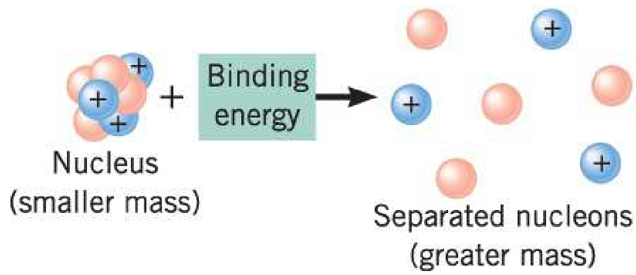

Figure 1.1: Illustration of the concept of binding energy of a nucleus. Typically, a nucleus has a lower energy than if its particles were free. Source of Figure 1.1: http://staff.orecity.k12.or.us/les.sitton/Nuclear/313.htm.

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- We can therefore write for the energy release of some reaction in terms of mass excesses:

$$Q_{aA} = [\Delta m(a) + \Delta m(A) - \Delta m(y) - \Delta m(Y)]. \tag{1.43}$$

- In general, the energy is released as kinetic energy to the resultant particles (as implied in Equation (1.39)), and sometimes in photons (and neutrinos, which isn't too important in the energy budget for normal reactions). The energy gets redistributed in the gas through collisions and the absorption of photons. The details don't matter because of energy equilibrium, but what matters is the total amount of heat added to the gas.

- Back to Equation (1.36), the energy generation rate per gram is the reaction rate multiplied by the energy for each reaction divided by density, so

$$\varepsilon_{aA} = \frac{r_{aA} Q_{aA}}{\rho}, \quad [\mathrm{erg\,g^{-1}\,s^{-1}}], \tag{1.44}$$

$$\varepsilon_{aA} = Q_{aA} \langle \sigma v \rangle_0 \frac{X_a X_A}{A_a A_A} \frac{\rho}{m_\mathrm{u}^2} \left(\frac{T}{T_0} \right)^n. \tag{1.45}$$

- One can in general write

$$\varepsilon = \varepsilon_0 \rho T^n, \tag{1.46}$$

for any reaction, absorbing most of the constant terms in ε_0.

1.1.5 Binding energy

- Do not confuse mass excess with nuclear binding energy, since they are very similar.
- The nuclear binding energy is the energy required to separate a stable nucleus into its constituent parts, as depicted in Figure 1.1. Note the following definitions:
  - $1\,\mathrm{amu} = 931.494\,\mathrm{MeV}/c^2 = m_\mathrm{u}$
  - $m_\mathrm{p} = 1.007825$ amu
  - $m_\mathrm{n} = 1.00867$ amu
  - $m_\mathrm{e} = 0.0005486$ amu
- Electron binding energy is usually referred to as *ionization energy*.
- Binding energy for neutral atoms is

$$E_\mathrm{bind} = 931.494\,\mathrm{MeV/c^2}\,(Z m_\mathrm{p} + N m_\mathrm{n} + Z m_\mathrm{e} - m)\,c^2 \qquad [\mathrm{MeV}], \tag{1.47}$$

where m is the atomic mass of the element or isotope in question. Often you'll see the binding energy per nucleon, E_bind/A. Electrons can be ignored when talking about nuclei. This is plotted in Fig. 1.2.

