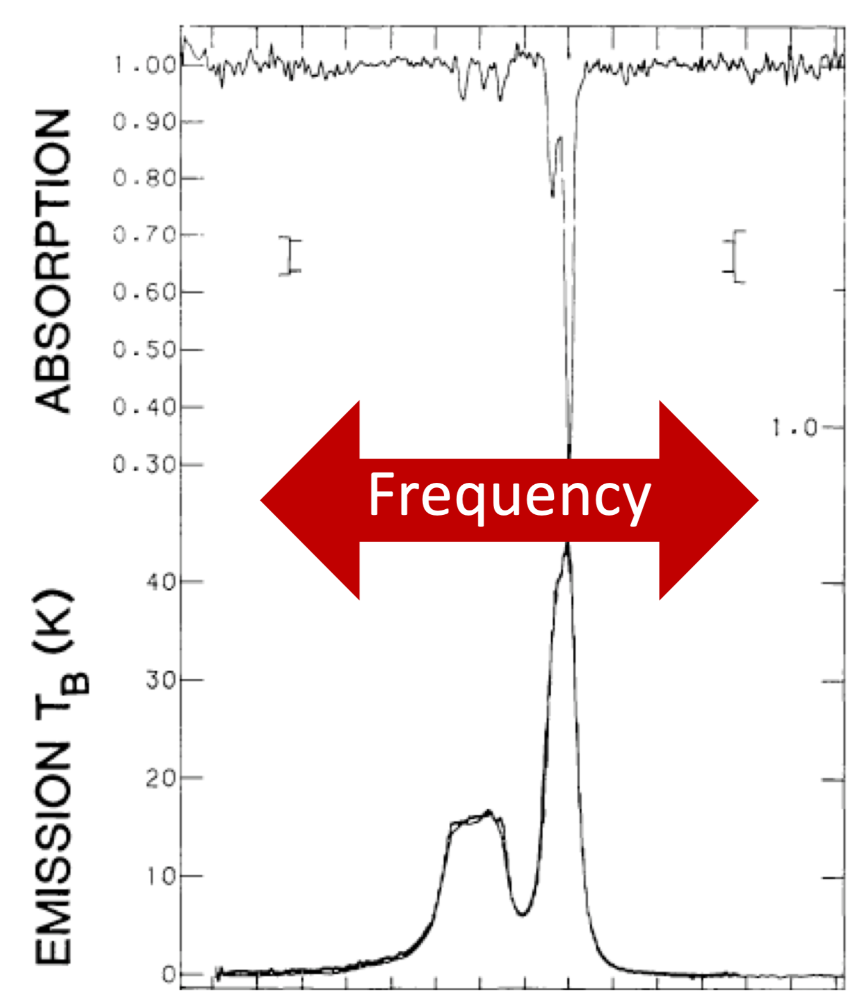
A closer look at the 21cm line



Absorption Lines

Emission Lines

ASTR 605
Joe Burchett
10/4/2021

Hyperfine splitting

The spins of the electron and proton in the nucleus of the hydrogen atom can couple

Ground state energy levels split based on whether spins are 'parallel' or 'anti-parallel'

