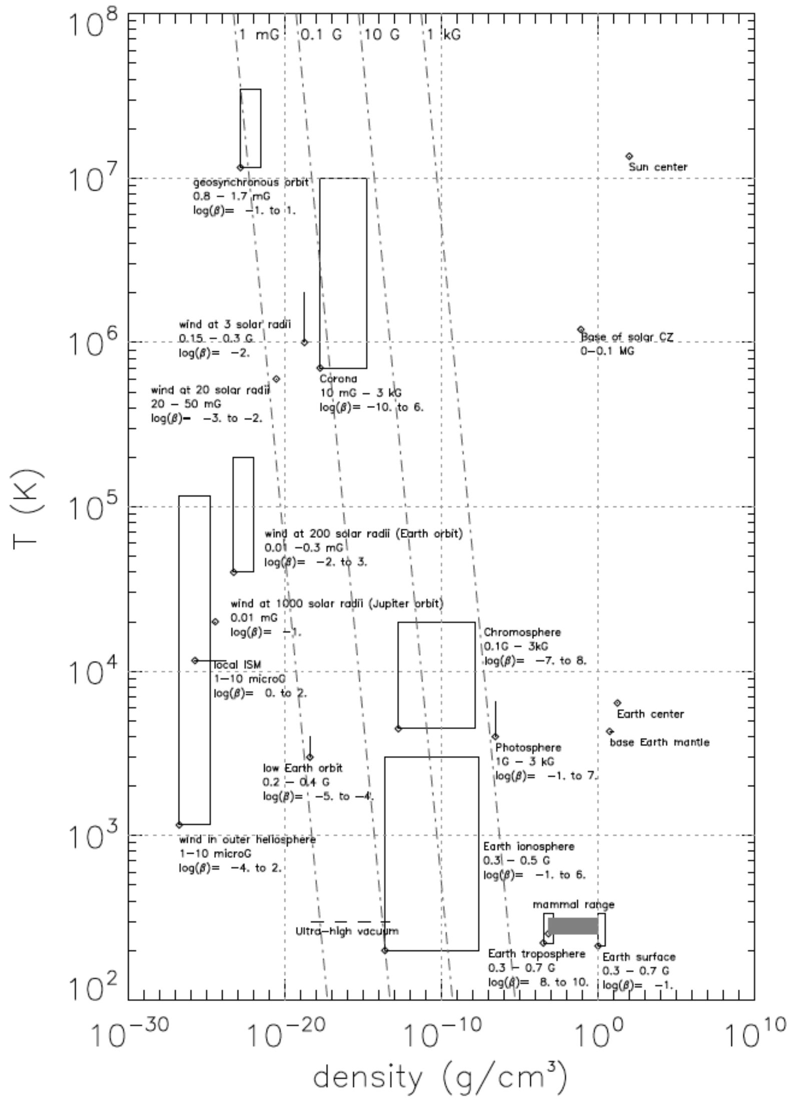
1.3 Plasma parameters

- **Recognize various locations of the Sun on a temperature - density scatter plot**
- **Calculate the wavelength of light at various temperatures.**
- **Describe the convective process in the Sun**
- **Calculate the up flow convective velocity and recognize its importance**
- **Formulate the ideal gas equation**
- **Formulate an equation for plasma β**
- **Show how the plasma β varies with density, temperature and magnetic field**
- **Recognize structural differences between a low β and high β plasma**
- **Calculate typical plasma β values throughout the solar atmosphere**

The normal ‘local’ cosmos exhibits an enormous diversity of conditions. The key objective of the astronomer is to describe and characterize that diversity using as few parameters as possible. One such set of parameters are density, temperature and magnetic field. The comparison of temperature and density help to visualize the thermal energy, and the magnetic field will let us compare that to the magnetic energy. All together, these physical parameters cover a wide range of states that include solids, liquids, gases, and ionized and magnetized particle ensembles called plasmas.

