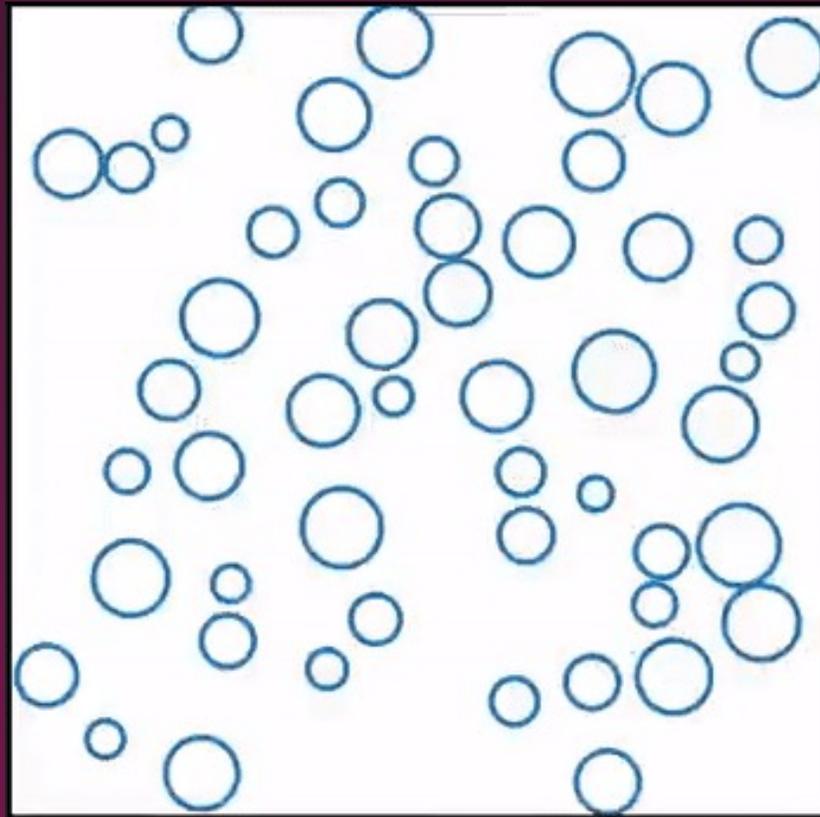


# Collisional processes, the two-level atom, and good ol' atomic physics



# Einstein coefficients and line emission

Equation of radiative transfer:  $dI_\nu = -I_\nu \kappa_\nu ds + j_\nu ds$

In case of only spontaneous emission – for line emission:

$$j_\nu d\nu = \frac{n_u A_{ul} h \nu_{ul}}{4\pi} \quad (\text{assuming isotropic emission})$$

For the term dependent on the radiation field:

$$I_\nu \kappa_\nu d\nu = \frac{h \nu_{lu} (n_l B_{lu} - u B_{ul}) I_{\nu lu}}{c}$$

# Einstein coefficients and line emission

Assume LTE, thus absorption = emission and levels are populated by Boltzmann eqn. Also, radiation field is Planck Law!

$$\frac{n_u A_{ul} h \nu_{ul}}{4\pi} = \frac{h \nu_{lu} (n_l B_{lu} - u B_{ul}) I_{\nu_{lu}}}{c}$$

$$\frac{n_u A_{ul} h \nu_{ul}}{4\pi} = \frac{h \nu_{lu} (n_l B_{lu} - u B_{ul})}{c} \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Boltzmann eqn. :

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-\frac{E_{ul}}{kT}}$$

# Relations between Einstein coefficients

NOTE: This holds under the assumption of LTE!

$$B_{ul} = \frac{c^3}{8\pi h\nu^3} A_{ul}$$

$$B_{lu} = \frac{g_u}{g_l} B_{ul} = \frac{g_u}{g_l} \frac{c^3}{8\pi h\nu^3} A_{ul}$$

# Collisional processes

Recall our collisional rate coefficient:

