ISM - Homework 1

Ezra Huscher – September 7, 2021

1 Visualizing Planck's Law with Bokeh

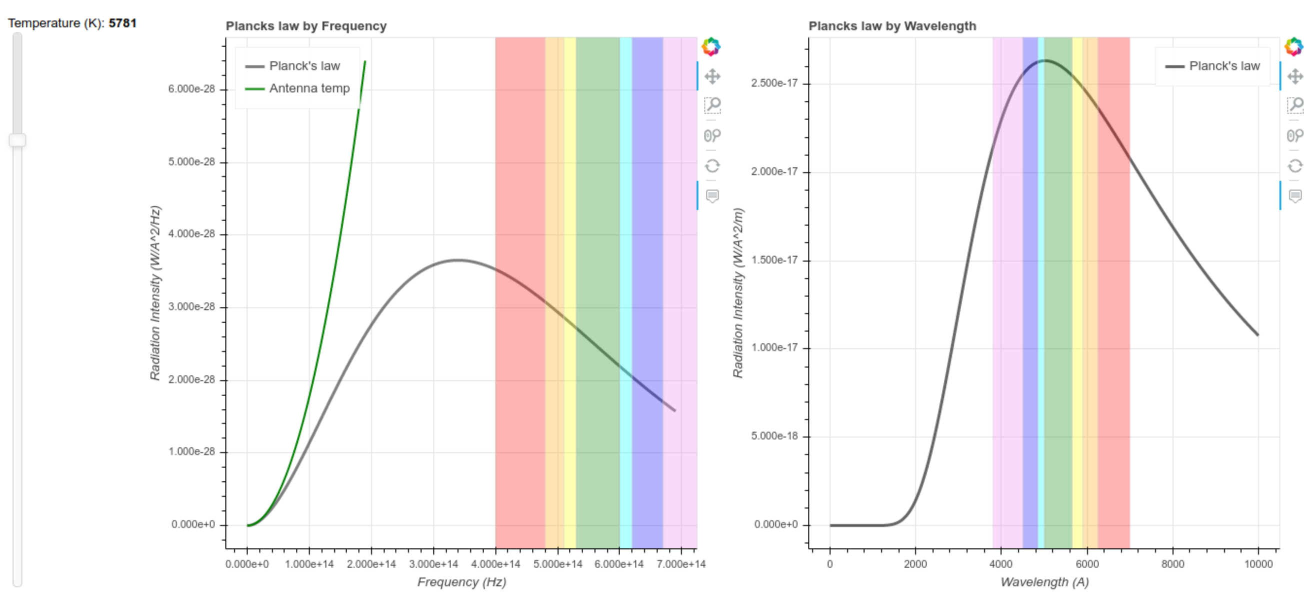
a.) Bokeh allows for a nice, interactive look at Planck's Law. This law can be used to derive the wavelength or frequency of peak emission for a given temperature. As a function of frequency, it is:

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{1}$$

b.) And as a function of wavelength:

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \tag{2}$$

c.) Making sure the result returns what we'd expect for the Sun's photosphere which has a temperature around 6000 K:



The peak wavelength is right around 500 nanometers. Perfecto! This is right in the middle of the visible spectrum.

d.) The phases of the interstellar medium are at a variety of temperatures. What peak wavelength will these emit radiation at?

Phases of the ISM				
Name	Temp (K)	Density (cm^{-3})	λ_{peak} (A)	Range of Peak
Molecular clouds	50	$10^2 - 10^6$	5×10^5	Infrared
Cold neutral medium	100	$10 - 1000$	2.8×10^5	Infrared
Warm neutral medium	5000	0.6	5.6×10^3	Visible
Warm ionized medium	10^4	0.3	2.8×10^3	UV
H II regions	10^4	$0.1 - 10^4$	2.8×10^3	UV
Hot, ionized medium	$> 3 \times 10^5$	$10^{-2} - 10^{-3}$	310	Far UV

e.) Choose one of these phases and describe in a few sentences some observations that detect the particular phase. A literature reference would be great, but at least include the telescope/instrument.

Though molecular clouds emit most in the infrared, this is more difficult to detect the a tracer these clouds would contain, such as carbon monoxide. Therefore, searching for CO at 2 or 3mm is a more effective way to find molecular H_2, per this 1975 paper:

Detection of Carbon Monoxide in the Large Magellanic Cloud
Huggins, et al
Monthly Notices of the Royal Astronomical Society, Volume 173, Issue 1, October 1975, Pages 69P–71P, https://doi.org/10.1093/mnras/173.1.69P

f.) If the frequency of the radiation is significantly small enough, such as in the radio, it can be useful to use the Rayleigh–Jeans approximation instead of the complete Planck's law.