1.3 Plasma parameters

1.3a Typical plasmas conditions in the Heliosphere



This plot summarizes the entire heliosphere in just 2 simple parameters - temperature and density. Note it spans 29 orders of magnitude of density, but only 6 orders of magnitude in temperature. Considering what we found in section 1.2, it will be no surprise to see that temperature, but itself, is insufficient to understand a plasma. Starting at the core of the Sun, the temperature is up to 10^7K. But note the temperature at locations far removed from the nuclear fusion process is almost the same - the solar corona is can be up to 10MK and the temperature above the earth in geosynchronous orbit can be even higher than the solar core. As the sun's core is clearly not 'the same' as the the solar corona or geosynchronous, the second parameter, density, is required to help us differentiate between these regions of the heliosphere. We may get the same temperature in the corona and near-Earth, but there are over 16 order of difference

1.3 Plasma parameters

in density. We'll see that the thermal pressure of a plasma is defined by the temperature and pressure, so the top right of this plot is high thermal pressure and this pressure falls off as either, or both, of temperature and density fall off. As thermal energy drops off, with either low pressure or low temperature, other forms of energy (e.g., magnetism) dominate the plasma.

Note the connection between temperature and type of light emitted, and the connection between density and the ability to observe that light. As an example of these two connections, consider the base of the convection zone and the corona. It takes photons 200,000 years to get through the random walk of the radiative zone and in doing so the photons have lost energy (and hence temperature).

$$E = hc/\lambda \approx kT$$
$$\lambda \approx ((7 \times 10^{-27})(3 \times 10^{10})) / ((1.4 \times 10^{-16})(1.5 \times 10^{6}))$$
$$\approx 10^{-6} \text{ cm which is about } 100\text{Å}$$

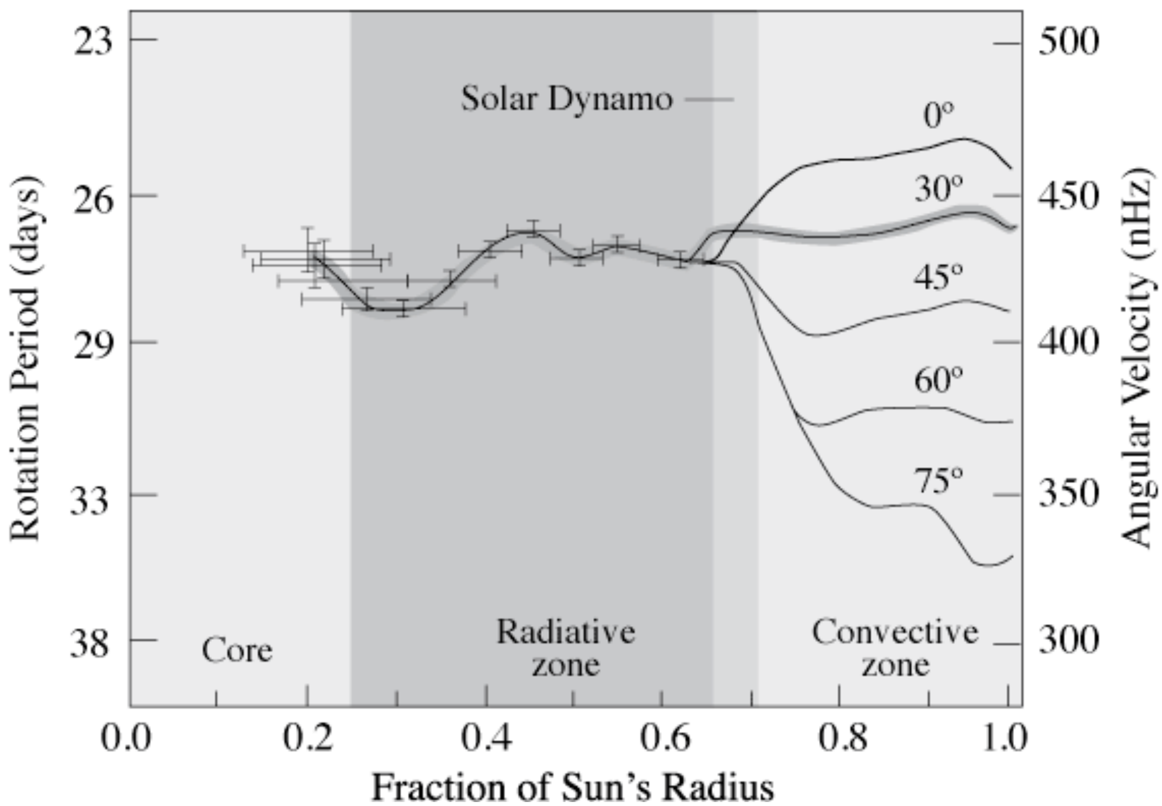
which is in the EUV. Note the Sun reaches a similar temperature in the corona and hence the corona also emits in the EUV. The fact we can see the corona in the EUV and that we cannot see the radiative zone to do with the density of the material in between. We can observe the corona as there is basically nothing between it and us. We cannot observe the top of the radiative zone, because the cool, thick photosphere gets in the way, and