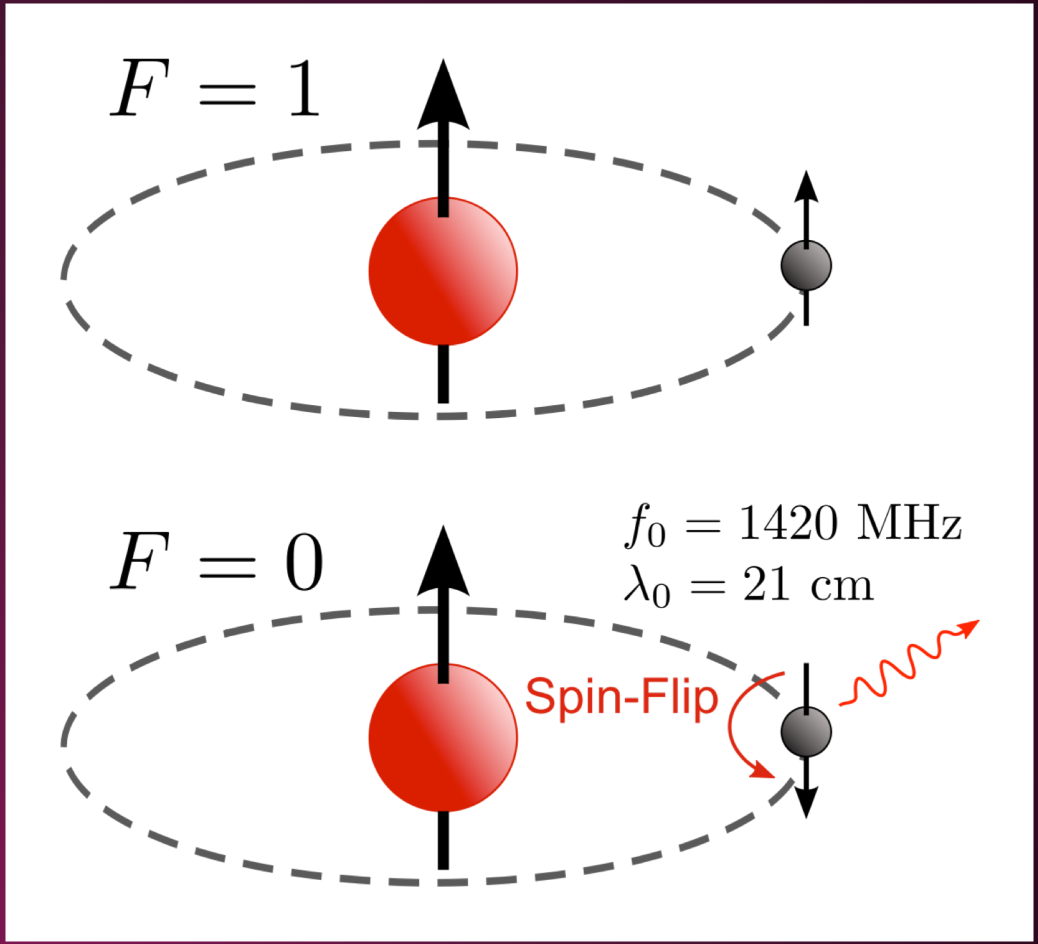
Lower state is that where spins are anti-parallel

$$\begin{aligned} J &\equiv [\text{electronic angular momentum}]/\hbar\,, \\ I &\equiv [\text{nuclear angular momentum}]/\hbar\,, \text{and} \\ F &\equiv [\text{total angular momentum}]/\hbar\,. \end{aligned}$$

