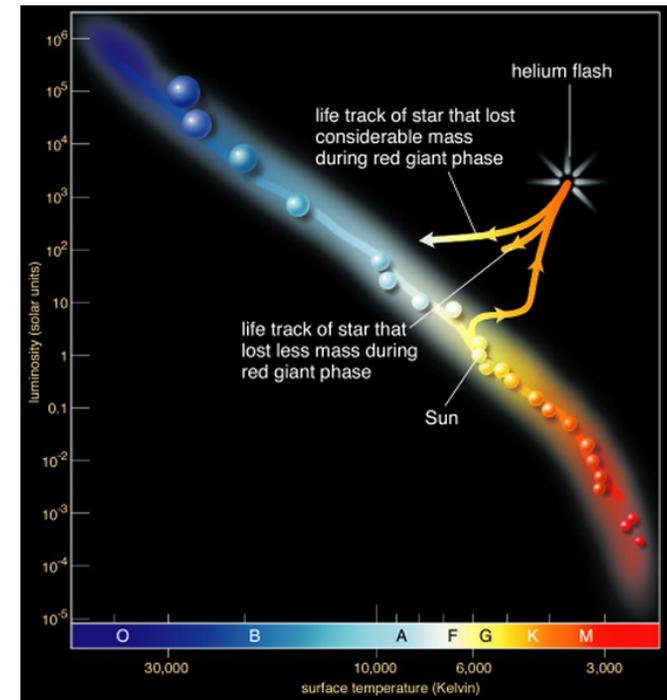
# The Helium Flash

Fusion ignition in degenerate matter is a bomb ready to explode

No thermostat! Core just gets hotter and hotter

Runaway Helium burning: **100 billion times the Solar output** in just a few seconds

## Helium Flash



# The Helium Flash

Fusion ignition in degenerate matter is a bomb ready to explode

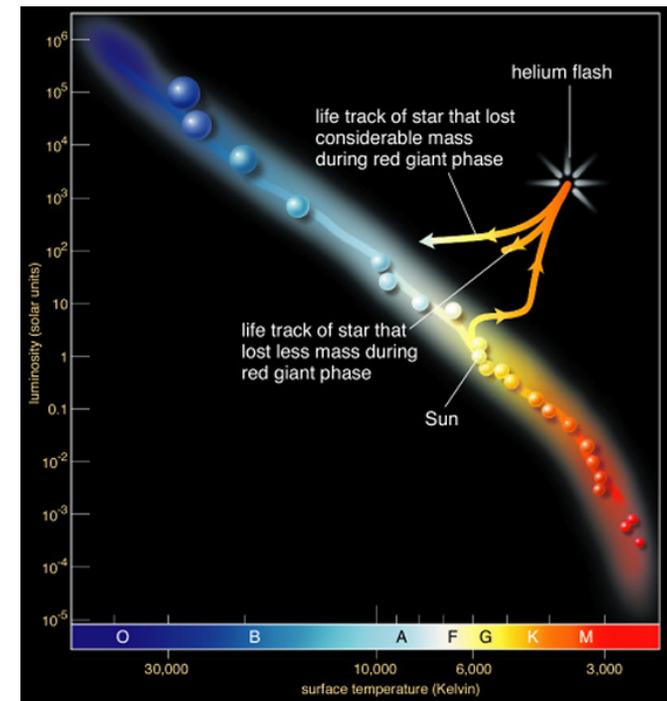
No thermostat! Core just gets hotter and hotter

Runaway Helium burning: **100 billion times the Solar output** in just a few seconds

## Helium Flash

Yet, nothing is seen

Why?



# The Helium Flash

Fusion ignition in degenerate matter is a bomb ready to explode

No thermostat! Core just gets hotter and hotter

Runaway Helium burning: **100 billion times the Solar output** in just a few seconds

## Helium Flash

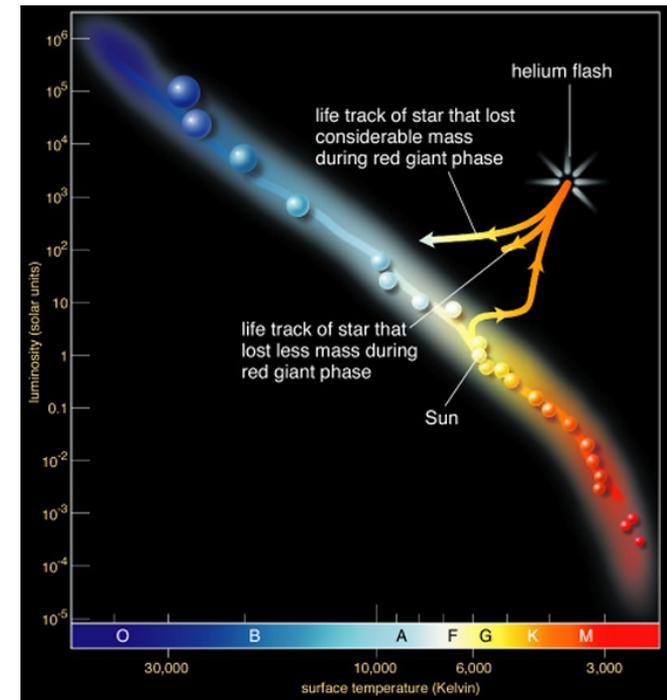
Yet, nothing is seen

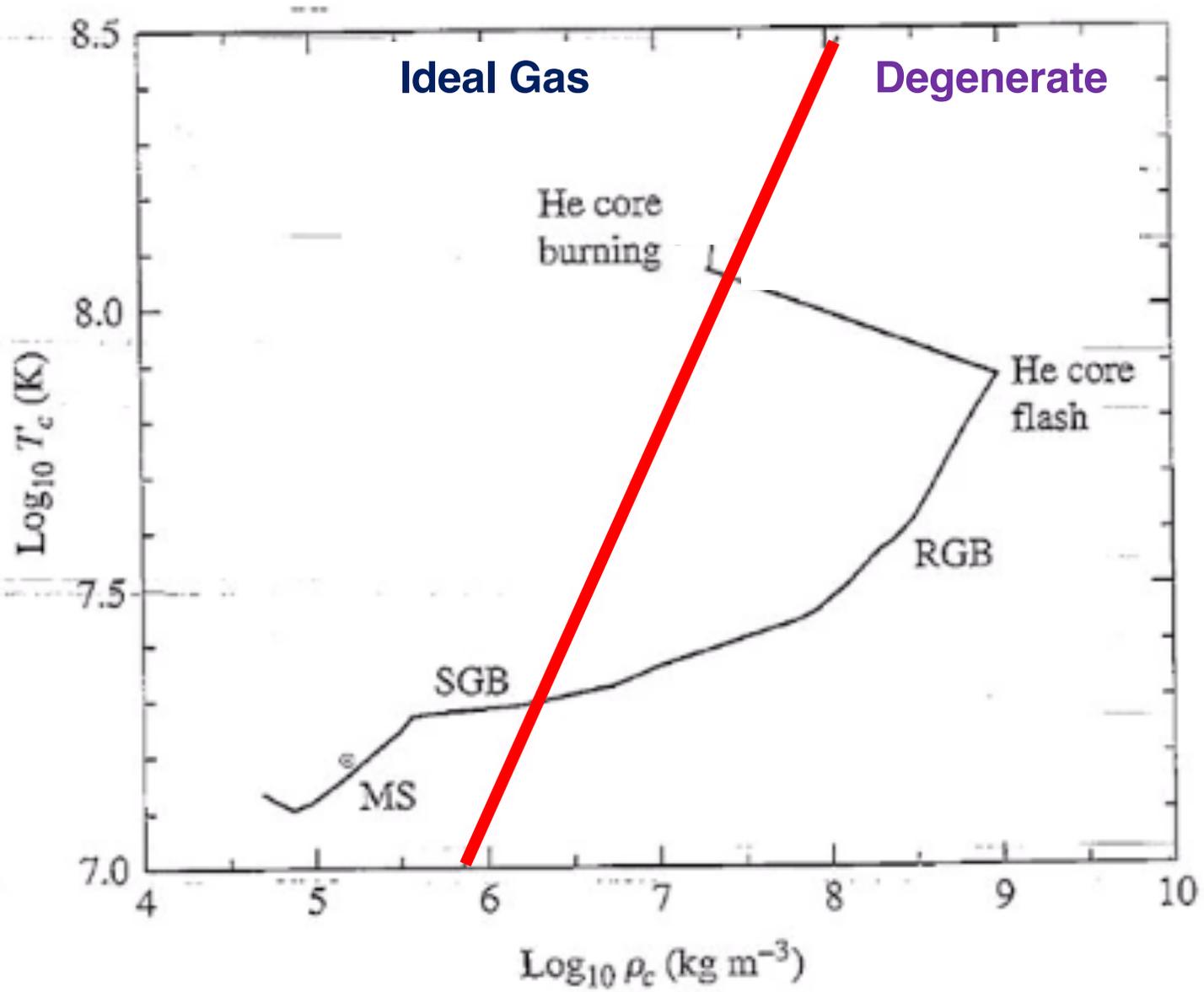
Why?

The energy is ALL used to lift the degeneracy

(i.e., to “melt” the degenerate core back into a normal gas)