By the time the photons that pass through the radiative zone reach the surface, they have lost more energy and so gets radiated as mostly optical light. This limits our direct observations to photosphere and above (chromosphere and corona), but there are other way of studying the interior, so let's start there.

1.3 Plasma parameters

1.3b From the Tachocline to the surface

The tachocline is the layer in the interior of the Sun where the solidly rotating core meets the differentially rotating convective envelope. From observations we see that the surface rotates differentially - i.e, the equator rotates faster than the poles. At the equator the rotation is, the rotation period is 25.7 days (450 / 2π nHz). At 60degrees, the rotation period is 31.2 days (370 / 2π nHz). From helioseismology we infer the convection zone also rotates differentially. But around a depth of 0.7 solar radii, the interior begins to rotate as a solid body. It is in this shearing layer just the convection zone, where solid rotation becomes differential rotation, that magnetism (flux tubes) are created and stored. Although naturally buoyant, downward convective down flows can keep the flux stored at this location until a returning up flow occasionally dredges up a flux tube. Lets being a journey there



Lets start just below the base of the convective envelope, and imagine an ion embedded in a magnetic bundle (flux tube). As the ion rises, opacity increases as the electrons recombine with H and He as the temperature drops. Consequently the ability for a rising volume of gas to exchange energy with its surroundings drops. This creates an adiabatic process. As the rising hot gas moves up into regions of lower pressure, it expands and the density of the rising hot gas decreases. Classical convection has now set in and creates an efficient means to transport the flux tube to the surface, where is can 'poke' through the surface. The fact that we see dipoles appearing at the surface has two critical implications.

1.3 Plasma parameters

(1) The flux tube must somehow stay intact and make it up through the turbulent shredding layer just below the photosphere. Just below the photosphere, the pressure drops off rapidly, so any emerging structure should expand and should be destroyed. But the flux tube does not expand, the structure is not destroyed. We can observe dipole structures emerge down to the resolution limit of modern telescopes (100km). Probably, the magnetic tension is the ultimate cause or the retention of small scale structure. Increased twist means more current and increased tension (we'll see this mathematically in Section 2), counteracting the urge to expand.

(2) The travel time through the convection zone must be similar to the rotation time of the Sun. If the travel time were much longer the flux distribution would be more mixed and if it were shorter we could not see any differential rotation on the surface. This also means the convective up flow velocity is fast enough to allow for the adiabatic convective process

<u>Calculate the convection zone up-flow timescale</u>

The largest convective cell we can observe on the Sun are supergranules. They are typically about 30,000km in size and have a lifetime of about 2 days

From, this the convective flow velocity is about

$$v \approx 3 \times 10^4 / (2 \times 8.4 \times 10^4) \text{ km/s}$$
$$\approx 0.18 \text{ km /s}$$