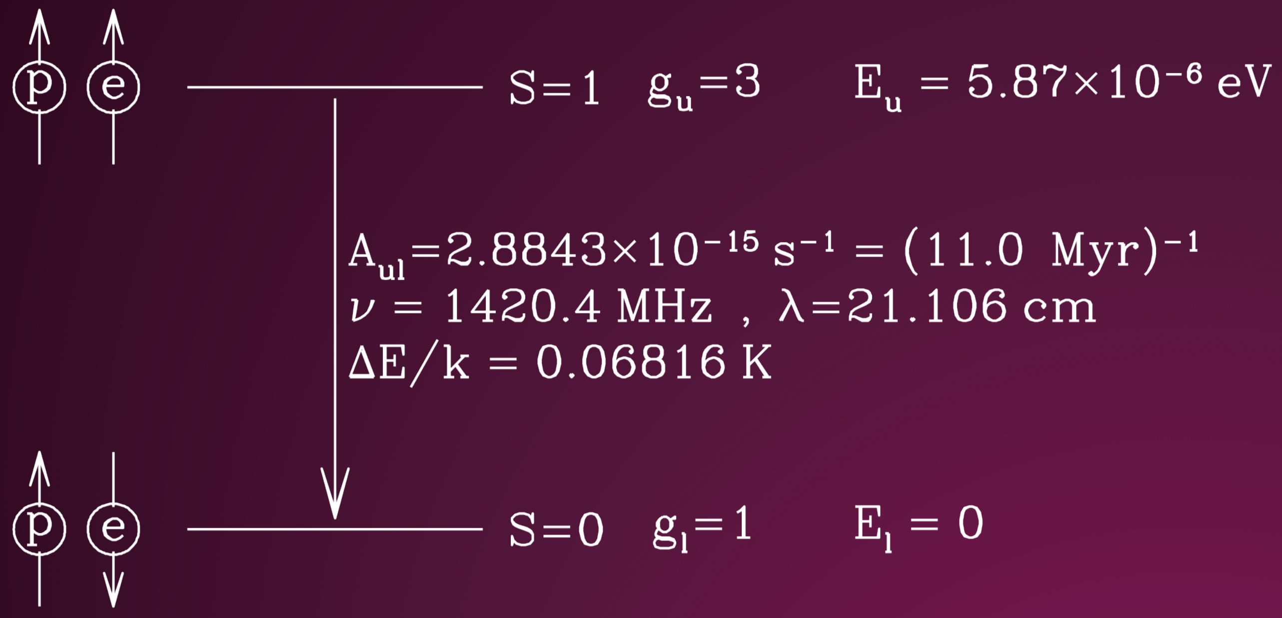
Spin of electron = ½
Spin of proton = ½

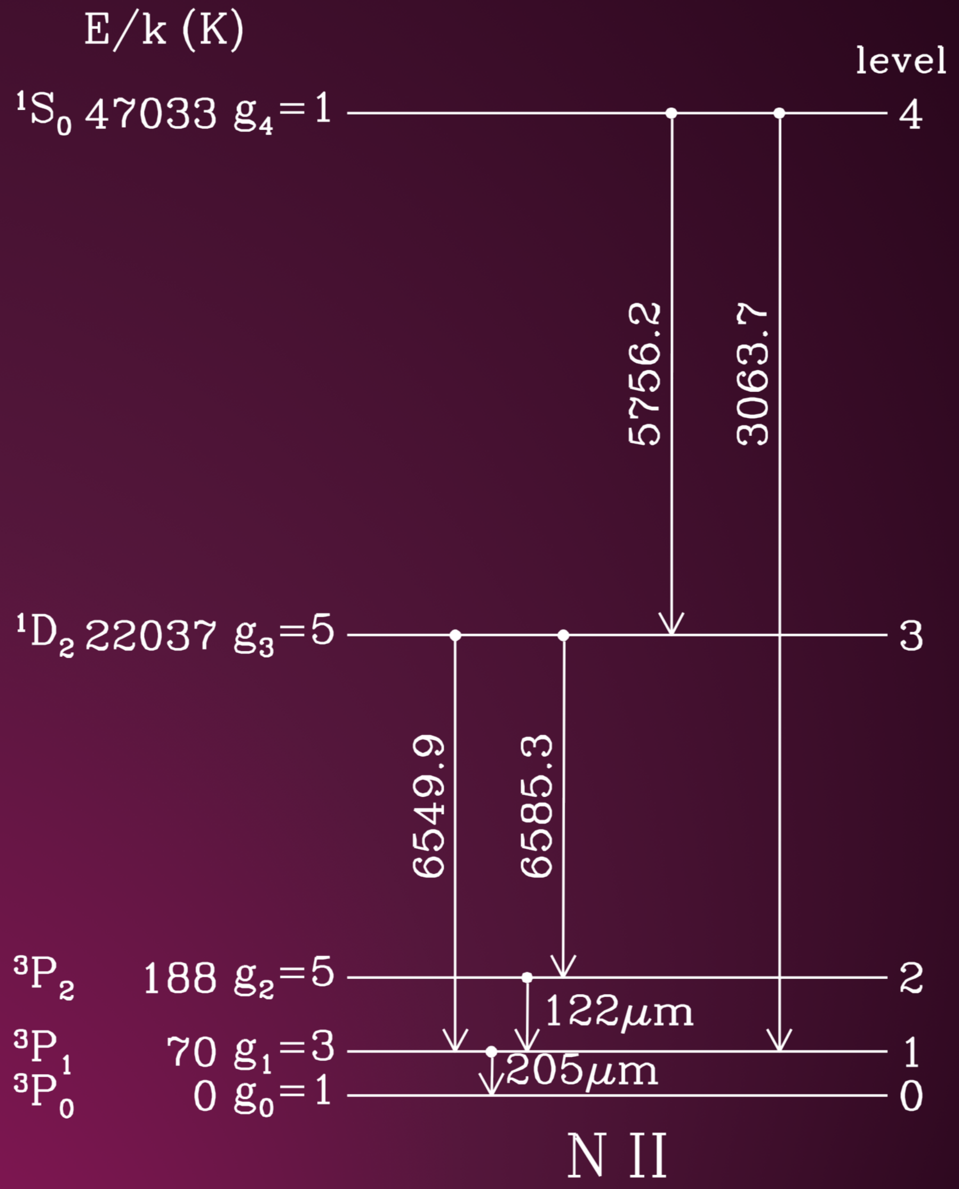
$^2S_{1/2}$

$F = J + I$
$F = 1$ or 0



21 cm transition





Population of states and emissivity

- Excitation temperature often called 'spin temperature' for hyperfine splitting of states
- Upper level excited by CMB photons, thus $T_{\rm exc} \equiv T_{\rm spin} \gg .0682\,{\rm K}$,