Helium then burns *steadily* in a core of ideal gas





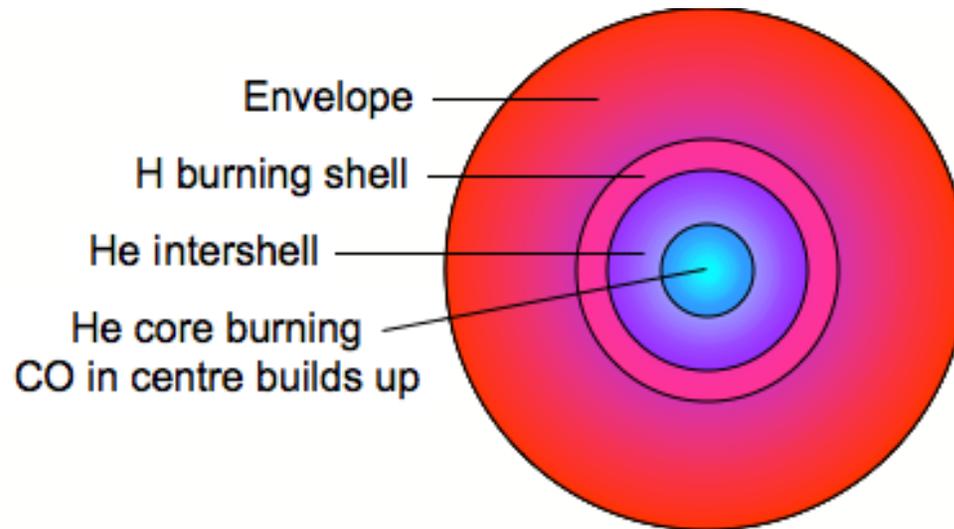
# The Horizontal Branch

Helium burning in the core

Hydrogen shell burning

In the HR diagram, the star sets in the **Horizontal Branch**

**The Horizontal Branch is the Helium Main Sequence**



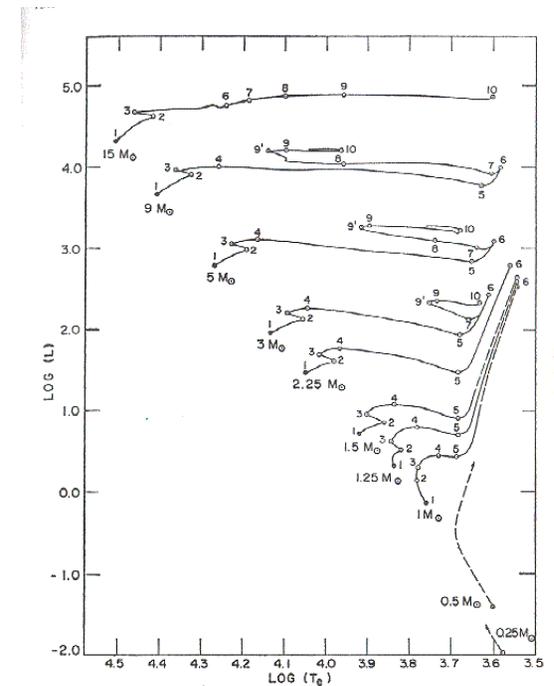
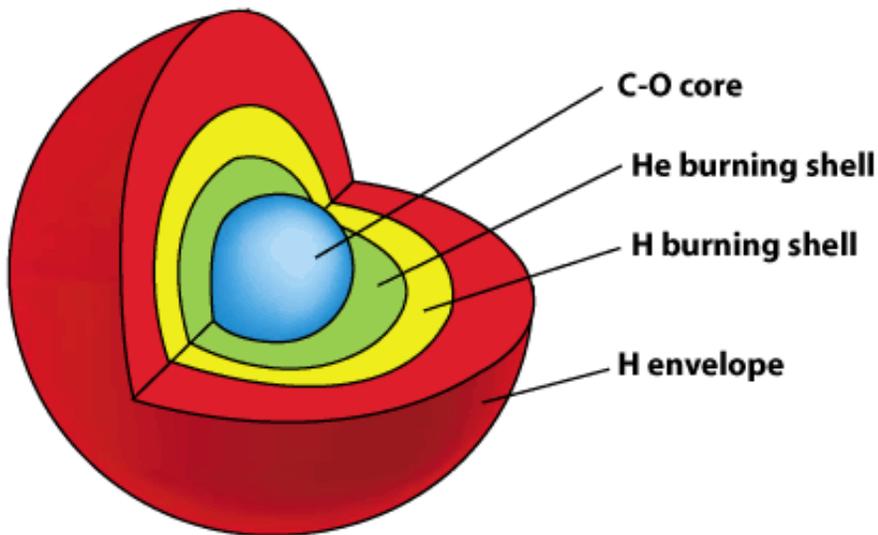
# Helium exhausted in the core

The Carbon-Oxygen core **contracts** and **heats** up.

## Helium shell burning

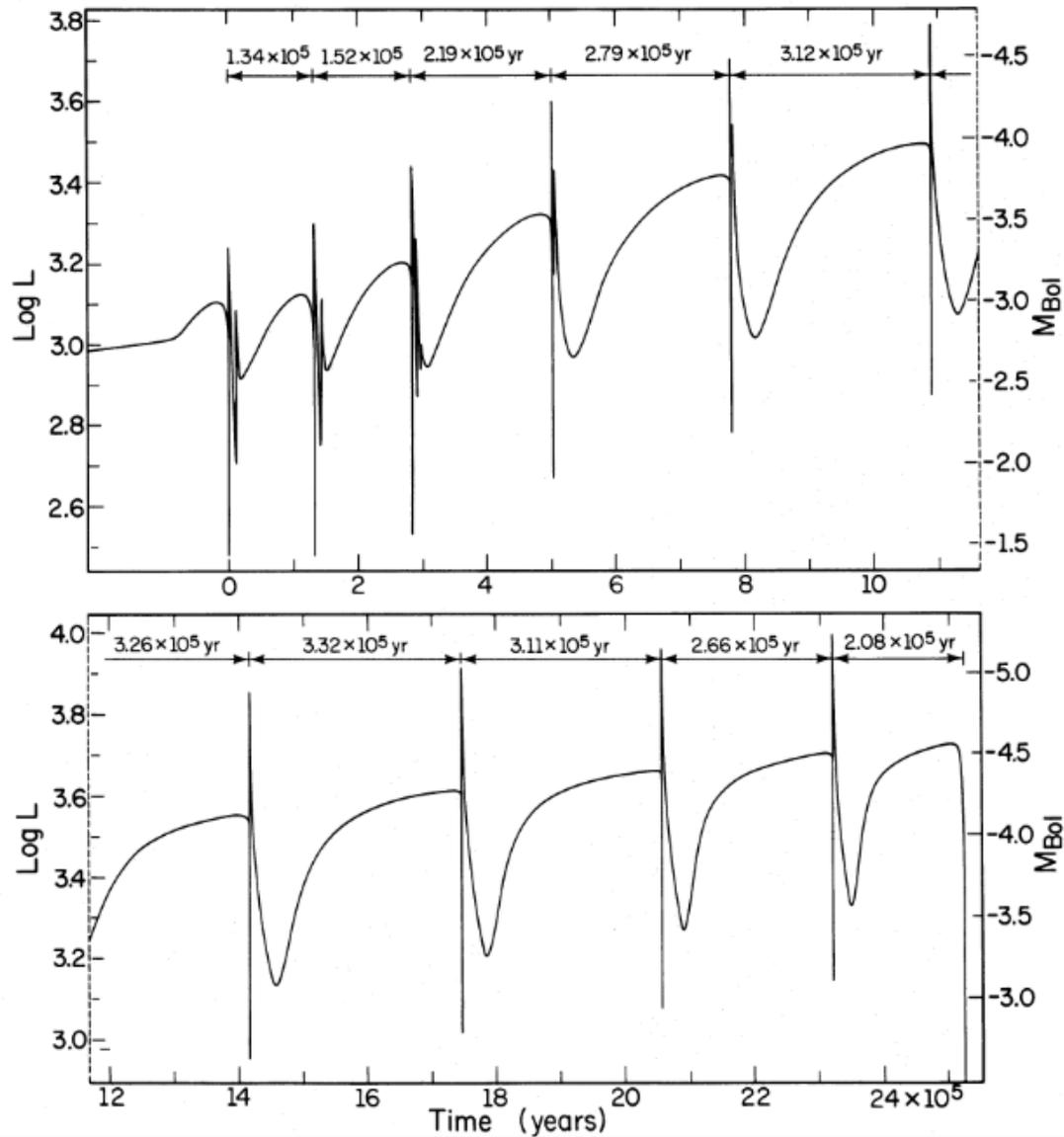
More energy is available, the star swells and becomes a red giant *again*