Let's assume (again incorrect assumption, but it will good enough for now and we'll come back to it later to assess the problems) that one big convection cell goes all the way through the entire convection zone and velocity is the same the whole way down.

$$\tau \approx \text{(distance from tachocline to photosphere) / (up-flow velocity)}$$
$$\approx (0.3 \times (7 \times 10^5)) / 0.18$$
$$\approx 12 \times 10^5 \text{ s}$$
$$\approx 15 \text{ days}$$

This estimate is pretty close to the actual rotation rate of the Sun, which we saw above is 25 to 31 days. However, we'll see later that as supergranules are much flatter than our 'column' assumption above, we need another larger cell, or multiple layered cells, to fully explain the convection zone. For, let's work with the timescale we've got, and, finally, at the surface, we can now continue our journey into the solar atmosphere. The emerging flux gets jostled by multiple surface flow, among these are granular cells, supergranular cells, differential rotation, and meridional advection at the surface. It is this jostling that leads to a build up of magnetic energy in some structures.

1.3 Plasma parameters

1.3c The plasma β

We've seen that convection parameters (velocity, density, timescales) played the most important role in the solar subsurface. But we still had to consider magnetism, as flux tubes were generated at the tachocline and the magnetic energy helped them remain a coherent structures as they passed through that subsurface turbulent layer. Now at the photosphere, a series of flows will make magnetism even more important. The two key parameter in describing the plasma for the rest of the journey through the solar atmosphere and into the heliosphere are the Plasma Beta, β and the Magnetic Reynolds number, R_M (which we'll see in section 1.4).

The single most important parameter in interpreting the behavior of any astrophysical plasma is the plasma β parameter. In this section we will derive an equation for the beta parameter, sketch how beta varies throughout the solar atmosphere, and compare our sketch to the real solution.

The plasma β parameter is the ratio of the thermal pressure to the magnetic pressure.

The equation for thermal pressure can be derived from the ideal gas law. Starting from the three empirical gas laws, let's derive the ideal gas equation. First of all state the three equations mathematically

Boyle's law: For a fixed amount of an ideal gas kept at a fixed temperature, P [pressure] and V [volume] are inversely proportional.

$$P = k_1 / V$$

Charles's law: At constant pressure, the volume of a given mass of an ideal gas increases or decreases by the same factor as its temperature on the absolute temperature scale (i.e. the gas expands as the temperature increases)

$$V = k_2T$$

Gay-Lussac's law (or Amonton's law): The pressure of a gas of fixed mass and fixed volume is directly proportional to the gas' absolute temperature.

$$P = k_3T$$

Combine them all to form the combined gas law.

$$P^2V = k_1k_2k_3 (T^2 / V)$$
$$P^2V^2 = k_1k_2k_3 T^2 \quad = k_4 T^2$$
$$PV = kT$$

The k constant is actually proportional to 'fixed amount' of ideal gas, so it is just the Universal Gas Constant, R multiplied by the 'fixed amount' (in number of moles) in the above 3 laws. It is more intuitive and more useful to equate $RN_{moles} = k_BN_{particles}$,where k_B is the Boltzmann constant and $N_{particles}$ is now the number of particles. We will drop the 'particles' subscript, and state that.

$$PV = Nk_BT$$

1.3 Plasma parameters

This is the more intuitive form of the ideal gas law as it relates macroscopic quantities on the left (volume, pressure) with the microscopic (N units of $k_B T$ energy each) on the right. In a plasma, all electrons are ionized. Hence the preferred quantity of study is the electron number density, n_e.

$$P_{th} = (N/V)\, k_B T = 2 n_e k_B T$$

where the factor of 2 means we include both electrons and ions, and I've included a 'thermal' subscript.

The equation for magnetic pressure will, for now, be defined in cgs units as

$$P_m = B^2 / (8\pi)$$

Note, as before, the pressure is the same as energy density. We'll find that any pressure is always proportional to the square of the appropriate field strength.