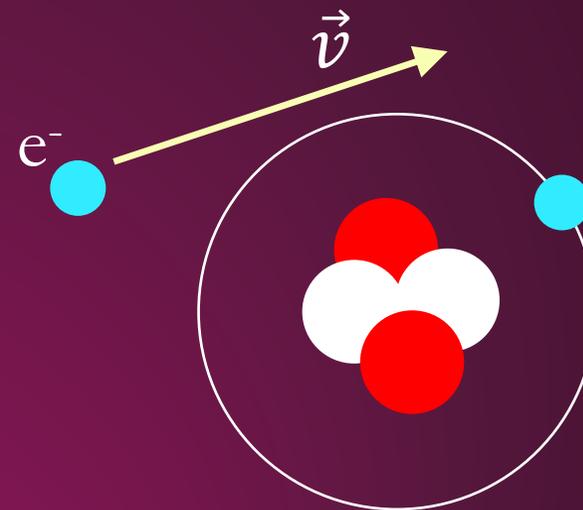
$$k = \langle \sigma v \rangle$$

units:

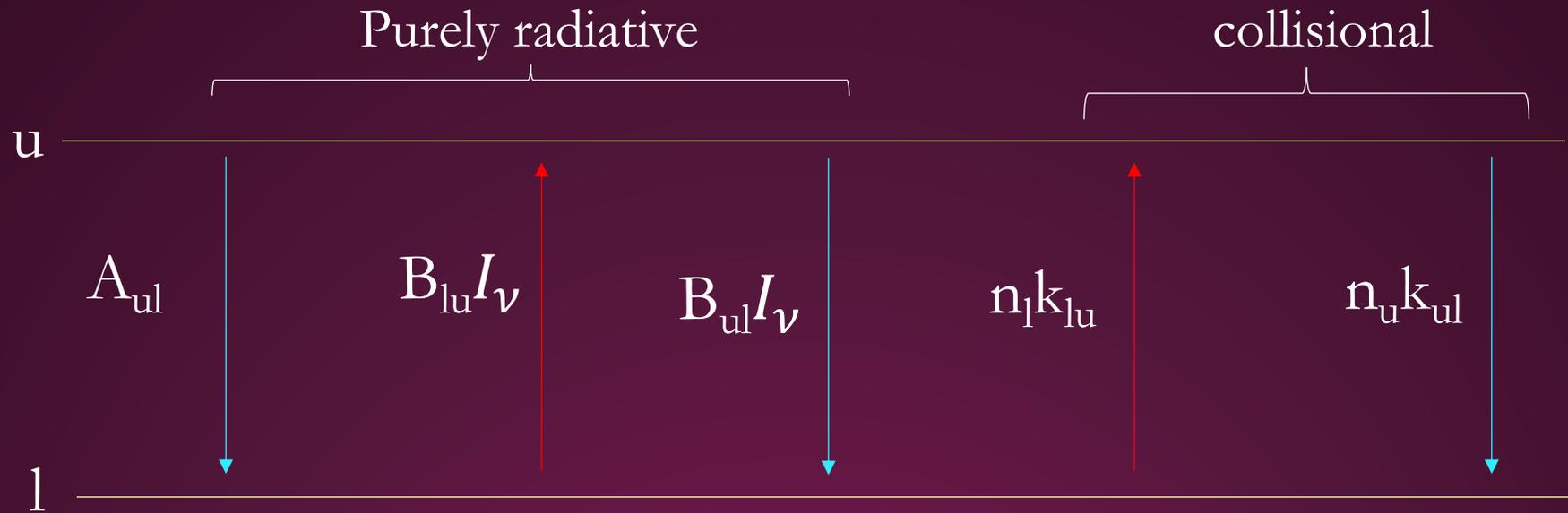
$$[k] = \text{cm}^3 \text{s}^{-1}$$

$$\text{collision rate per unit volume} = n_a n_b k [\text{cm}^{-3} \text{s}^{-1}]$$

Note: one of these densities is very often electrons from ionized gas ( $n_e$ )!