The star reaches the ***Asymptotic Giant Branch***

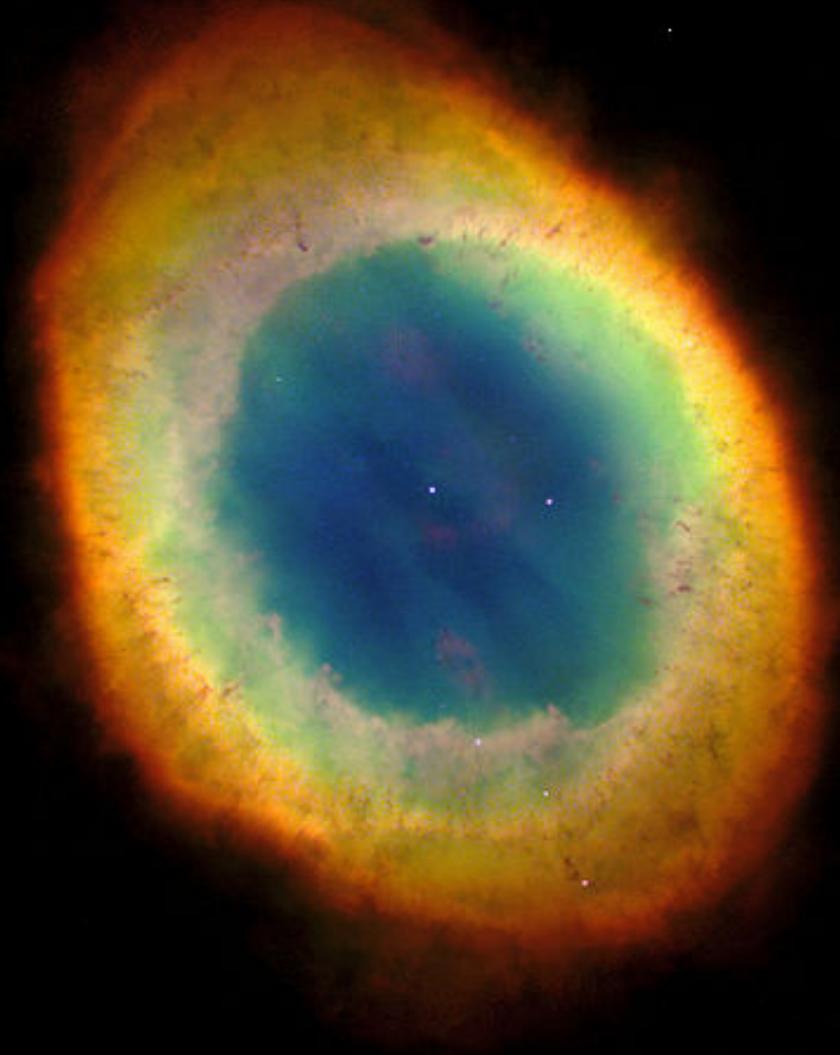


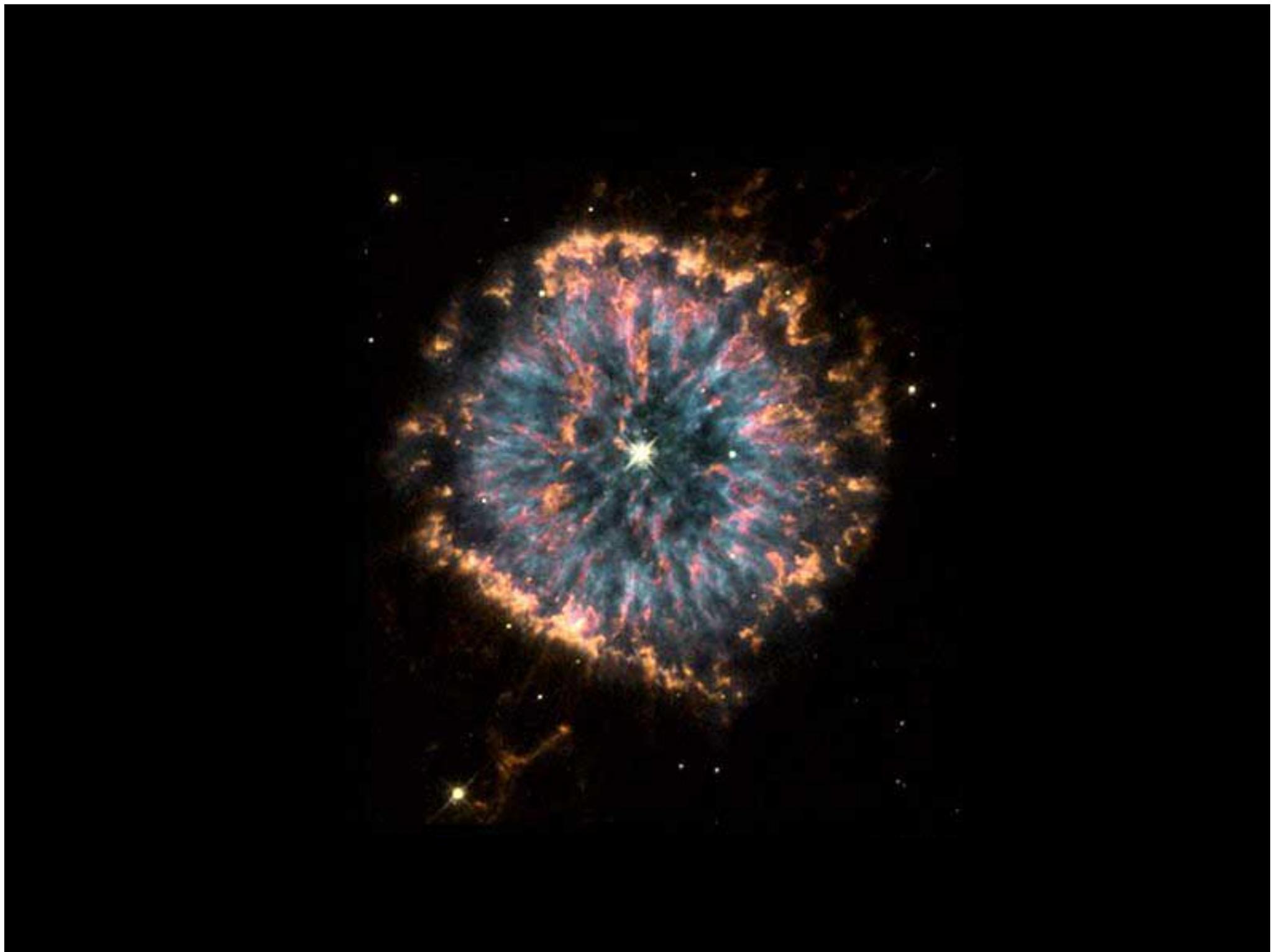
# Thermal pulses in AGB stars

A series of Helium flashes



**PLANETARY NEBULA**  
**The gracious death of low mass stars**

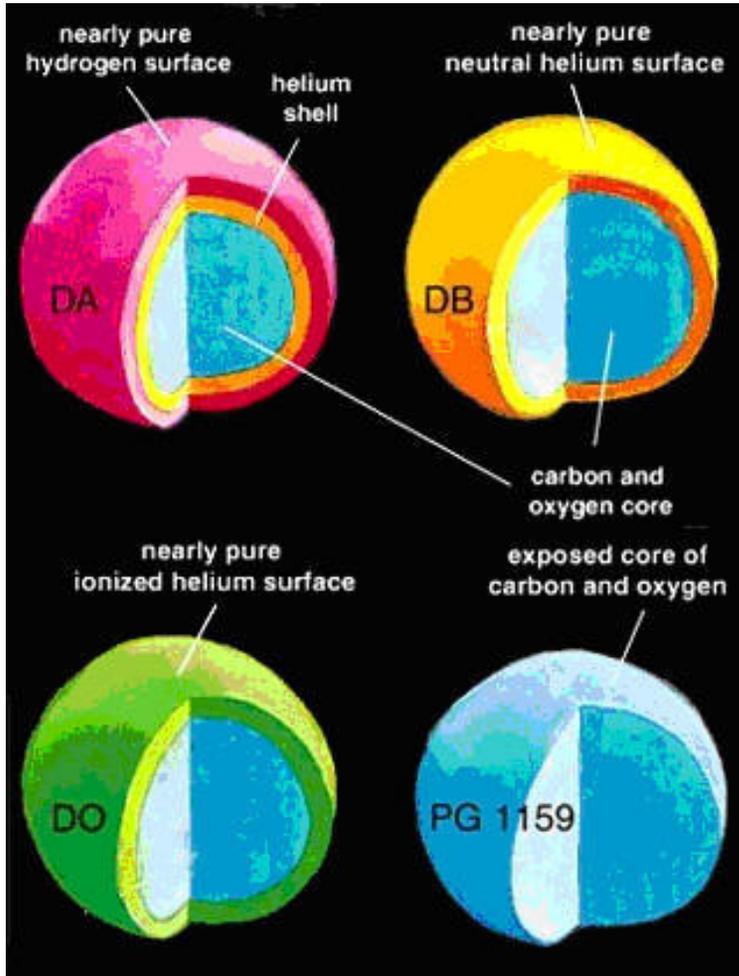




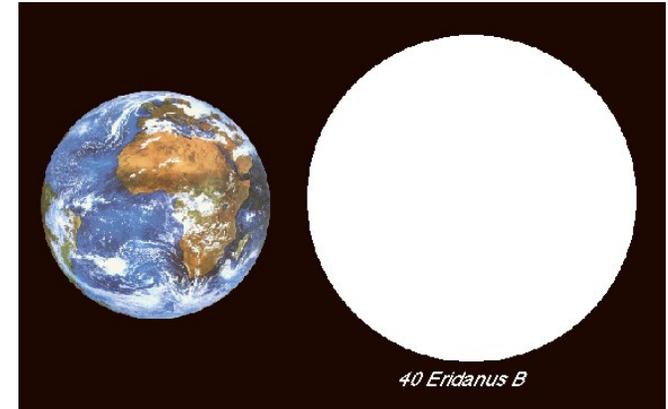


# White dwarfs

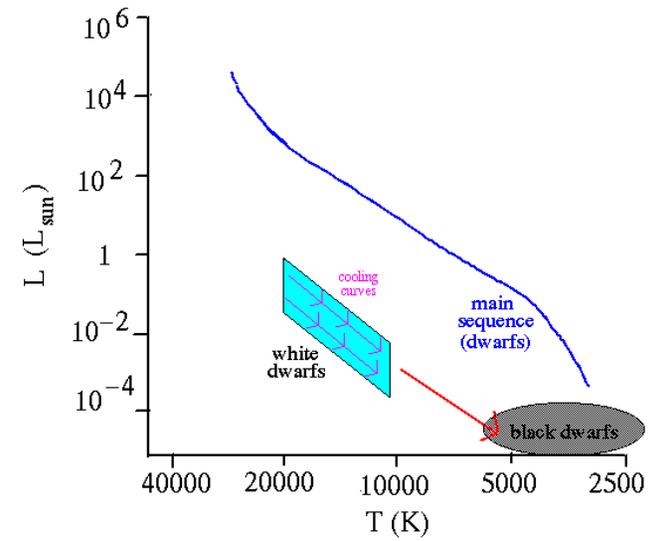
White dwarfs are the exposed degenerate core of the star



Types of white dwarfs



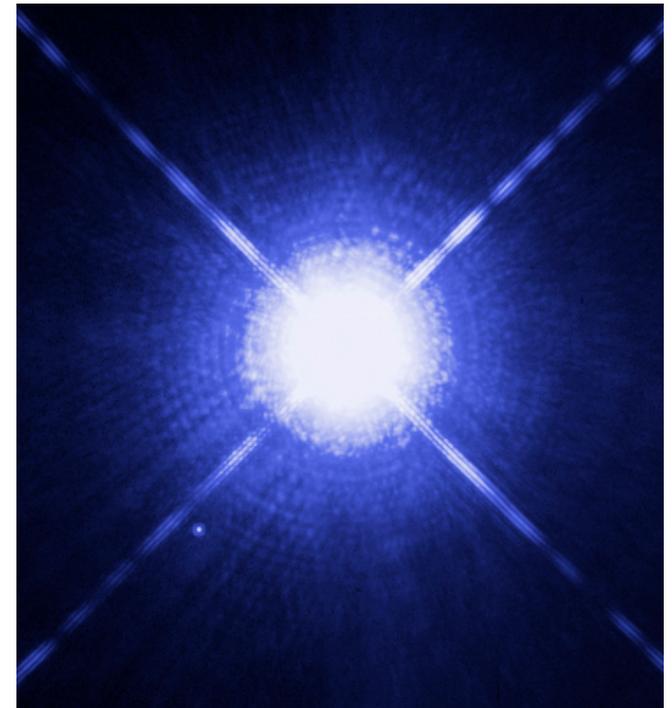
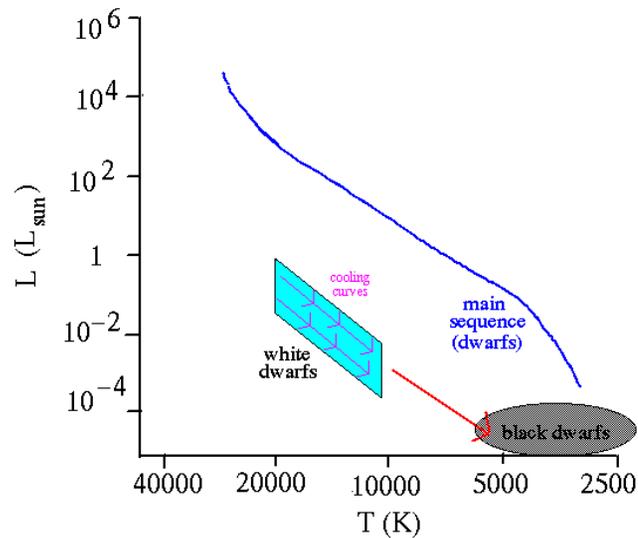
White dwarfs have planetary dimensions...



... and they do little but cooling.