It is trivial to combine the two equations above to produce an equation for the plasma β. In doing so let's combine all the constants and state the equation in terms of mass density instead of number density. This will let us refer to the figure in chapter 1.3a above.

<u>Show how β varies with ρ, T, and B</u>

$$\beta = (2 (\rho / m_p)\, k_B T\, 8\pi) / B^2$$
$$\approx (16\, \pi\, (1.4 \times 10^{-16}) / (1.7 \times 10^{-24}))\ (\rho\, T / B^2)$$
$$\approx (50 \times 10^{-16+24})\ (\rho\, T / B^2)$$
$$\approx (5 \times 10^{9})\ (\rho\, T / B^2)$$

$$\beta \approx (10^{10}\, \rho\, T / 2\, B^2)$$

We see that the magnetic field plays a more dominant role (i.e, it is squared) than either of density or temperature. Clearly at any location of constant density and temperature in the Sun's atmosphere, a few small changes in the magnetic field can make a big difference. Where magnetism dominates we expect to see linear coherent, twisted, structures and where thermal pressure and gravity dominates we expect to see more homogenous plasmas. One further thing to consider is that density and temperature tend not to vary much <u>within one layer</u> of the atmosphere, whereas the magnetic field can vary horizontally by 3-4 orders of magnitude.

1.3 Plasma parameters

Now have a look once again at the figure in 1.3above, lets adopt sensible values for density and temperature, and calculate the magnetic field that we would need to get to β =1

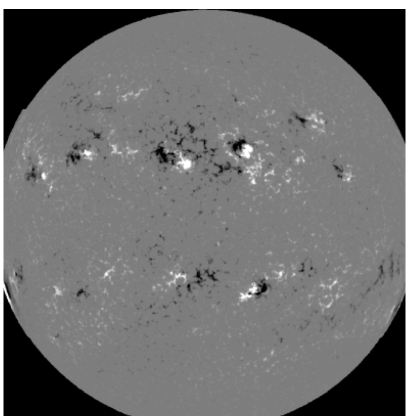
i.e., lets ask ourselves the following key question

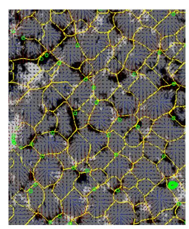
What is the minimum magnetic field strength at which magnetism will being to dominate the behavior of the plasma?

For the photosphere, lets take $\rho \approx 10^{-7}$ g cm^{-3}, $T \approx 5 \times 10^{3}$ K,

$$\beta \approx (10^{10})(10^{-7})(5 \times 10^{3}) / (2 B^{2}) \approx 2 \times 10^{6} / B^{2}$$

So, the all-important β ≈ 1 occurs at around 1kG. Typically, non-magnetic quiet Sun areas have a field strength of a few 50-100G. So the photosphere can be considered thermally dominated. However inside sunspots and at the boundaries of both granular and supergranular flows, the magnetic field can be 1kG or higher, and the temperature (and sometimes, density) can be lower, and so these areas are magnetically dominated.





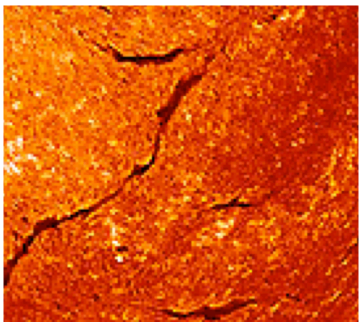
The photosphere 'looks' mostly thermal, i.e., featureless. But when the field is kG or above, as in sunspots or at junctions of supergranules, magnetism dominates.