$$\frac{n_u}{n_\ell} \equiv \frac{g_u}{g_\ell} {\rm e}^{-h\nu_{u\ell}/kT_{\rm exc}} = 3\,{\rm e}^{-.0682\,{\rm K}/T_{\rm spin}} \approx 3$$

$$n_u \approx \frac{3}{4} n({\rm H\,I}), \quad n_\ell \approx \frac{1}{4} n({\rm H\,I})$$

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell}\,\phi_\nu \approx \frac{3}{16\pi} A_{u\ell}\,h\nu_{u\ell}\,n({\rm H\,I})\,\phi_\nu$$

Emissivity independent of spin temperature!

21 cm absorption coefficient

Contributions from pure absorption and stimulated emission:

$$\kappa_\nu = n_\ell \sigma_{\ell u} - n_u |\sigma_{u\ell}|$$

Recall relationships between Einstein coefficients and with the cross section

$$B_{u\ell} = \frac{c^3}{8\pi h \nu^3} A_{u\ell} \qquad B_{\ell u} = \frac{g_u}{g_\ell} B_{u\ell} = \frac{g_u}{g_\ell} \frac{c^3}{8\pi h \nu^3} A_{u\ell}$$

$$\left(\frac{dn_u}{dt}\right)_{\ell \to u} = n_\ell \int d\nu \, \sigma_{\ell u}(\nu) c \frac{u_\nu}{h\nu} \approx n_\ell u_\nu \frac{c}{h\nu} \int d\nu \, \sigma_{\ell u}(\nu)$$

$$B_{\ell u} = \frac{c}{h\nu} \int d\nu \, \sigma_{\ell u}(\nu)$$

$$\sigma_{\ell u}(\nu) = \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{\ell u}^2} A_{u\ell} \phi_\nu$$

21 cm absorption coefficient

$$\kappa_\nu = n_\ell \sigma_{\ell u} - n_u |\sigma_{u\ell}|$$
$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \phi_\nu \left[1 - \frac{n_u}{n_\ell} \frac{g_\ell}{g_u}\right]$$
$$= n_\ell \frac{g_u}{g_\ell} \frac{A_{u\ell}}{8\pi} \lambda_{u\ell}^2 \phi_\nu \left[1 - \mathrm{e}^{-h\nu_{u\ell}/kT_{\mathrm{spin}}}\right]$$

$h\nu \sim 10^{-6}$ eV, $h\nu / k \sim 0.0682$

Recall key assumption: Recall $T_{spin} \ll 0.0682$ K

$$\kappa_\nu \approx \frac{3}{32\pi} A_{u\ell} \frac{hc\lambda_{u\ell}}{kT_{\mathrm{spin}}} n(\mathrm{H\,I})\, \phi_\nu$$

21 cm optical depth

$$\kappa_\nu \approx \frac{3}{32\pi} A_{u\ell} \frac{hc\lambda_{u\ell}}{kT_{\rm spin}}\, n(\mathrm{H\,I})\, \phi_\nu$$

Assume Gaussian line profile

$$\phi_\nu = \frac{1}{\sqrt{2\pi}} \frac{c}{\nu_{u\ell}} \frac{1}{\sigma_V} \mathrm{e}^{-u^2/2\sigma_V^2}$$

$$\kappa_\nu = \frac{3}{32\pi} \frac{1}{\sqrt{2\pi}} \frac{A_{u\ell}\lambda_{u\ell}^2}{\sigma_V} \frac{hc}{kT_{\rm spin}} \mathrm{e}^{-u^2/2\sigma_V^2}$$

$$\tau_\nu = 2.190 \frac{N(\mathrm{H\,I})}{10^{21}\,\mathrm{cm}^{-2}} \frac{100\,\mathrm{K}}{T_{\rm spin}} \frac{\mathrm{km\,s^{-1}}}{\sigma_V} \mathrm{e}^{-u^2/2\sigma_V^2}$$

$$N(\mathrm{H\,I}) \equiv \int ds\, n(\mathrm{H\,I})$$

Column densities often exceed 10^{21} cm^{-2} - self-absorption important!