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{3}$$

Because $h\nu << kT$:

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{kT}{h\nu} \tag{4}$$

Leaving us with a temperature radio astronomers like to use:

$$= \frac{2\nu^2 kT}{c^2} \tag{5}$$

Rearranging, radio astronomers like to use a quantity called the antenna temperature:

$$T_A = \frac{c^2 B_\nu}{2\nu^2 k} \tag{6}$$

or

$$T_A = \frac{\lambda^2 B_\lambda}{2k} \tag{7}$$

g.) Express the antenna temperature as an intensity as a function of frequency and temperature (quite possibly an intermediate step in f above). Now, add this curve to your Bokeh Planck law plots.

Done! Without the exp component, it simply rises with increased frequency. So, I read this is akin to "noise" of a radio telescope, but that's a little unclear to me. Does this not need to include the effective area of the dish?

2 Maxwellian distribution, the three-dimensional velocity distribution of free electrons in a gas

This is the probability, per unit velocity, of finding the electron with a speed near v, and is written as:

$$f_v dv = 4\pi(\frac{m}{2\pi kT})^{3/2} e^{-\frac{mv^2}{2kT}} v^2 dv \tag{8}$$

Let's imagine an H II region with an electron temperature, $T_e = 10^4$ K.

a.) First, find the minimum velocity an electron needs to be moving to ionize a Hydrogen atom (e.g. assuming the gas is in kinetic equilibrium).

The kinetic energy equation is:

$$E = \frac{1}{2} m_e v^2 \tag{9}$$

The ionization energy of hydrogen is 13.6 eV, or 2.18×10^{-18} Joules.

Rearranging, with the electron's mass as 9.1×10^{-31} kg:

$$v_{min} = \sqrt{\frac{2E}{m_e}} \tag{10}$$

$$v_{min} = \sqrt{\frac{2(2.18 \times 10^{-18})}{9.1 \times 10^{-31}}} \tag{11}$$

$$v_{min} = 2.19 \times 10^6 \frac{m}{s} \tag{12}$$

b.) Next, integrate the Maxwellian to compute the fraction of electrons that have sufficient energy (or, are moving at high enough velocities) to keep hydrogen atoms ionized.

To find this fraction, let's integrate the velocity distribution from the minimum ionization velocity to the speed of light:

$$f_v = \int_{2\times10^6}^{3\times10^8} 4\pi(\frac{m}{2\pi kT})^{3/2} e^{-\frac{mv^2}{2kT}} v^2 dv \tag{13}$$

To simplify the integral, let's set $b = m/2kT$:

$$b = \frac{m}{2kT} \tag{14}$$

With m as the reduced mass: $m = m_e m_p / m_e + m_p$:

$$m = \frac{(9.11 \times 10^{-31})(1.67 \times 10^{-27})}{(9.11 \times 10^{-31}) + (1.67 \times 10^{-27})} \tag{15}$$

$$m = 9.09 \times 10^{-31} kg \tag{16}$$

And so:

$$b = \frac{9.09 \times 10^{-31}}{2(1.38 \times 10^{-23})(10^4)} \tag{17}$$

$$b = 3.30 \times 10^{-12} \tag{18}$$

$$f_v = \int_{2\times10^6}^{3\times10^8} 4\pi(\frac{b}{\pi})^{3/2} e^{-bv^2} v^2 dv \tag{19}$$

$$f_v = 4\pi(\frac{b}{\pi})^{3/2} \int_{2\times10^6}^{3\times10^8} e^{-bv^2} v^2 dv \tag{20}$$

Using integration by parts and being careful with the exotic error function leaves:

$$f_v \sim 6 \times 10^{-7} \tag{21}$$

c.) What can you conclude about how the H II region is ionized based on this calculation?

The electrons do not seem to be energetic enough to overcome hydrogen's 13.6eV valence electron's binding energy, as only a tiny fraction of the free electrons have the required kinetic energy. So, the HII regions at this temperature must be ionized from a different mechanism, such as UV radiation from nearby young stars.

Now, imagine the hot halo gas surrounding a galaxy with $T_e = 10^6$ K.

e.) Repeat your calculation, above, to determine whether electron collisions are able to keep this "coronal gas" ionized.

Re-running this calculation with an updated b value:

$$b = \frac{9.09 \times 10^{-31}}{2(1.38 \times 10^{-23})(10^6)} \tag{22}$$

$$b = 3.30 \times 10^{-14} \tag{23}$$

$$f_v = \int_{2\times10^6}^{3\times10^8} 4\pi(\frac{b}{\pi})^{3/2} e^{-bv^2} v^2 dv \tag{24}$$

$$f_v = 0.96 \tag{25}$$

There appears to be a 96% probability at an electron temperature of 10^6 K that the electrons will have sufficient velocity to ionize a hydrogen atom. Thus, the hot halo surrounding a galaxy will assuredly keep the gas ionized, regardless of any present electromagnetic radiation.