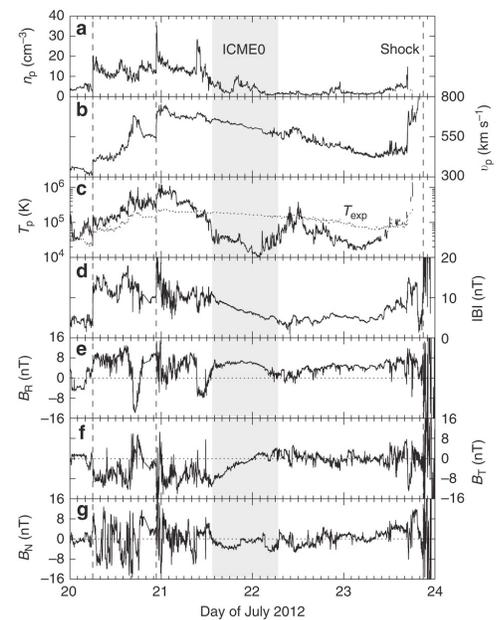


1.2 The Heliosphere

- Derive the equation for the speed of sound in any adiabatic, steady-state, gas and show this depends mostly on temperature
- Calculate the ‘speed of sound’ in space
- Recognize and describe the flaws in calculating the speed of sound in space
- Calculate the escape velocity of the Sun and show the solar wind is supersonic
- Show that ‘energy density’ is equivalent to pressure
- Calculate the location of the heliopause
- Define the locations of the termination shock, heliosheath, heliopause, and bow shock relative to the planets and the Oort cloud
- Derive timescales of space weather and space climate

A large part of both civilian and military operations now relies on the stability of the space environment that surrounds us, known as the heliosphere. When this (usually stable) environment turns hostile, space operators need to know why, when, and how this stability will be disrupted. The stable space environment is governed by the quiet Sun (Unit 1) and the smooth solar wind (Unit 2). The hostile space environment is produced by solar flares and coronal mass ejections (CMEs) (Unit 3). You can see typical in-situ (at 1AU) values for the stable environment and the hostile environment in the plot on the right. Early on 20 July, 2012, space was quiet. Then the front of a CME arrived, the body of the CME arrived, and late on 23 July, another shock occurs. If magnetism (See Unit 1.1) is our paint, then this quasi-steady heliosphere is our canvas. So, before progressing any further, we should clearly define time scales and size scales of the heliosphere, recognize what determines these, and use this to define the regions that exist inside it, and terms that we’ll use over the semester.



1.2 The Heliosphere

1.2a The characteristic speed of space plasma

The first step in considering any gas or plasma is to calculate the characteristic speed of the environment. This tells us the rate at which information is transferred through the medium, and is commonly just the sound speed. The behavior of any flow through this gas is dramatically different if the speed of the flow is less than, or greater than, the sound speed of the gas. Of course, in space “No One Can Hear You Scream”, so the idea of a ‘sound speed’ is somewhat of a misnomer, but we should derive the equation for sound speed and calculate that value anyway. Then we’ll revisit the obvious flaws in our assumptions that we’ve adopted

Starting with the usual equation for the speed of sound, c_s

$$c_s^2 = \partial P / \partial \rho, \text{ where } P \text{ is pressure and } \rho \text{ is mass density}$$

and assume adiabatic expansion $P \propto \rho^\gamma$, where γ is specific heat ratio

$$\begin{aligned} P &= k \rho^\gamma \\ \partial P / \partial \rho &= k \gamma \rho^{\gamma-1} \\ &= (k \rho^\gamma) \rho^{-1} \gamma \\ &= \gamma P \rho^{-1} \end{aligned}$$

And so substituting back into the equation for the speed of sound,

$$\begin{aligned} c_s^2 &= \gamma P \rho^{-1} \\ &= \gamma (n k_B T) (m n)^{-1}, \text{ where } P = n k_B T \text{ for a steady state gas, } n \text{ is number density} \\ &= \gamma k_B T m^{-1} \end{aligned}$$

And so the speed of sounds depends (almost entirely) on Temperature. Our in-situ measurements show that the electron temperature in the heliosphere is about $T_e \approx 10^5 \text{K}$, and so by adopting cgs standards for the Boltzmann constant, k_B , and proton mass

$$\begin{aligned} c_s^2 &\approx (5/3)(1.4 \times 10^{-16})(10^5) / (1.7 \times 10^{-24}) \text{ cm}^2 \text{ s}^{-2} \\ &\approx 1 \times 10^{13} \text{ cm}^2 \text{ s}^{-2} \\ &\approx 10 \times 10^{12} \text{ cm}^2 \text{ s}^{-2} \end{aligned}$$

$$\begin{aligned} c_s &\approx 3 \times 10^6 \text{ cm s}^{-1} \\ c_s &\approx 30 \text{ km s}^{-1} \end{aligned}$$

Now lets look at our bad assumptions.