The optically thin case

Optically thin column density regime: $N(\mathrm{H\,I}) \lesssim 10^{20}\,\mathrm{cm}^{-2}\,\frac{T_{\rm spin}}{100\,\mathrm{K}}\,\frac{\sigma_V}{\mathrm{km\,s^{-1}}}$

This allows us to neglect absorption:

$$I_\nu = I_\nu(0) + \int j_\nu ds$$
$$= I_\nu(0) + \frac{3}{16\pi} A_{u\ell}\, h\nu_{u\ell}\, \phi_\nu\, N(\mathrm{H\,I})$$

$I_\nu(0)$ is the intensity measured in an 'off' pointing: $\int [I_\nu - I_\nu(0)]\, d\nu = \frac{3}{16\pi} A_{u\ell}\, h\nu_{u\ell}\, N(\mathrm{H\,I})$

Instead of I_ν, express in terms of 'antenna temperature' T_A:

$$\int [T_A - T_A(0)]\, du = \int \frac{c^2}{2k\nu^2}\,[I_\nu - I_\nu(0)]\,\frac{c}{\nu} d\nu$$
$$= \frac{3}{32\pi}\,\frac{hc\lambda_{u\ell}^2}{k}\, A_{u\ell}\, N(\mathrm{H\,I})$$
$$= 54.89\,\mathrm{K\,km\,s^{-1}}\,\frac{N(\mathrm{H\,I})}{10^{20}\,\mathrm{cm}^{-2}}$$

Masses of optically thin clouds

Mass in frequency units:

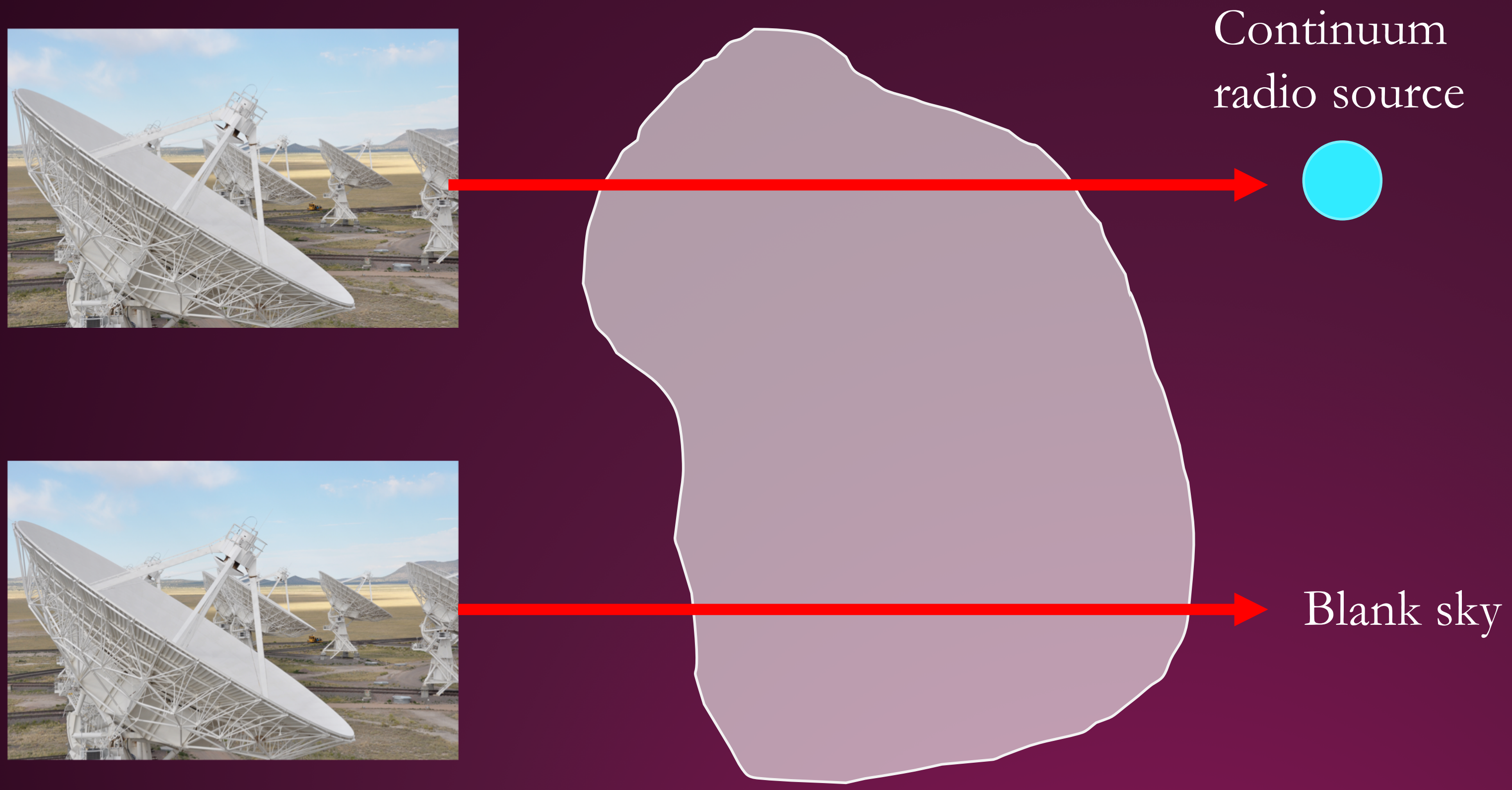
$$M_{\mathrm{H\,I}} = \frac{16\pi m_{\mathrm{H}}}{3A_{ul}h\nu_{u\ell}} D_L^2\, F_{\mathrm{obs}}$$
$$= 4.945 \times 10^7\, M_\odot \left(\frac{D_L}{\mathrm{Mpc}}\right)^2 \left(\frac{F_{\mathrm{obs}}}{\mathrm{Jy\ MHz}}\right),$$