# White dwarfs

White dwarfs are the exposed degenerate core of the star



**Sirius A** (Main Sequence star)  
and **Sirius B** (White Dwarf)

No energy production  
Supported by degenerate pressure

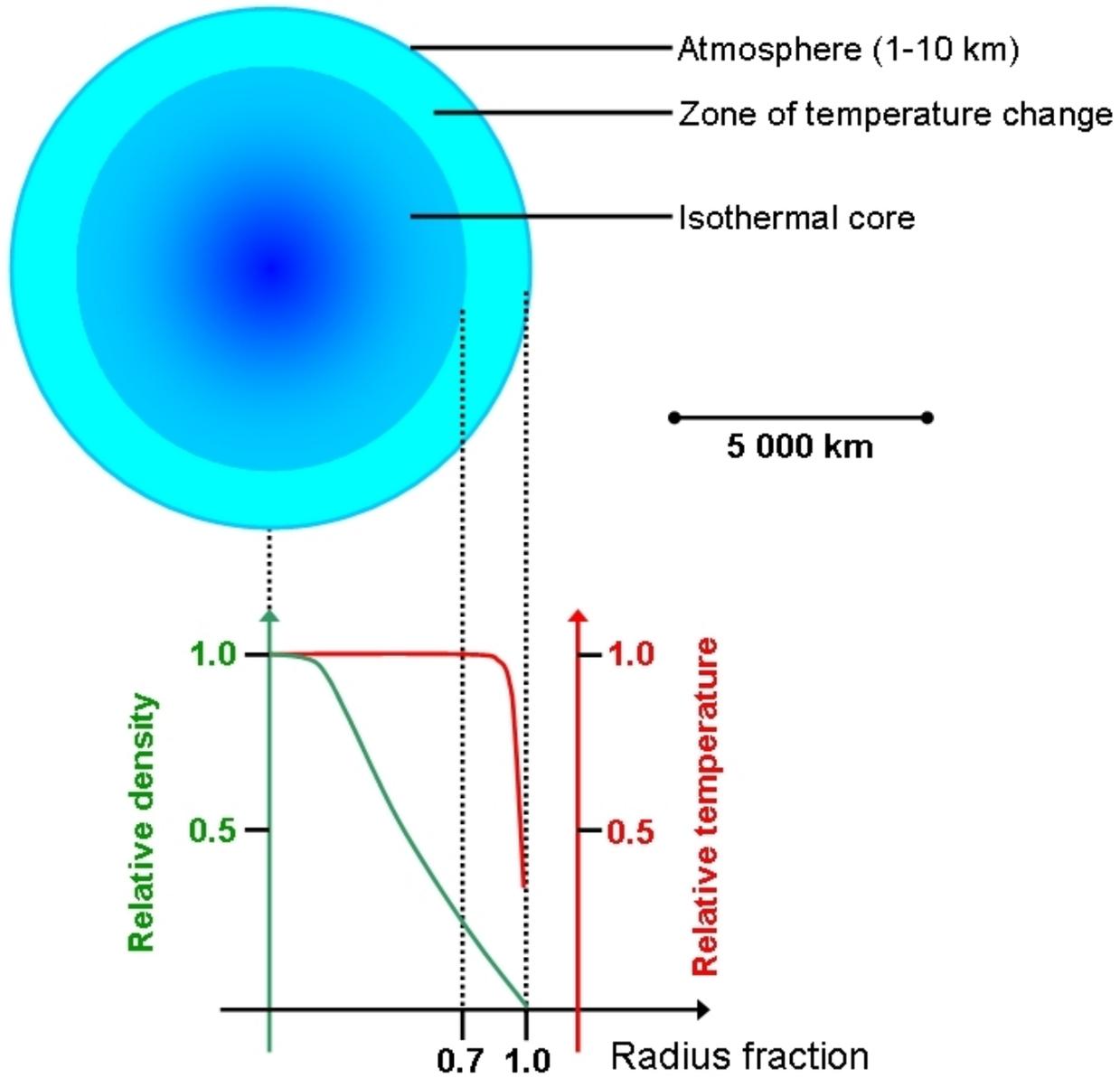
Cooling takes a long time

**$10^{15}$  yr** to cool down to background temperature

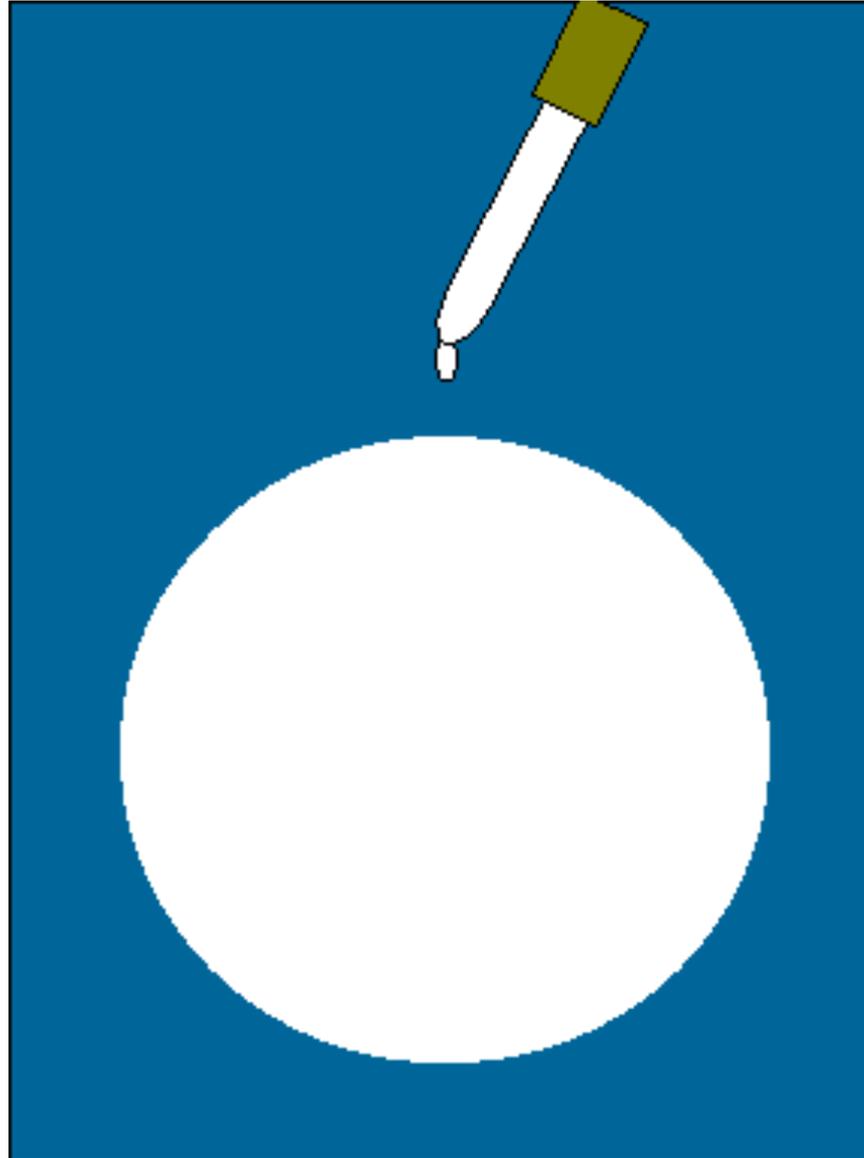
**The universe is not old enough to have black  
dwarfs**

Coldest white dwarfs ~5000 K.

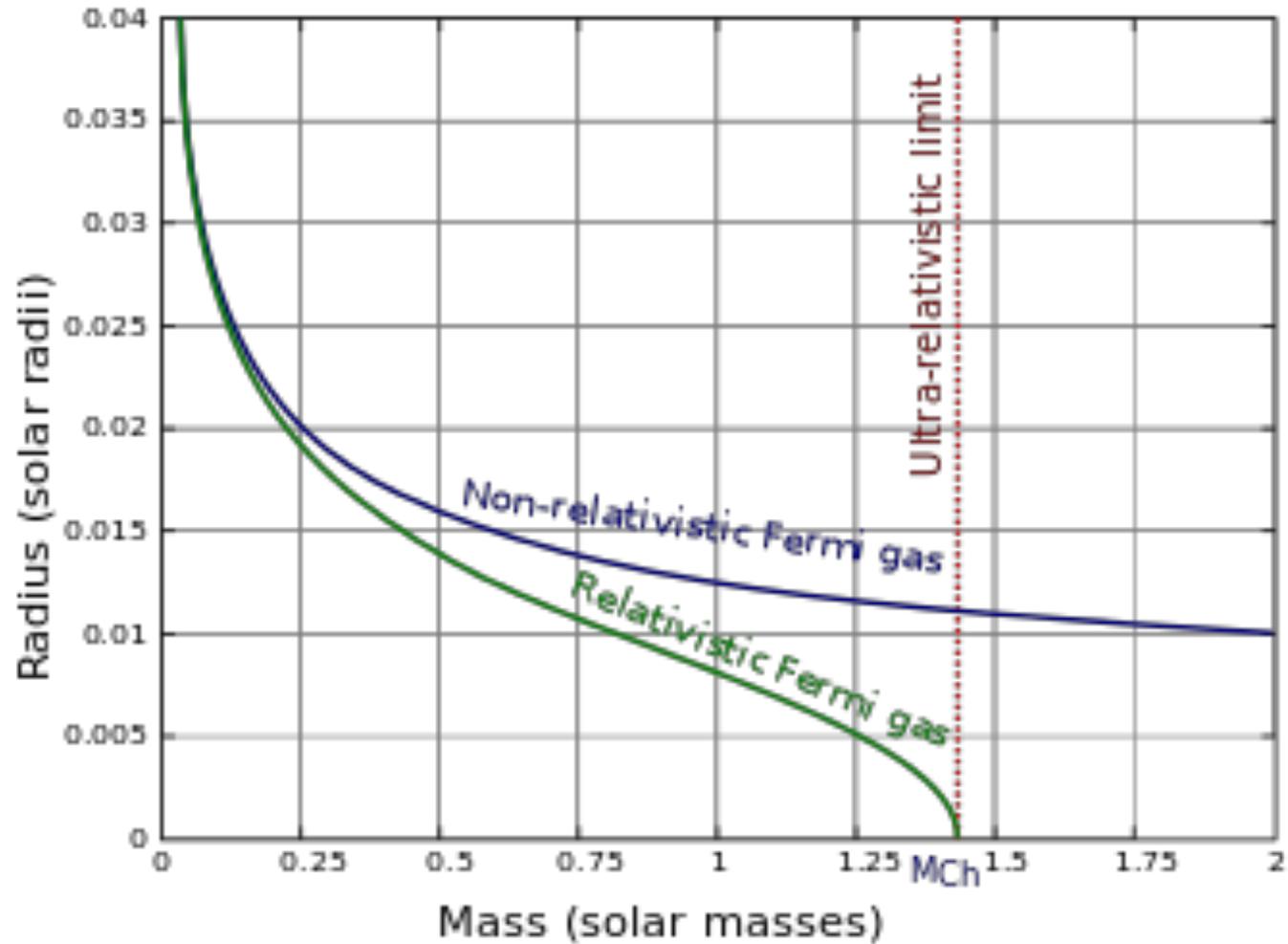
# White Dwarf Structure



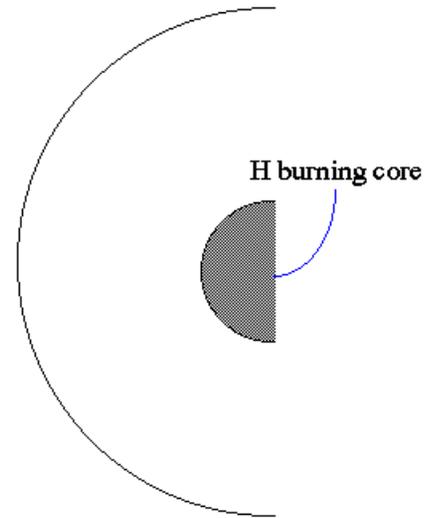
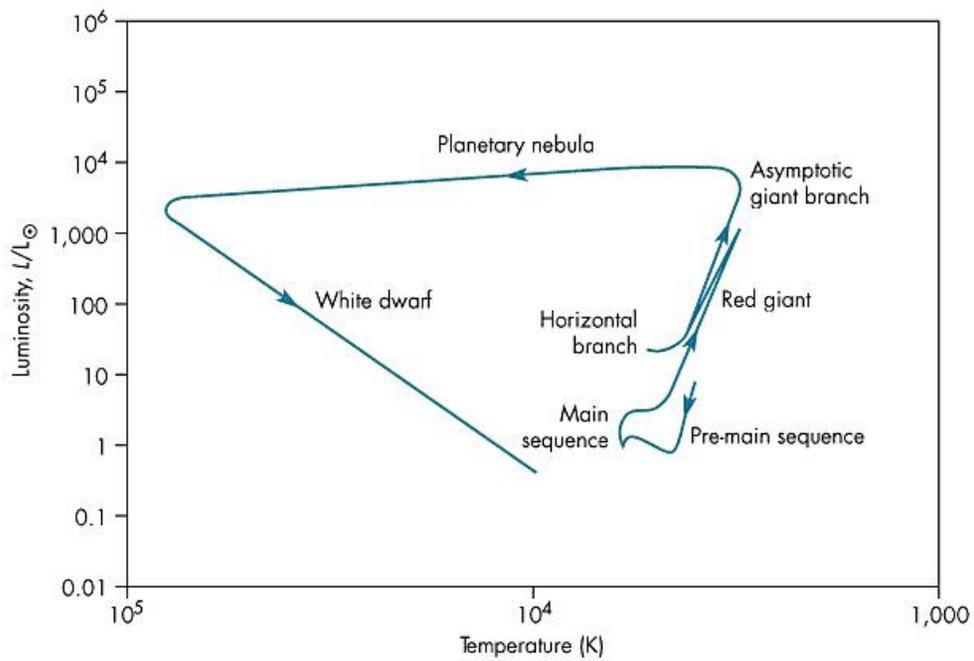
# White dwarf Mass-Radius relationship