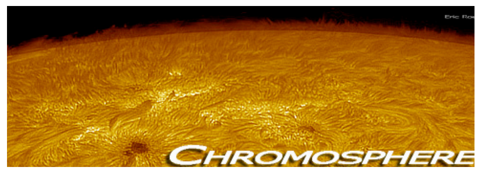
1.3 Plasma parameters

When we reach the chromosphere, we note that the density plummets by 5-6 orders of magnitude and temperature rises slightly, so lets take $\rho \approx 10^{-12}$ g cm^{-3}, $T \approx 10^4$ K

$$\beta \approx (10^{10})(10^{-12})(10^4) / (2 B^2) \approx 50 / B^2$$

So, the all-important $\beta \approx 1$ occurs at a few G. Although the magnetic field strength has dropped, it is still typically a few 10's of Gauss everywhere. So the chromosphere can be considered entirely magnetically dominated. No wonder then that chromospheric images are so full of tube-like structures.



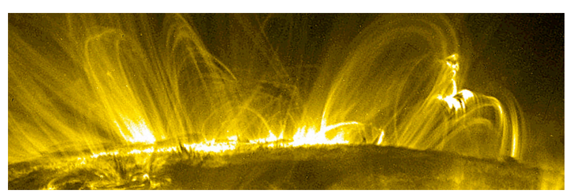


The chromosphere 'looks' almost entirely magnetic, i.e., it is full of long, linear, filamentary features - tubes and sheets. Such features can only be formed when magnetism dominates

For the corona, density has fallen by another 4-5 orders of magnitude, but the temperature profile has reversed back up to a few million K, so lets take $\rho \approx 10^{-16}$ g cm^{-3}, $T \approx 10^6$ K

$$\beta \approx (10^{10})(10^{-16})(10^6) / (2 B^2) \approx 0.5 / B^2$$

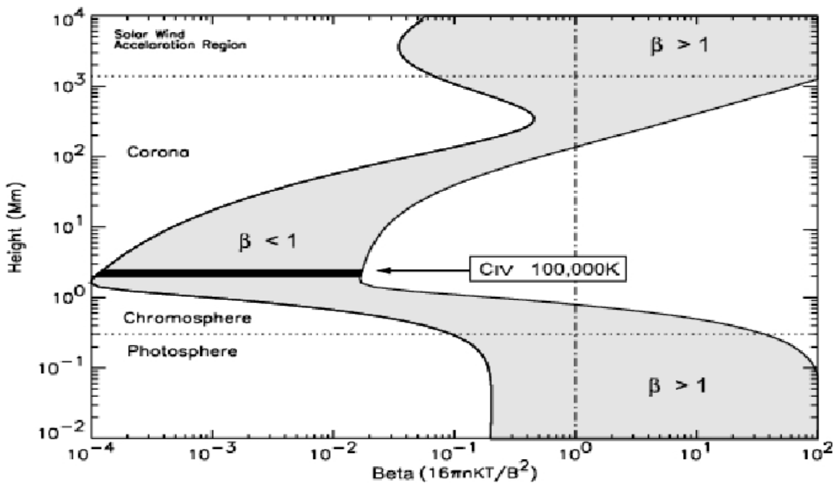
So, the all-important $\beta \approx 1$ occurs around 0.1 G. The magnetic field in the corona is usually greater than this (around 10G) and so the corona is mostly magnetically dominated. Again this is expected as we see images of loop-like structures. To find a thermally-dominated region in the corona, we would need a higher temperature or density and it just-so-happens that regions of higher temperature and density also have stronger field strength. However there are some areas where the magnetic field may drop off and temperature remains high enough for thermal pressure to dominate and we'll revisit these areas in our study of the solar wind.



The corona 'looks' almost entirely magnetic, i.e., it is full of long, linear, filamentary features - loops, sheets, arches. Such features can only be formed when magnetism dominates

1.3 Plasma parameters

So the complete picture of the Sun's atmosphere looks likes this.



It is worth repeating that whereas the magnetic field strength can vary several orders of magnitude across any layer (i.e, horizontally), it tends to vary slowly as we move up through the Sun's atmosphere. In contrast, density and temperature tend to vary rapidly with height, but they exhibit less horizontal variation across any layer.