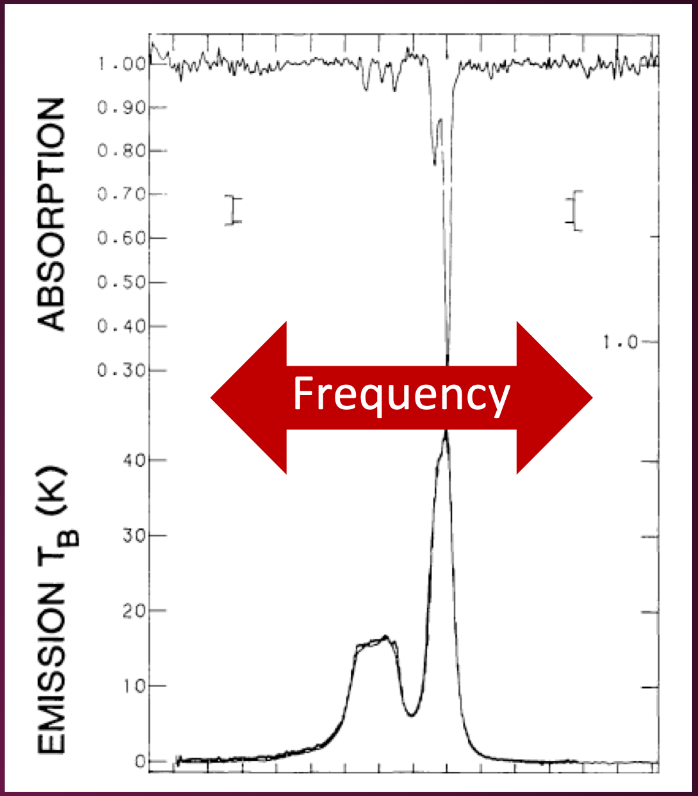
Mass in velocity units:

$$M_{\mathrm{H\,I}} = \frac{16\pi m_{\mathrm{H}}}{3A_{ul}hc} D_L^2 \int F_\nu dv$$
$$= 2.343 \times 10^5\, M_\odot (1+z) \left(\frac{D_L}{\mathrm{Mpc}}\right)^2 \left(\frac{\int F_\nu dv}{\mathrm{Jy\,km\,s^{-1}}}\right),$$

D_L = luminosity distance, z = redshift of object; 1 Jansky (Jy) = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Measuring spin temperature