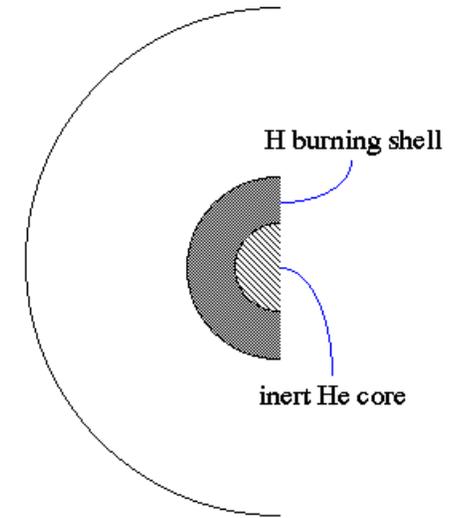
# White dwarf Mass-Radius relationship



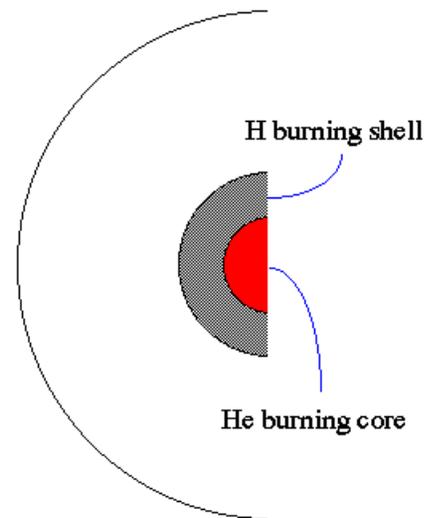
# Evolution of a low mass star



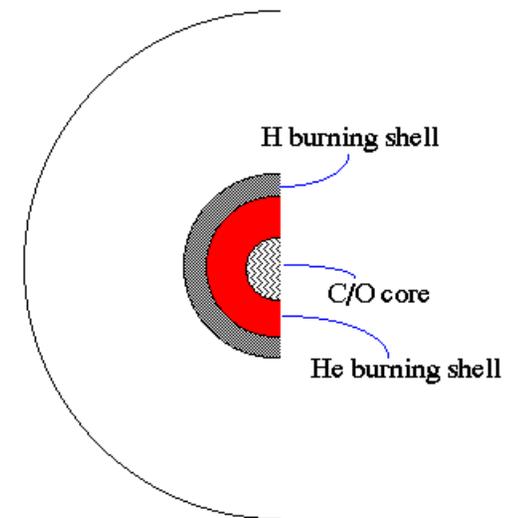
Main Sequence Star



Red Giant Star

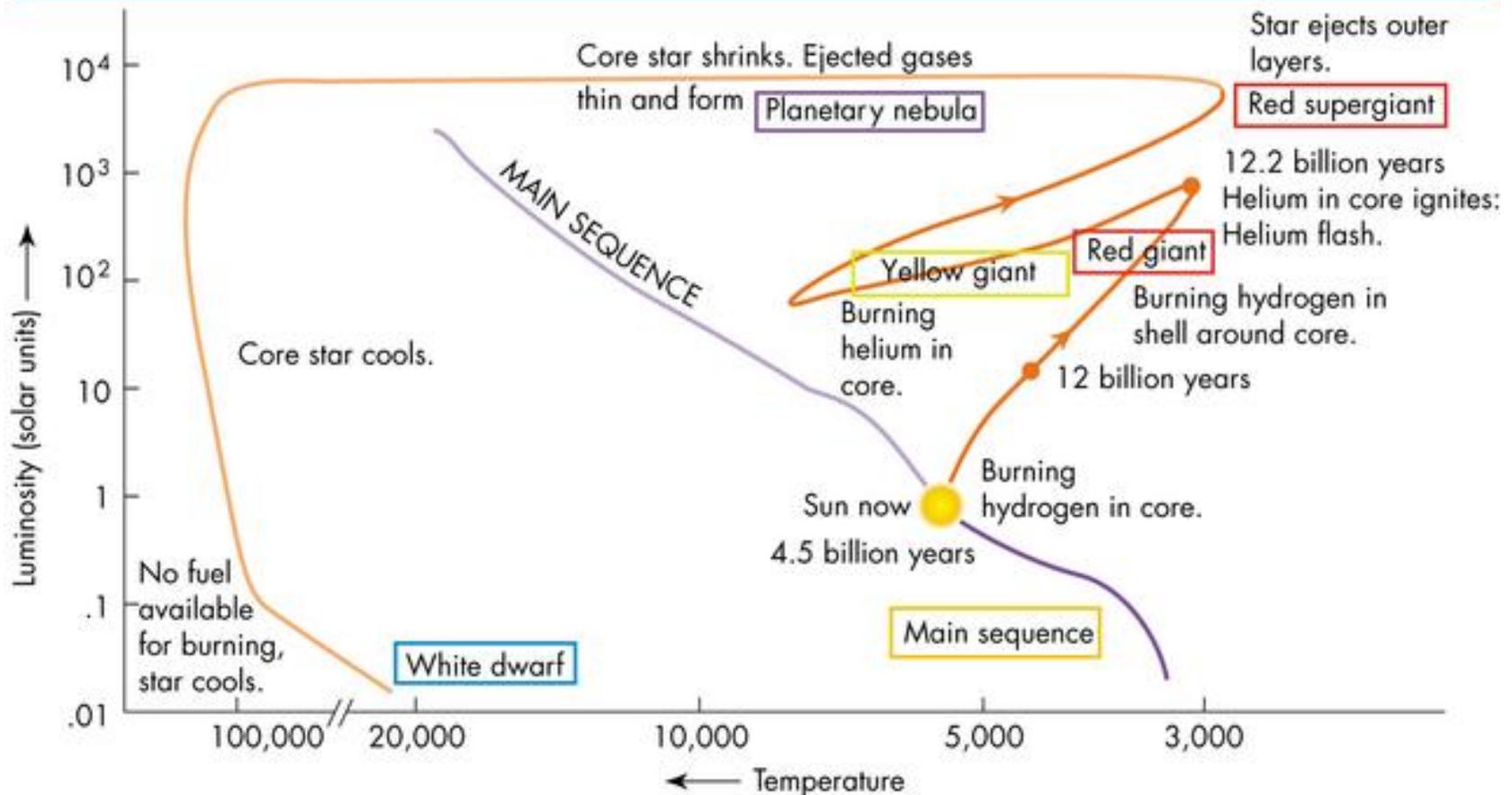
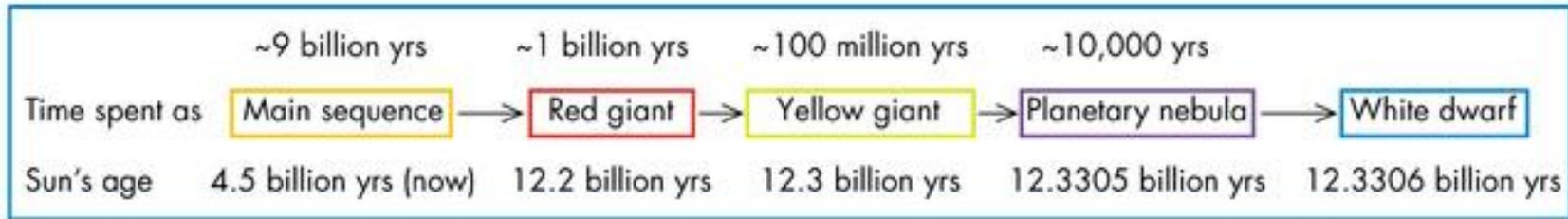


Horizontal Branch Star



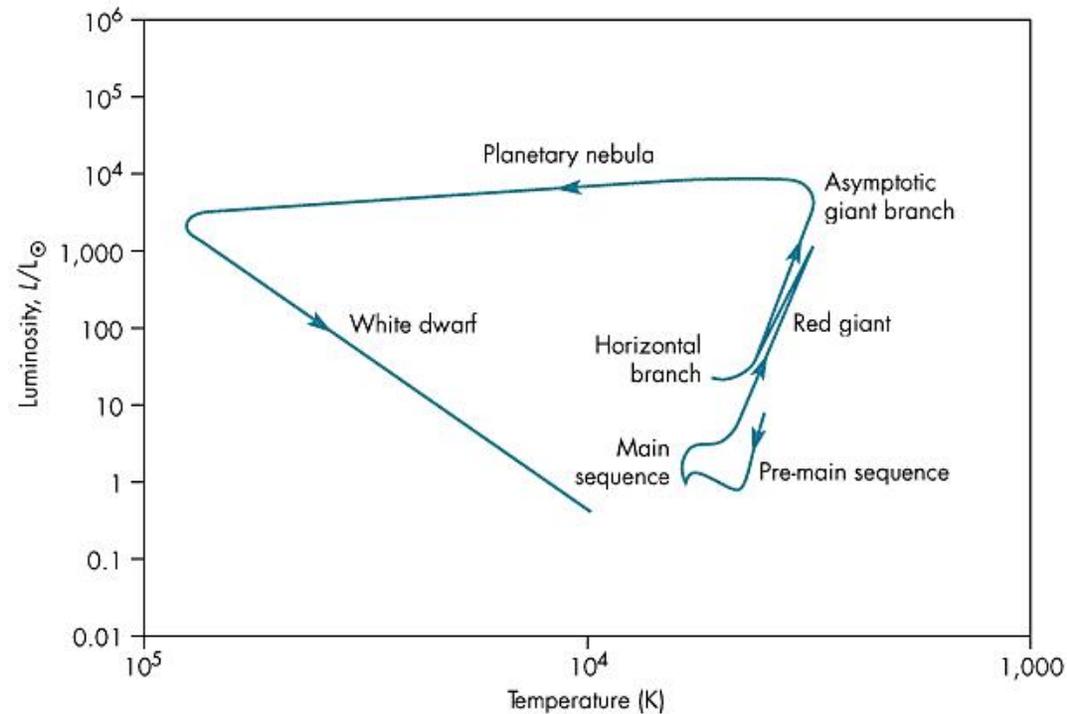
Asymptotic Giant Branch Star

# Post-Main Sequence Evolution - Timescales



# Evolution of high mass stars

The evolution we covered in last class is for low mass stars ( $M < 4 M_{\odot}$ )



High mass stars differ basically due to the ***temperature of the core***.

# Evolution of high mass stars ( $4 < M/M_{\odot} < 8$ )

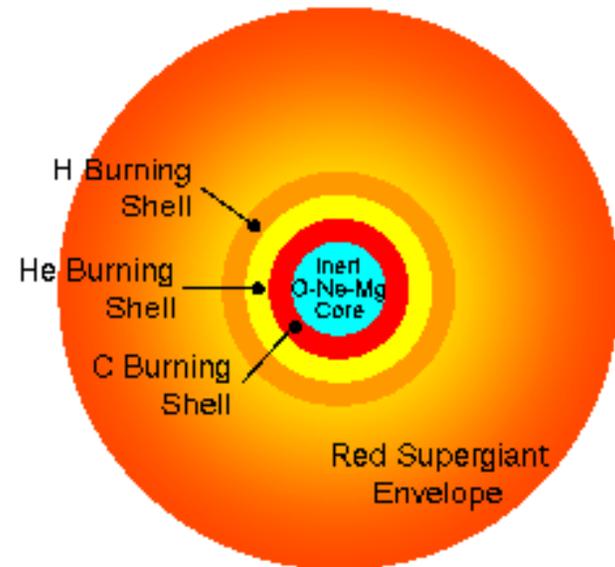
## *The Helium Flash never happens*

The star reaches Helium burning temperatures *before* the core becomes degenerate

They also reach temperatures hot enough to burn **Carbon**

**600 million K**

Leaves a O-Ne-(Mg)  
white dwarf.



# Evolution of high mass stars

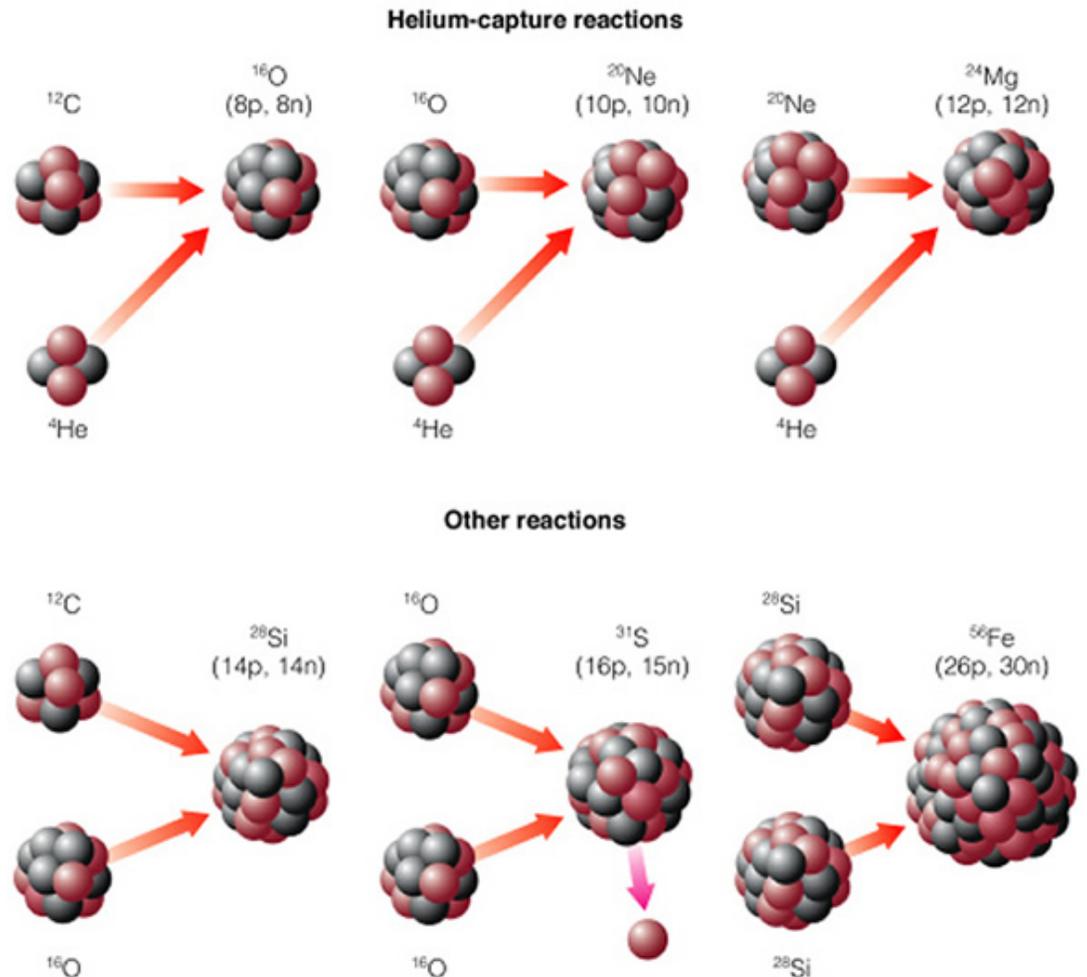
$$M > 8 M_{\odot}$$

Carbon  $\rightarrow$  O, Ne, Mg (600 million K)