# Two-level atom

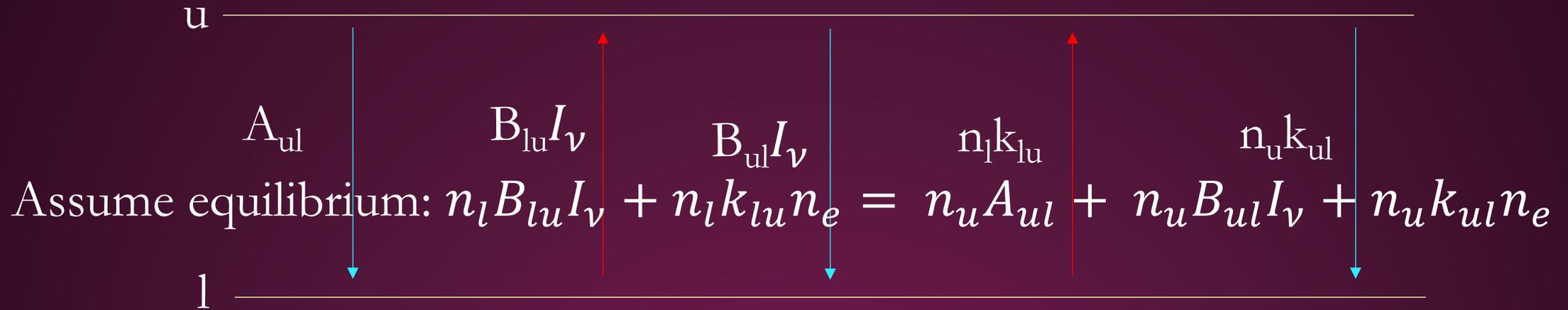


Volumetric collisional rates:

excitation:  $n_e n_l k_{lu}$

deexcitation:  $n_e n_u k_{ul}$

# Two-level atom



Solve for ratio of populations:

$$\frac{n_u}{n_l} = \frac{B_{lu} I_\nu + k_{lu} n_e}{A_{ul} + B_{ul} I_\nu + k_{ul} n_e}$$

# Two-level atom

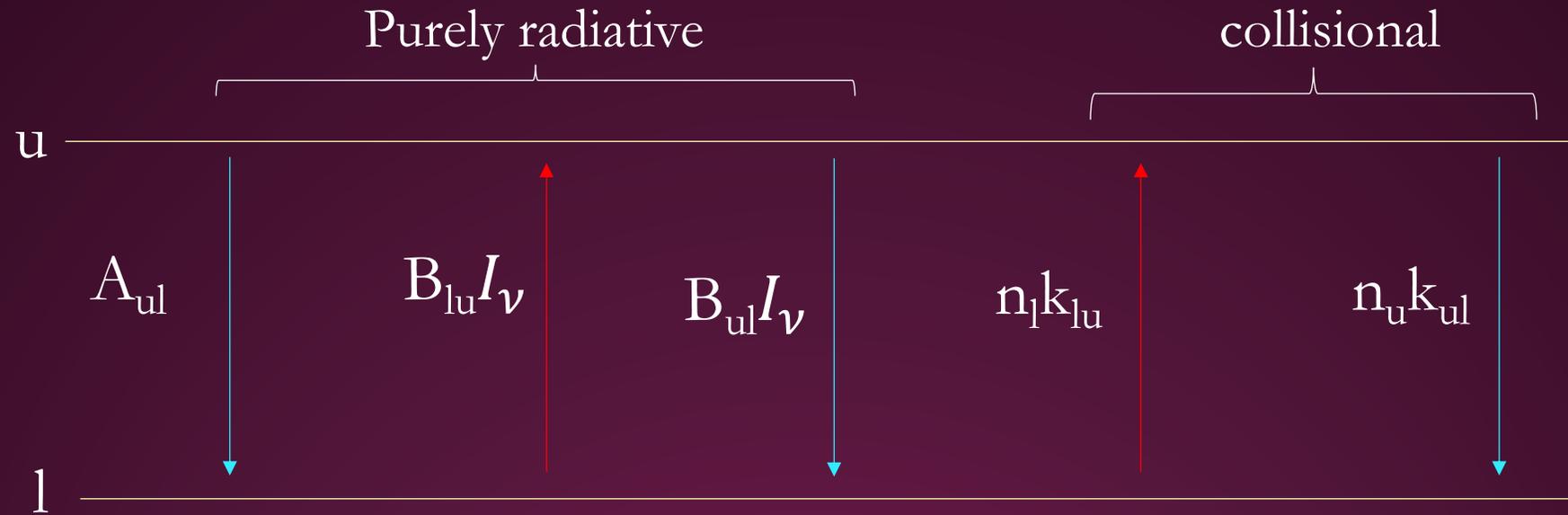
Ratio of populations:

$$\frac{n_u}{n_l} = \frac{B_{lu}I_\nu + k_{lu}n_e}{A_{ul} + B_{ul}I_\nu + k_{ul}n_e}$$

Employ relations between the Einstein coefficients, definition of excitation temperature, and Planck function (def. of radiation temperature):

$$e^{\frac{-h\nu}{kT_{exc}}} = \frac{A_{ul} \left[ e^{\frac{h\nu}{kT_{rad}}} - 1 \right]^{-1} + n_e k_{ul} e^{\frac{h\nu}{kT_{kin}}}}{A_{ul} \left[ 1 - e^{\frac{h\nu}{kT_{rad}}} \right]^{-1} + k_{ul} n_e}$$

# Critical density



Define the critical density:  $n_{cr} = A_{ul}/k_{ul}$

If  $n \gg n_{cr}$ , then  $T_{exc} \rightarrow T_{kin}$

If  $n \ll n_{cr}$ , then  $T_{exc} \rightarrow T_{rad}$

# Periodic table of the elements

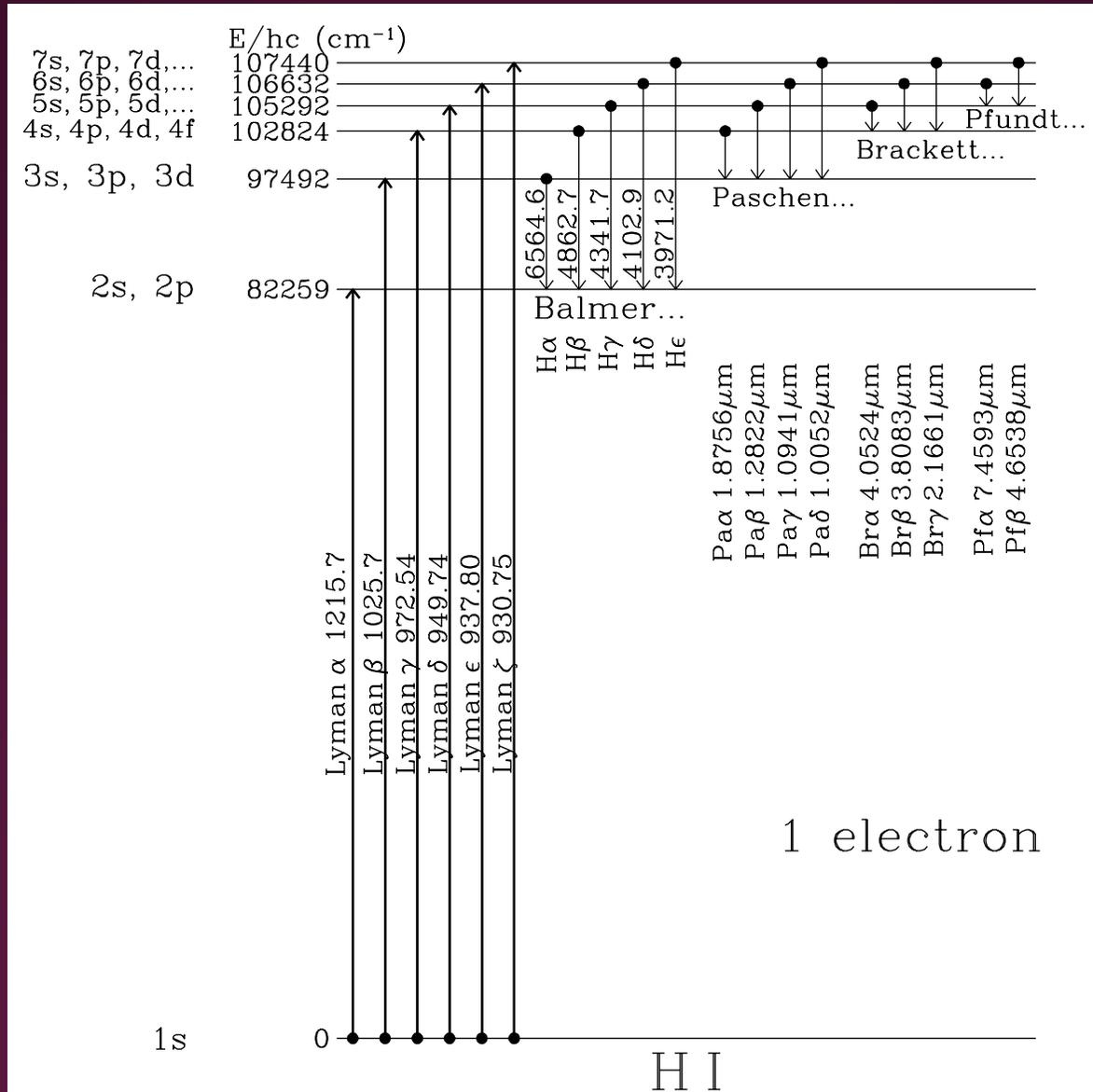
group																	18	
1*													13	14	15	16	17	18
1	<b>H</b>	2											5	6	7	8	9	10
2	<b>Li</b>	<b>Be</b>											<b>B</b>	<b>C</b>	<b>N</b>	<b>O</b>	<b>F</b>	<b>Ne</b>