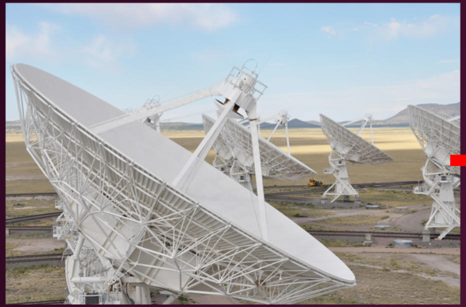




In general, we detect WNM in emission only

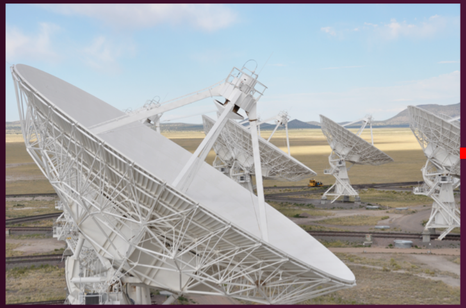
CNM in emission and absorption

Measuring spin temperature



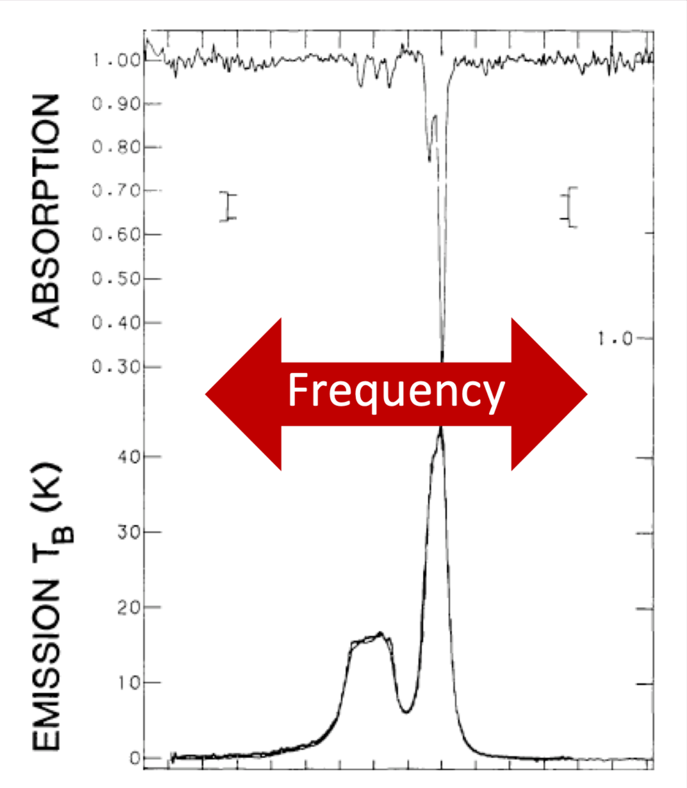
Continuum radio source

$$T_A^{\mathrm{on}}(v) = T_{\mathrm{RS}}\mathrm{e}^{-\tau_v} + T_{\mathrm{spin}}(1 - \mathrm{e}^{-\tau_v})$$



$$T_A^{\mathrm{off}}(v) = T_{\mathrm{sky}}\mathrm{e}^{-\tau_v} + T_{\mathrm{spin}}(1 - \mathrm{e}^{-\tau_v})$$