Neon  $\rightarrow$  O, Mg (1.5 Billion K)

Oxygen  $\rightarrow$  Si, S, P (2.1 Billion K)

Silicon  $\rightarrow$  Fe, Ni (3.5 Billion K)

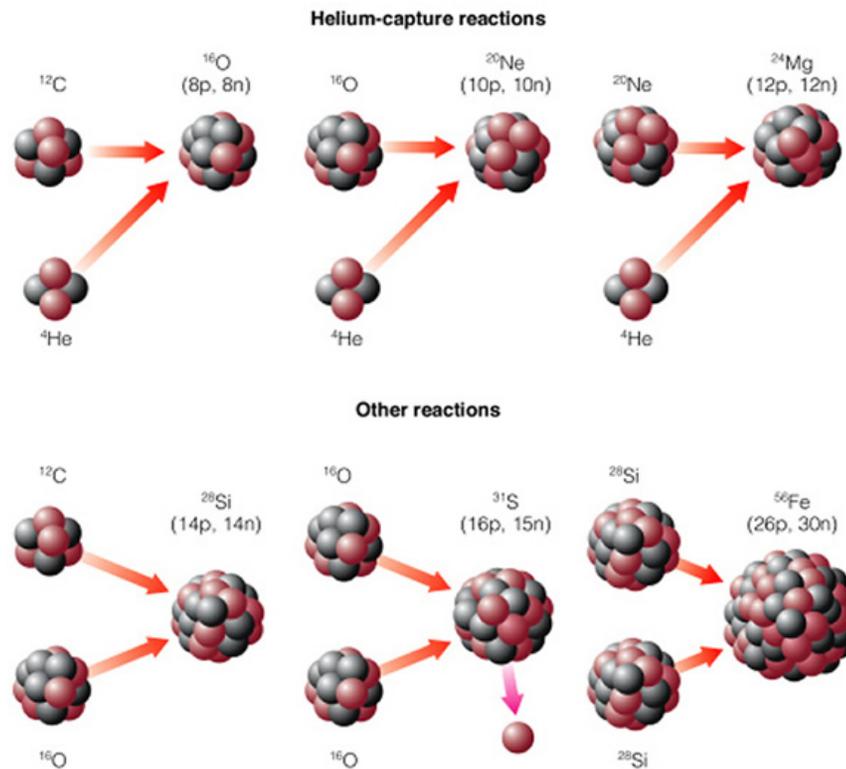


# Alpha ladder

Low mass stars produce elements up to Carbon and Oxygen

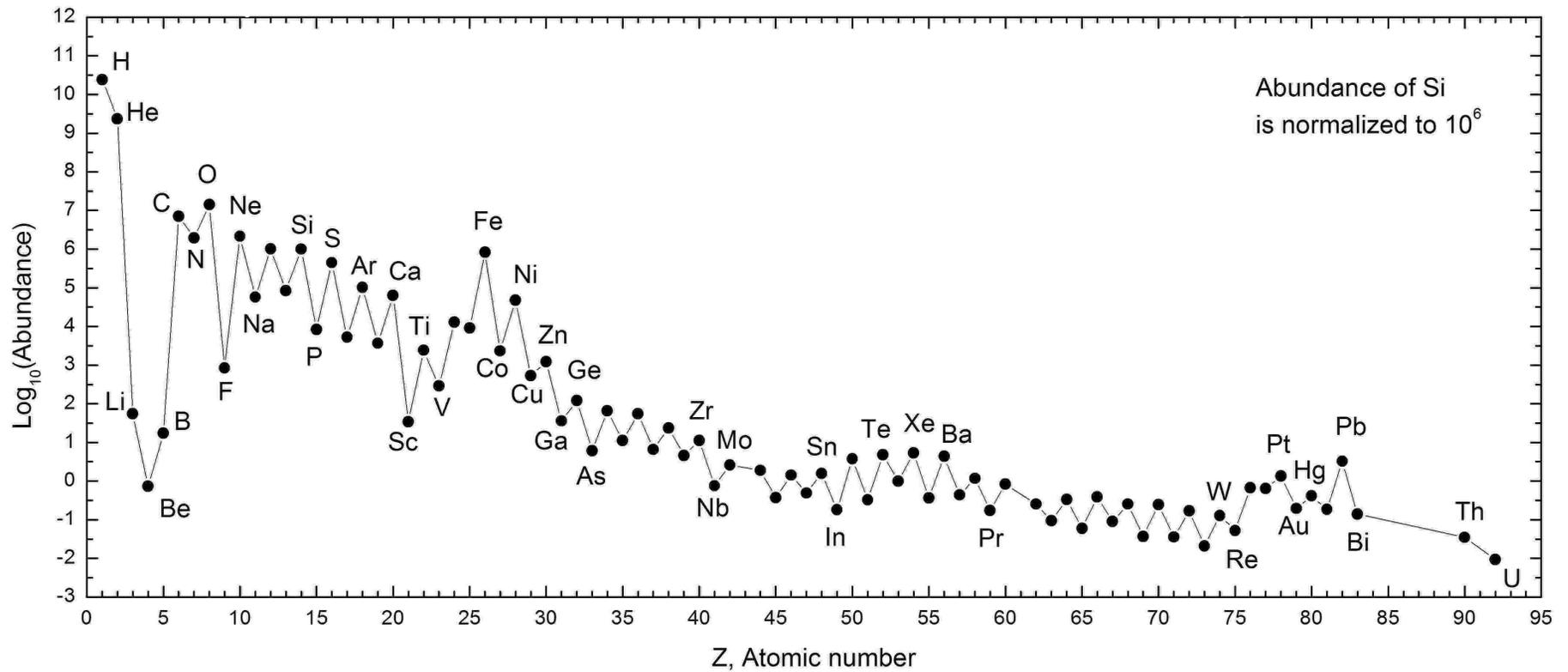
***High mass stars produce all the rest of the periodic table***

Up to Iron we have basically alpha reactions



# The Sun's abundance pattern

Because of the alpha ladder, elements with even atomic number are more abundant than those with odd



# Evolution of high mass stars

$$M > 8 M_{\odot}$$

## ***TIMESCALES FOR NUCLEAR BURNING***

Hydrogen – 10 Myr

Helium – 1 Myr

Carbon – 1000 yr

Neon ~ 10 yr

Oxygen ~ 1 yr

Silicon ~ 1 day

# Evolution of high mass stars $M > 8 M_{\odot}$

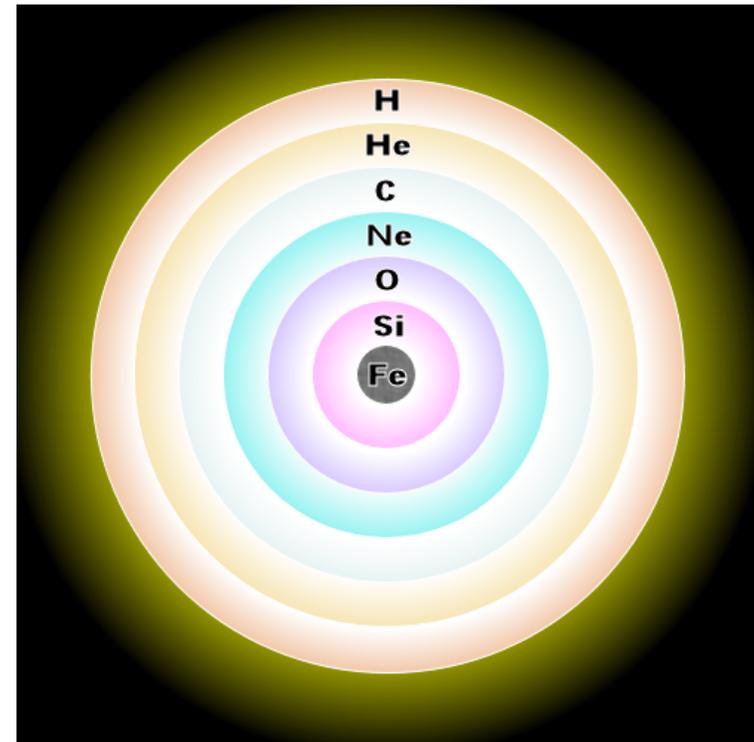
The star develops an “**onion layers structure**” of burning shells

Carbon  $\rightarrow$  O, Ne, Mg (600 million K)

Neon  $\rightarrow$  O, Mg (1.5 Billion K)

Oxygen  $\rightarrow$  Si, S, P (2.1 Billion K)

Silicon  $\rightarrow$  Fe, Ni (3.5 Billion K)