3	<b>Na</b>	<b>Mg</b>	3	4	5	6	7	8	9	10	11	12	<b>Al</b>	<b>Si</b>	<b>P</b>	<b>S</b>	<b>Cl</b>	<b>Ar</b>
4	<b>K</b>	<b>Ca</b>	<b>Sc</b>	<b>Ti</b>	<b>V</b>	<b>Cr</b>	<b>Mn</b>	<b>Fe</b>	<b>Co</b>	<b>Ni</b>	<b>Cu</b>	<b>Zn</b>	<b>Ga</b>	<b>Ge</b>	<b>As</b>	<b>Se</b>	<b>Br</b>	<b>Kr</b>
5	<b>Rb</b>	<b>Sr</b>	<b>Y</b>	<b>Zr</b>	<b>Nb</b>	<b>Mo</b>	<b>Tc</b>	<b>Ru</b>	<b>Rh</b>	<b>Pd</b>	<b>Ag</b>	<b>Cd</b>	<b>In</b>	<b>Sn</b>	<b>Sb</b>	<b>Te</b>	<b>I</b>	<b>Xe</b>
6	<b>Cs</b>	<b>Ba</b>	<b>La</b>	<b>Hf</b>	<b>Ta</b>	<b>W</b>	<b>Re</b>	<b>Os</b>	<b>Ir</b>	<b>Pt</b>	<b>Au</b>	<b>Hg</b>	<b>Tl</b>	<b>Pb</b>	<b>Bi</b>	<b>Po</b>	<b>At</b>	<b>Rn</b>
7	<b>Fr</b>	<b>Ra</b>	<b>Ac</b>	<b>Rf</b>	<b>Db</b>	<b>Sg</b>	<b>Bh</b>	<b>Hs</b>	<b>Mt</b>	<b>Ds</b>	<b>Rg</b>	<b>Cn</b>	<b>Nh</b>	<b>Fl</b>	<b>Mc</b>	<b>Lv</b>	<b>Ts</b>	<b>Og</b>