First the concept of ‘sound speed’ depends on a high density of particles that each particle can ‘feel’ its neighbor within some short distance. In the heliosphere, densities are about 1 cm^{-3} (contrast with $10^{19} - 10^{20} \text{ cm}^{-3}$ on Earth), leading to a mean free path (distance a particle can travel before it has a 100% likelihood of interacting with another particles) of about 1AU (10^{13}cm). This is why no can hear you

1.2 The Heliosphere

scream, as $v \approx (3e6) / (10^{13}) \approx 10^{-7}\text{Hz}$, is far below our audible lower limit of 20Hz. The region of space between the Sun and Earth is almost collisionless.

Second, in such a low density environment, our intuition of temperature is correct. Although space may have a temperature of $\sim 10^5\text{K}$, the density is so low that we would not feel hot (note astronauts doing spacewalks at the ISS do not boil up, despite the ‘temperature’ of a few thousand degrees outside the space station). As the density is low, the effective pressure that you would feel in space is tiny, so the amount of overall kinetic energy transferred to you from the space plasma electron is small. We have to consider ‘Temperature’ as a measure of the average kinetic energy of each type of particle in space, and so, as we’ll see below, each ion and charge state of each species will have its own temperature.

Third, the heliosphere is not ‘steady state’ and is not ‘adiabatic’. As we’ll see later in the course, the solar wind is actually accelerated and flows from the Sun through the heliosphere.

Despite these three obvious flaws, we’ve actually stumbled upon the real value of the characteristic speed of space plasma. It is actually given by the ion-acoustic speed,

$$C_s^2 = \frac{k_B (Z_i \gamma_e T_e + \gamma_i T_i)}{m_i + m_e}$$

calculated individually for each species and for each charge state (Z), and where $\gamma = 1 + 2/(\text{dof}^1)$.

Where the plasma is almost all ionized hydrogen, $Z=1$, $T_e \gg T_i$ (as electrons are easier to move than protons) and $m_i \gg m_e$, then $c_s \approx \gamma k_B T_e m_p^{-1} \approx 10 \text{ km s}^{-1}$ as we calculated above. This can be considered as the characteristic speed of space plasma, due to the pressure of the electrons and the mass of the ions.

¹ dof is degrees of freedom, set $\text{dof}=3$ and $\gamma \approx 1.6$

1.2 The Heliosphere

1.2b Everything is supersonic

It is rather trivial to calculate the escape velocity of the Sun as that speed required for any particles to obtain sufficient kinetic energy to overcome the gravitational pull of the Sun².

$$\begin{aligned}v_{\text{esc}} &= (2GM/R)^{0.5} \\ &\approx (2(7 \times 10^{-8})(2 \times 10^{33}) / (7 \times 10^{10}))^{0.5} \\ &\approx (4 \times 10^{15})^{0.5} \\ &\approx (40 \times 10^{14})^{0.5} \\ &\approx 6 \times 10^7 \text{ cm s}^{-1} \\ &\approx 600 \text{ km s}^{-1}\end{aligned}$$

The key point is that the escape velocity of the Sun is much larger than the characteristic speed of interplanetary space. Anything leaving the Sun is automatically going to be supersonic. So we have a heliospheric bubble around the Sun that is completely dominated by supersonic outflow.

² Set KE = PE, $(1/2)mv^2 = GMm/R$, then 'm' cancels so the escape velocity does not depend on particle mass.

1.2 The Heliosphere

1.2c The heliopause

As the Sun moves around the galactic center, it will carry around a bubble of its own particles - an outflow of particles from the Sun forms a protective sphere around the Sun. The outflowing supersonic solar particles carry along a ram pressure that pushes against the in-situ pressure of the interstellar medium (ISM). We would like to know how far into the interstellar space the Sun can win this battle against the ISM.