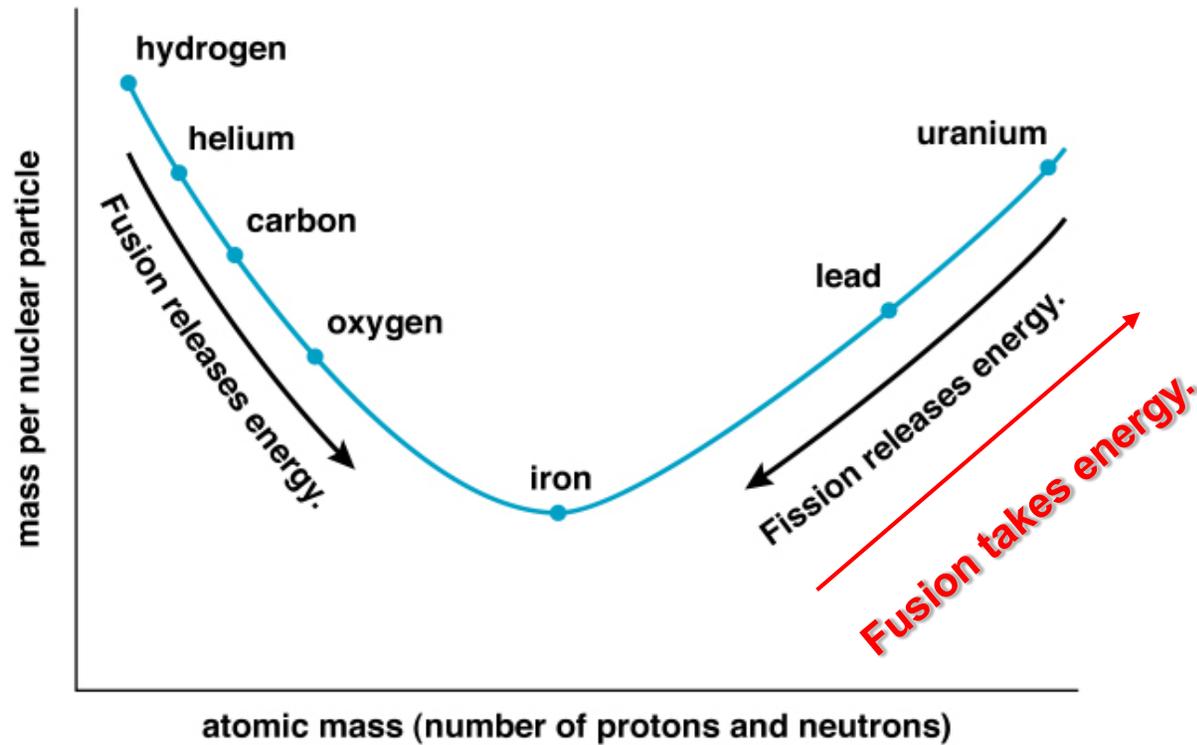


*But **Iron** is a **DEAD END** !!*

# Iron is a dead end

Iron is the most tightly bound element

***Fusion beyond Iron TAKES energy***

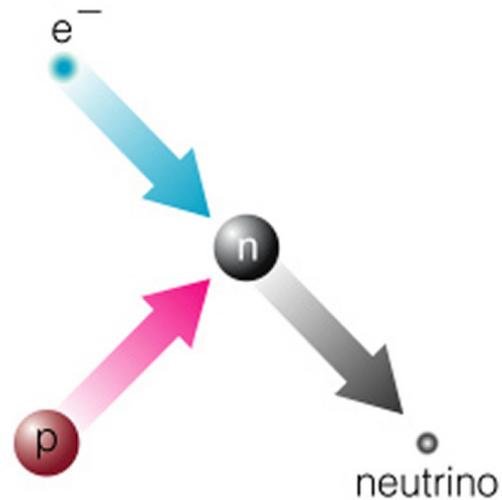


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***No fusion reactions left to yield energy!!***

# Neutronization

At densities of  $10^{10}$  g/cm<sup>3</sup>  
(remember: nuclear densities are  $\sim 10^{14}$  g/cm<sup>3</sup>)

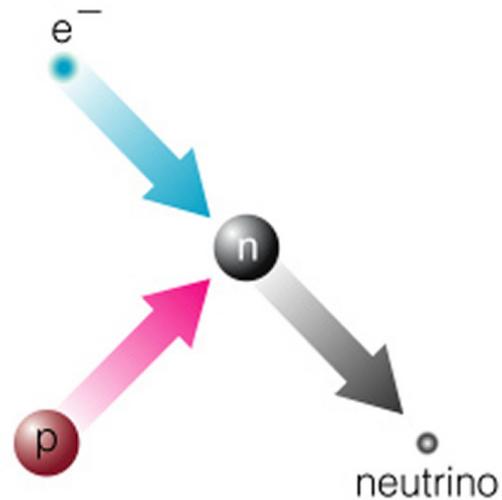


Proton + electron  $\longrightarrow$  neutron + neutrino



# Neutronization

At densities of  $10^{10}$  g/cm<sup>3</sup>  
(remember: nuclear densities are  $\sim 10^{14}$  g/cm<sup>3</sup>)



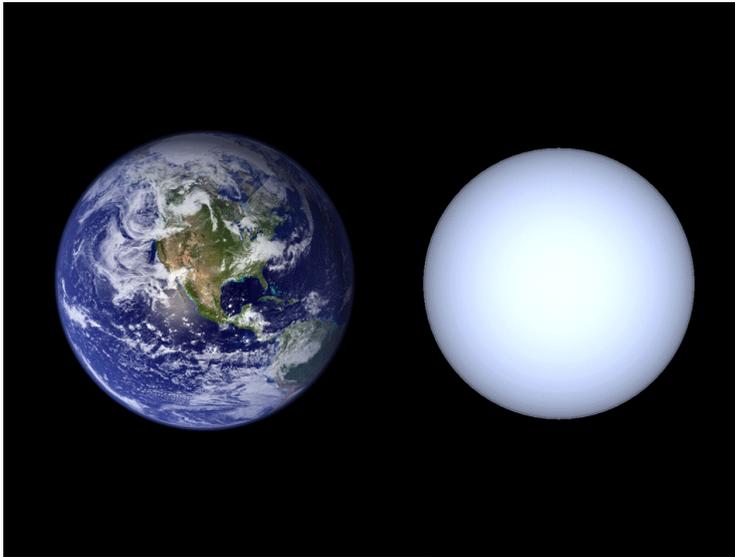
***Electrons lost!***

***electron degeneracy  
pressure is gone!!***

Proton + electron  $\longrightarrow$  neutron + neutrino



# Core collapse



*A second later*



*Collapse speed: 0.25c*



6000 km  
 $10^{10} \text{ g/cm}^3$

10 km  
 $10^{14} \text{ g/cm}^3$



**Nuclear densities!**

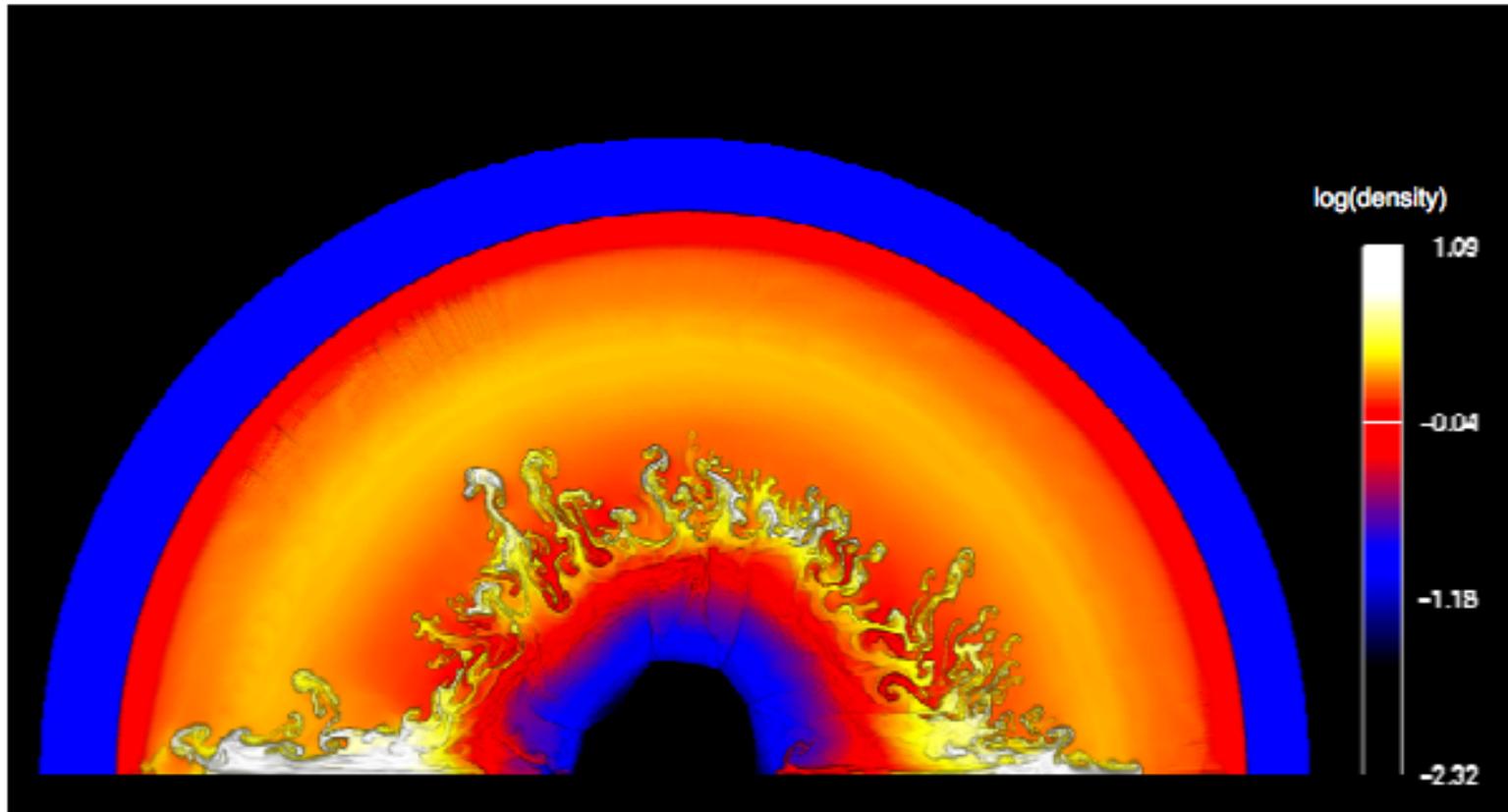
*Neutron degeneracy  
provides support against gravity*

# The Thermonuclear Shock Wave

The sudden release of gravitational energy generates a **shock wave**

The blastwave generates **explosive nuclear reactions** along its path

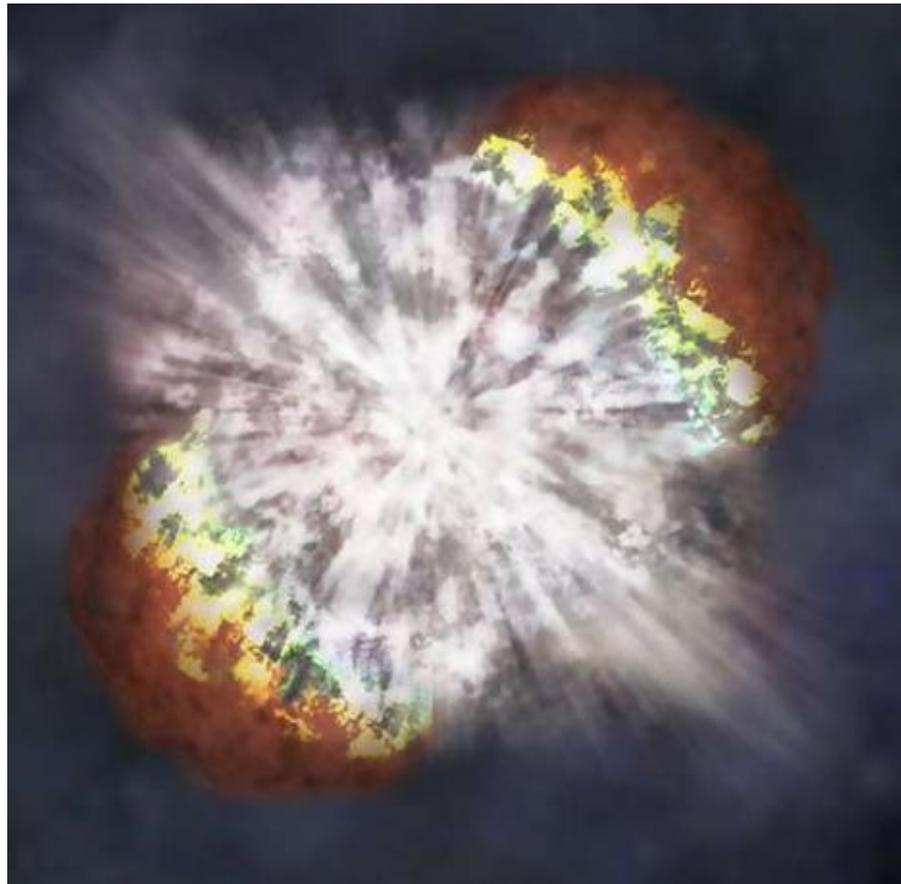
Violently heats and accelerates the stellar envelope



# Supernova!

In a few hours, the shockwave reaches the surface

From the outside, the star is seen to explode.



# Supernova 1987A

## Confirmation of the theory

A **burst of neutrinos** 4 hours before the event

The progenitor had a mass of  **$20 M_{\odot}$** .