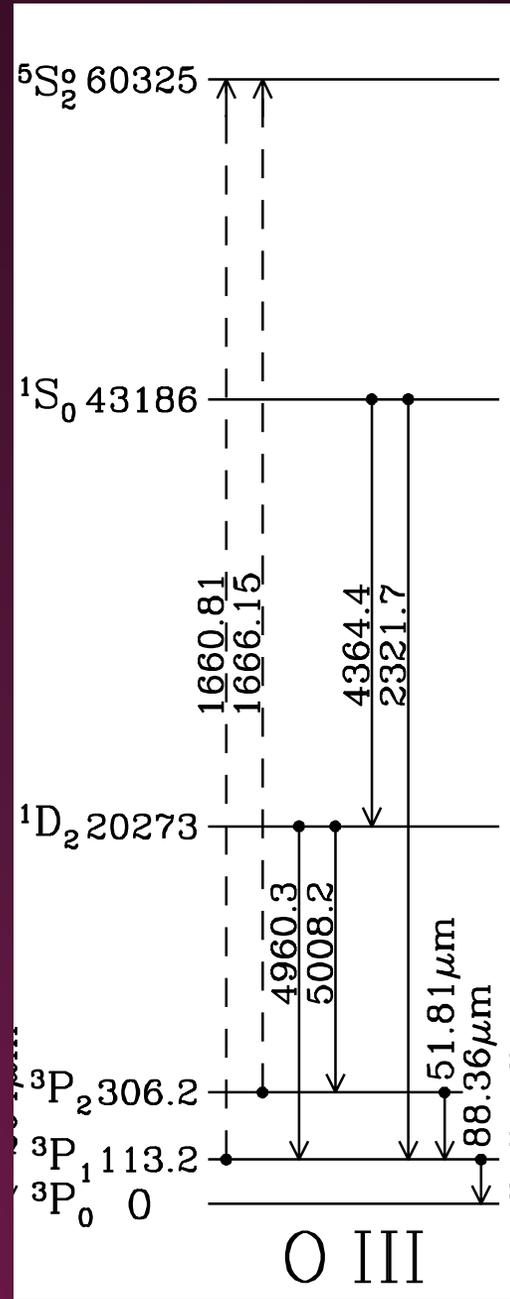
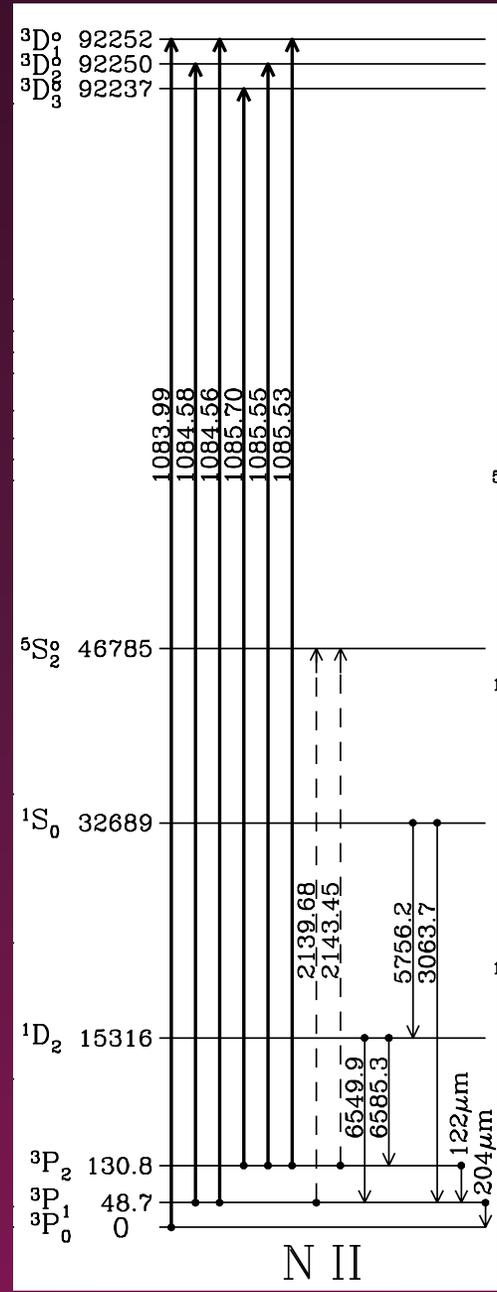
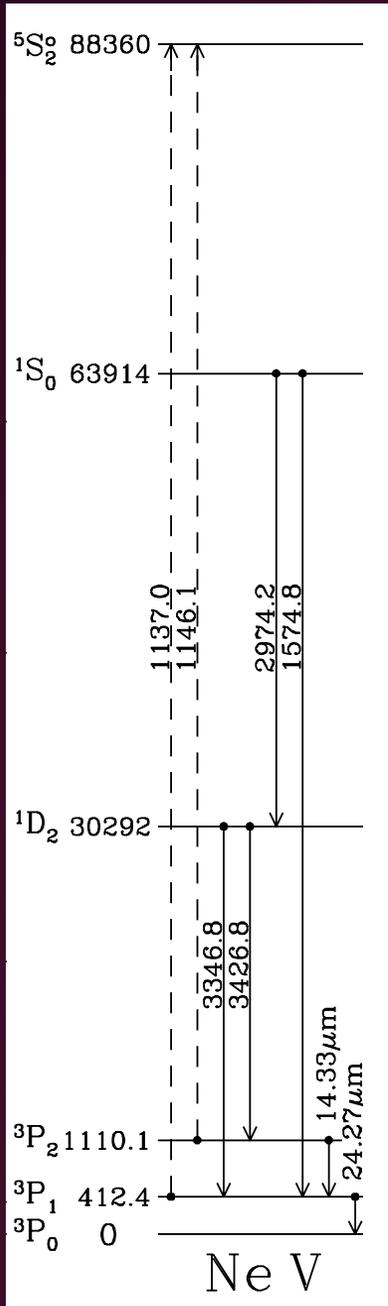
- Alkali metals
- Alkaline-earth metals
- Transition metals
- Other metals
- Other nonmetals
- Halogens
- Noble gases
- Rare-earth elements (21, 39, 57–71) and lanthanoid elements (57–71 only)
- Actinoid elements