Blank sky



Measuring spin temperature

$$T_A^{\rm on}(v) = T_{\rm RS}e^{-\tau_v} + T_{\rm spin}(1 - e^{-\tau_v})$$

$$T_A^{\rm off}(v) = T_{\rm sky}e^{-\tau_v} + T_{\rm spin}(1 - e^{-\tau_v})$$

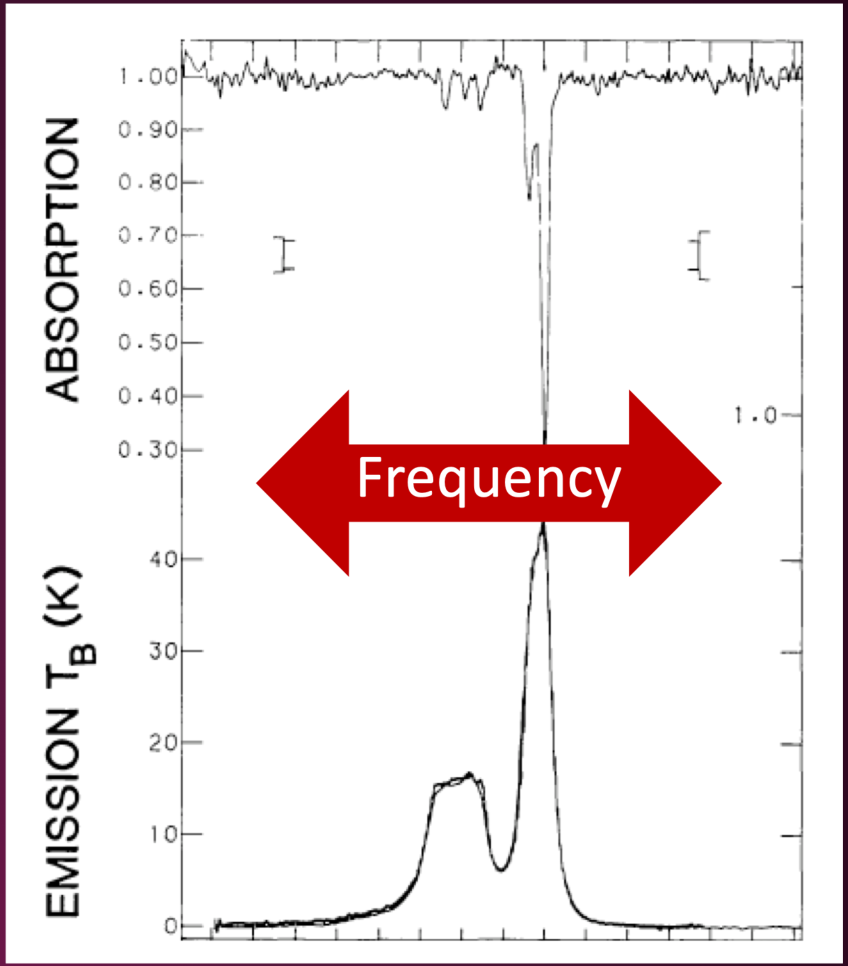
Solve two equations with two unknowns:

$$\tau(v) = \ln\left[\frac{T_{\rm RS} - T_{\rm sky}}{T_A^{\rm on}(v) - T_A^{\rm off}(v)}\right] ,$$

$$T_{\rm spin}(v) = \frac{T_A^{\rm off}(v)T_{\rm RS} - T_A^{\rm on}(v)T_{\rm sky}}{(T_{\rm RS} - T_{\rm sky}) - [T_A^{\rm on}(v) - T_A^{\rm off}(v)]}$$

In case of absorption, $(T_{\rm RS} - T_{\rm sky}) > [T_A^{\rm on}(v) - T_A^{\rm off}(v)]$

We can then solve for $T_{\rm spin}$ and τ!



Using spin temperature to measure N(H I)

Recall that for optically thin clouds:

$$\int [T_A - T_A(0)]\, du = \int \frac{c^2}{2k\nu^2}\, [I_\nu - I_\nu(0)]\, \frac{c}{\nu} d\nu$$
$$= \frac{3}{32\pi} \frac{hc\lambda_{u\ell}^2}{k} A_{u\ell}\, N(\mathrm{H\,I})$$
$$= 54.89\,\mathrm{K\,km\,s^{-1}} \frac{N(\mathrm{H\,I})}{10^{20}\,\mathrm{cm^{-2}}}$$

Therefore:

(if $\tau < 0.1$)

$$\frac{dN(\mathrm{H}\,I)}{dv} \approx \frac{32\pi}{3\lambda^2} \frac{k}{hcA_{u\ell}} [T_A^{\mathrm{on}}(v) - T_{\mathrm{sky}}(v)]$$
$$= 1.813\, \frac{T_A^{\mathrm{on}}(v) - T_{\mathrm{sky}}(v)}{\mathrm{K}} \times \frac{10^{18}\,\mathrm{cm^{-2}}}{\mathrm{km\,s^{-1}}}$$

If $\tau > 0.1$ and we've measured *T*spin:

$$\frac{dN(\mathrm{H\,I})}{dv} = \frac{32\pi}{3\lambda^2} \frac{k}{hcA_{u\ell}} T_{\mathrm{spin}}(v)\tau(v)$$
$$= 1.813\, \frac{T_{\mathrm{spin}}(v)\tau(v)}{\mathrm{K}} \times \frac{10^{18}\,\mathrm{cm^{-2}}}{\mathrm{km\,s^{-1}}}$$

Recall, in general:

$$\tau_\nu = 2.190 \frac{N(\mathrm{H\,I})}{10^{21}\,\mathrm{cm^{-2}}} \frac{100\,\mathrm{K}}{T_{\mathrm{spin}}} \frac{\mathrm{km\,s^{-1}}}{\sigma_V} \mathrm{e}^{-u^2/2\sigma_V^2}$$