The thermal pressure in the ISM can vary (a lot) depending on whether we are discussing hot / cold clouds, diffuse / dense, HI/HII, etc. It also can be supplemented by a magnetic pressure in the ISM. However, adopting average values of $n_{\text{ISM}} \sim 10\text{-}20 \text{ cm}^{-3}$, and $T \sim 500\text{K}$, then the interstellar pressure is

$$\begin{aligned} P_{\text{ISM}} &\approx nK_{\text{B}}T \\ &\approx (15)(1.4 \times 10^{-16})(500) \\ &\approx 10 \times 10^{-13} \\ &\approx 10^{-12} \text{ g cm}^{-1} \text{ s}^{-2} \quad (\text{which you'll see in the literature as } 10^{-13}\text{Pa in SI})^3 \end{aligned}$$

This is the pressure that the solar wind will have to meet and overcome to form the protective bubble of the heliosphere. Note how small this is: by simple definition, take the force of 1N that accelerates a 1kg mass by 1 m s^{-2} , and spread that out of 1 m^2 and you've got a 1Pa pressure (the pressure of a 1 dollar bill on the surface of a table). Our ears can detect down to 10^{-4}Pa , ultra-high vacuum in labs are 10^{-7}Pa

Now we introduce the concept that 'energy density' is the same as 'pressure' as we're going to come across this terminology several times throughout this course. The simplest way of seeing this connection is via a series of basic definitions.

Energy density = energy / volume = force x distance / volume = force / area = pressure.

So consider 'pressure' as the density of (any type of) energy in any plasma. In the solar wind, the kinetic energy is just $\sim m_p v_{\text{sw}}^2$ and so kinetic energy density (KED) is then $n_p m_p v_{\text{sw}}^2$

So with typical values as measured at 1AU (See figure above)⁴

$$\begin{aligned} \text{KED} &= m_p \times n_p \times v_{\text{sw}}^2 \\ &\approx (1.7 \times 10^{-24}) \times (5) \times (4 \times 10^7)^2 \\ &\approx (100 \times 10^{-10}) \text{ g cm}^{-1} \text{ s}^{-2} \end{aligned}$$

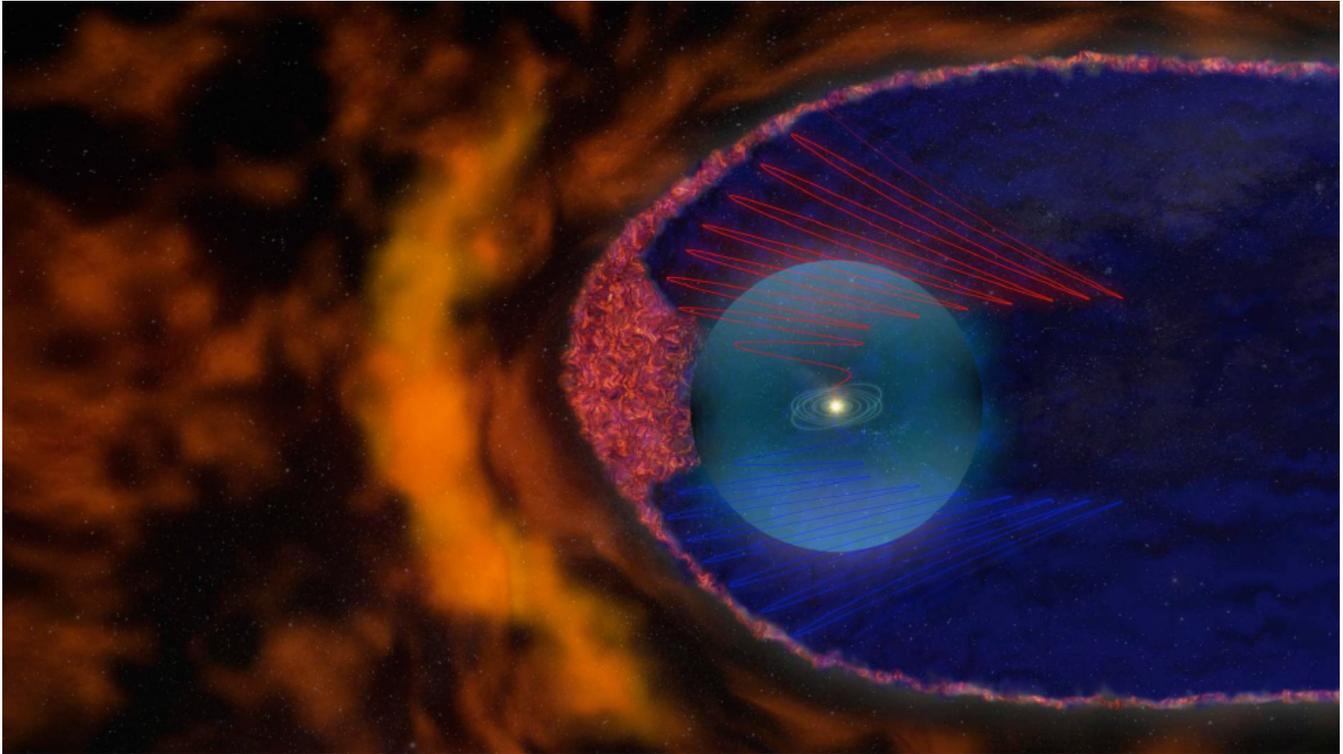
Assuming the kinetic energy density of the solar wind falls off with increasing distance from the Sun, then the dynamic pressure of the solar wind will eventually equalize with the interstellar medium. If we assume that the kinetic energy density of the solar wind falls as r^{-2} , then this point at which the pressure balances with the ISM pressure will be around 100AU. This is the heliopause.