lanthanoid series 6	58	59	60	61	62	63	64	65	66	67	68	69	70	71
	<b>Ce</b>	<b>Pr</b>	<b>Nd</b>	<b>Pm</b>	<b>Sm</b>	<b>Eu</b>	<b>Gd</b>	<b>Tb</b>	<b>Dy</b>	<b>Ho</b>	<b>Er</b>	<b>Tm</b>	<b>Yb</b>	<b>Lu</b>
actinoid series 7	90	91	92	93	94	95	96	97	98	99	100	101	102	103
	<b>Th</b>	<b>Pa</b>	<b>U</b>	<b>Np</b>	<b>Pu</b>	<b>Am</b>	<b>Cm</b>	<b>Bk</b>	<b>Cf</b>	<b>Es</b>	<b>Fm</b>	<b>Md</b>	<b>No</b>	<b>Lr</b>

\*Numbering system adopted by the International Union of Pure and Applied Chemistry (IUPAC). © Encyclopædia Britannica, Inc.

# Energy level diagram for hydrogen





# Nomenclature for lines

- “Allowed” transitions

- $A_{ul} \sim 10^8 \text{ s}^{-1}$



- “Semi-forbidden” transitions

- $A_{ul} \sim 10^2 \text{ s}^{-1}$



- “Forbidden transitions

- $A_{ul} \sim 20 \text{ min lifetime}$