# Urca process

***Free neutrons are unstable!***

Neutron  $\longrightarrow$  Proton + electron + neutrino



Proton + electron  $\longrightarrow$  neutron + neutrino



# Urca process

***Free neutrons are unstable!***

## ***Beta decay***

Neutron  $\longrightarrow$  Proton + electron + neutrino



## ***Inverse Beta Decay***

Proton + electron  $\longrightarrow$  neutron + neutrino



# Urca process

*Free neutrons are unstable!*

## *Beta decay*

Neutron  $\longrightarrow$  Proton + electron + neutrino



## *Inverse Beta Decay*

Proton + electron  $\longrightarrow$  neutron + neutrino



# Urca process

***Free neutrons are unstable!***

## ***Beta decay***

Neutron  $\longrightarrow$  Proton + electron + ***neutrino***



## ***Inverse Beta Decay***

Proton + electron  $\longrightarrow$  neutron + ***neutrino***



***A flood of neutrinos!!***

# Urca process



Mario Schenberg



George Gamow



## *Urca Casino*

***“The energy disappears from the core of the star as quickly as the money disappeared at that roulette table”***

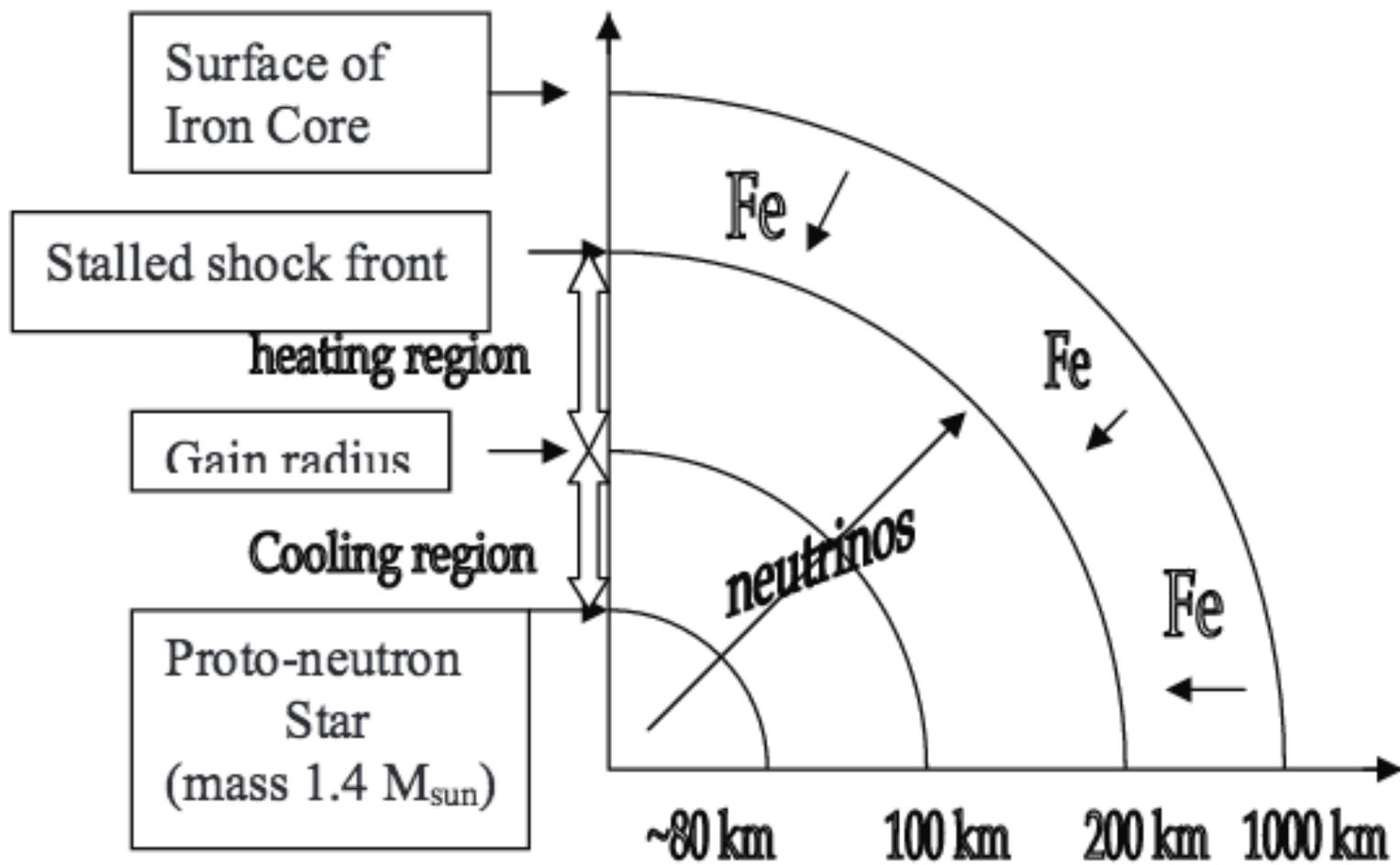


Figure 1. Core collapse Supernova

# Neutron capture

Beyond the Iron peak, nucleosynthesis occur by neutron capture and beta decay



The process is classified according to the neutron flux

## **S-process**

(slow neutron capture)

Neutron capture occurs  
**slower**  
than beta decay

Works up to bismuth (Z=83)

Where?

**AGB stars + Supernovae**

## **R-process**

(rapid neutron capture)

Neutron capture occurs  
**faster**  
than beta decay

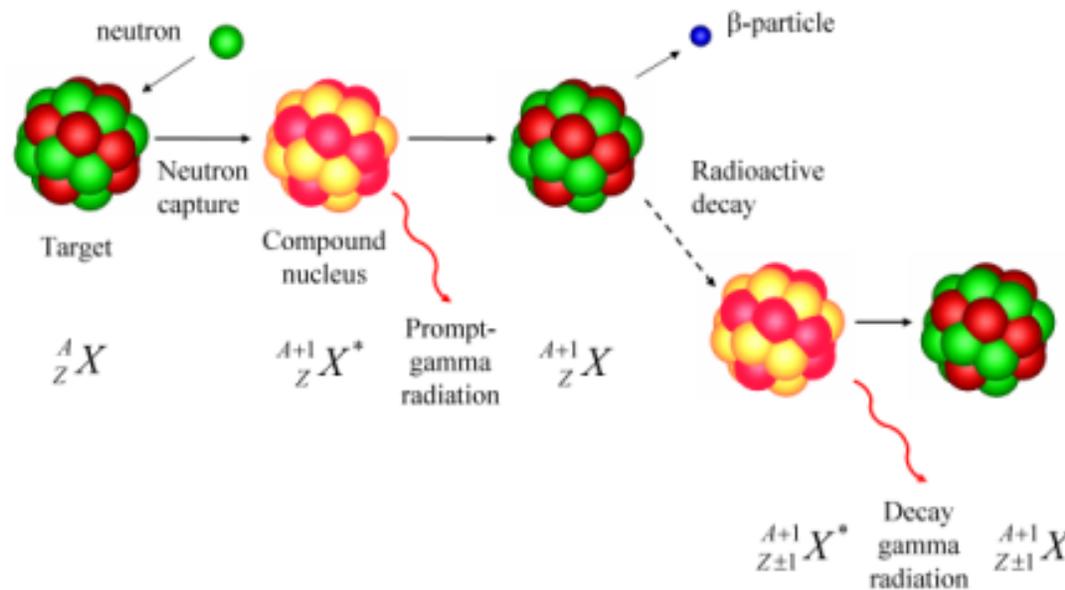
Really heavy stuff  
All the way to Uranium

Where?

**Supernovae**

# Neutron capture

Beyond the Iron peak, nucleosynthesis occurs by *neutron capture* and *beta decay*



Neutron capture produces isotopes

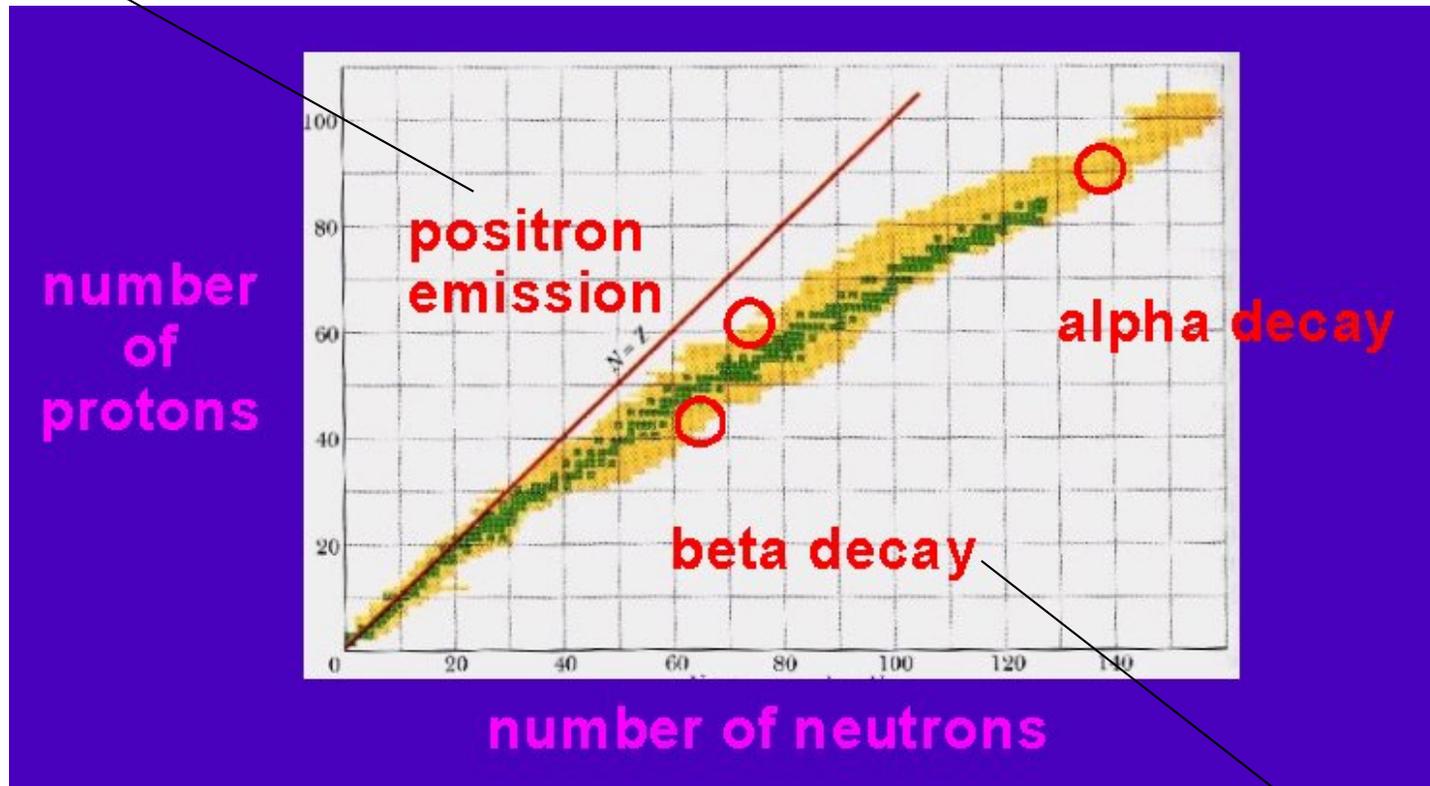
Neutron capture proceeds until the nuclide goes unstable (radioactive)

If a proton decays, the atomic number decreases

**But if a neutron decays, the atomic number increases!**

# Climbing the periodic table

Proton decays



Neutron decays



# Nucleosynthesis summary

Element	# of Protons	Site
H	1	Big Bang
He, C, O	2,6,8	Big Bang + Low and High Mass stars
Ne - Fe	10-26	High mass stars
Co - Bi	27-83	S and R process, AGB and SN
Po - U	84-92	R process in SN