³ Our nearby ISM is actually quite hot and dense. More typical interstellar values in the Milky Way are about 10^{-5}Pa

⁴ As the solar wind is accelerated over several solar radii above the surface, it does not have to reach the escape velocity we calculated above.

1.2 The Heliosphere

1.2d A brief tour of the heliosphere



Once viewed as a strange mix of aeronomy, astronomy and astronautics, solar physics sat outside astrophysics. Indeed NSF and NASA still both have Heliophysics divisions separate from astrophysics. This separation at the funding agencies derives from this historical split. We now use the phrase of ‘heliosphere’ to mean everything inside the part of space dominated by the Sun. The heliosphere spans the gambit of space from the solar interior, out through the solar atmosphere and all way to the heliopause at about 100AU.

As the heliopause (red cloudy material above) barrels along thru the ISM, it builds up and deflects the ISM plasma ahead of it, around and into a tear-shaped structure. At the very front of this turbulent region (orange above, around 230AU) there may be a bow-shock, where the ISM plasma is slowed down to being subsonic. However if this region of the ISM is not moving fast enough, or if the heliosphere hits it obliquely (such that the effective speed of the ISM is too slow), or if the ISM density is too low, or too cold, there will be no bow shock. Note even here at several hundred AU we are not at the end of the Sun’s gravitational influence - the Oort Cloud resides at 5000-100,000AU), and only then do we arrive half way to Alpha-Centauri (~300,000 AU).

Inside the heliopause, there is another region of turbulence (red and blue squiggles) and another shock. As the solar wind moves away from the Sun, its speed remains pretty constant. However it eventually slows down and becomes sub-sonic. The point where the solar wind becomes slower than the ion-acoustic velocity is called the termination shock. Data from Voyager 1 showed that it crossed the termi-

1.2 The Heliosphere

nation shock and entered the Heliosheath in the middle of December 2004, at a distance of 94 AU. Beyond the (almost isotropic) termination shock, the solar particles enter this turbulent Heliosheath region, which is then defined to end at the heliopause. NASA announced in 2013 that Voyager 1 had encountered the Heliopause on 25 August 2012, when the spacecraft measured a sudden 40-fold increase in plasma density. It is now beyond the Heliopause, and into the ISM.

Heliophysics deals with all physics from size scales on electron-electron interactions up to the Heliopause, and from rapid ionization timescales of microseconds up to many solar cycles. Consider the length of time it takes for a solar proton in a solar wind of 400 km s^{-1} to traverse the 100AU size scale of the heliosphere.

$$\begin{aligned}\tau &\approx (100)(1 \times 10^{13}) / (4 \times 10^7) \\ &\approx 0.25 \times 10^8 \text{ s} \\ &\approx 1 \text{ year}\end{aligned}$$

This is our canvas on which we're now going to build our magnetic structures. Anything occurring on timescales less than 1 year (e.g., the day-to-day interactions between the Earth's magnetosphere and the constituents of interplanetary-space) are commonly referred to as *space weather*. *Space weather* is a consequence of the magnetic behavior of the Sun as well as the magnetic nature of a planet and its location in the solar system. Our attempts to explain variations in these interactions are commonly referred to as space physics.

Anything occurring on longer timescales is more commonly referred to as *space climate*, e.g., long term solar emissions and their effect on the heliosphere and the planets inside it. As a practical matter space weather often refers to a disturbed situation whereas space climate often refers to a typical or background condition.