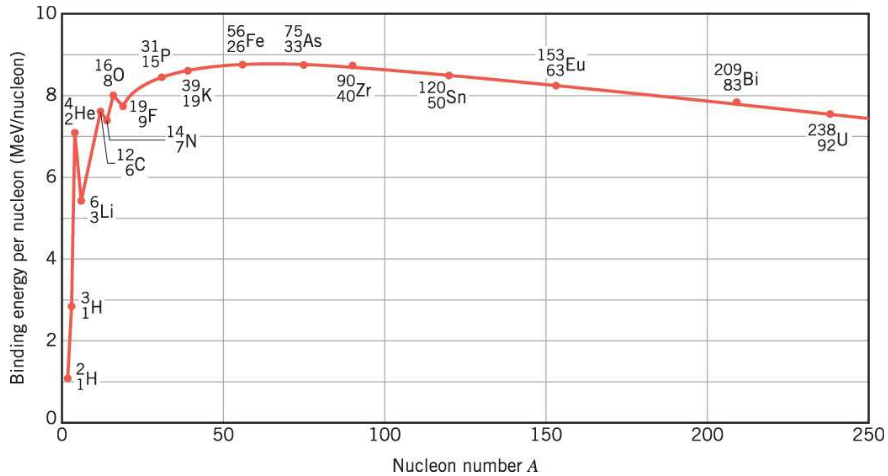

Figure 1.2: The binding energy of isotopes per nucleon. After iron, energy must be used to fuse its nucleons. Source of Figure 1.1: http://staff.orecity.k12.or.us/les.sitton/Nuclear/313.htm.

- The atomic mass of ^{4}He is 4.0026 amu. It's binding energy per nucleon is $931.494 * (2m_\mathrm{p} + 2m_\mathrm{n} - 4.0026)/4 = 7.07$ MeV/nucleon.
- The large binding energy of ^{4}He is important in stellar physics, as we'll see.

1.2 Hydrogen burning

- Historically, considering a star of H and He, all major 2-particle interactions produce unstable nuclei

$$\begin{aligned} \mathrm{p}+\mathrm{p} &\longrightarrow {}^2\mathrm{He}\ (\text{unstable}) \longrightarrow \mathrm{p}+\mathrm{p} \\ \mathrm{p}+^4\mathrm{He} &\longrightarrow {}^5\mathrm{Li}\ (\text{unstable}) \longrightarrow \mathrm{p}+^4\mathrm{He} \\ {}^4\mathrm{He}+^4\mathrm{He} &\longrightarrow {}^8\mathrm{Be}\ (\text{unstable}) \longrightarrow {}^4\mathrm{He}+^4\mathrm{He}. \end{aligned} \tag{1.48}$$

- It was Hans Bethe who first showed that the weak force plays a role in all this, in the form of beta decay (see below).
- Anyway, the general idea of hydrogen fusion is always

$$4^1\mathrm{H} \longrightarrow ^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu_\mathrm{e}, \tag{1.49}$$

where 2 positrons are needed to keep charge conserved, and 2 electron neutrinos conserve lepton number (from the 2 anti-lepton positrons). This does not happen all at once, but along certain "paths," see below.

- The atomic mass of H is 1.007852 amu and of ^{4}He is 4.002603 amu. So 0.0288 mass units are lost. The mass fraction that is turned into energy is thus $0.0288/4 = 0.007$, or 0.7%.
- Using Δmc^2, we have $(0.0288)(931.494\,\mathrm{MeV}/c^2)c^2 = 26.8\,\mathrm{MeV}$.
- Using the mass excesses in the units of MeV we can compute the energy liberated in yet another way

$$Q = 4(7.289) - 2.4248 - 2(0.263) = 26.21\,\mathrm{MeV}, \tag{1.50}$$

where this time we take into account the energy carried away by the neutrinos (see table in Fig. 1.3). Note, as mentioned before, that the values are from atomic mass excesses, not *nuclear* mass excesses, so electrons are implicitly in there (including . That is why we do not take into account the ~ 0.5 MeV from the positrons (since those, plus the 2 electrons on the RHS implicit in the the He, cancel energetically with the 4 electrons implicit on the LHS).

Table 5-1 Reactions of the PP chains

Reaction	Q value, Mev	Average ν loss, Mev	S_0, kev barns	$\frac{dS}{dE}$, barns	B	τ_{12}, years†
$H^1(p,\beta^+\nu)D^2$	1.442	0.263	3.78×10^{-22}	4.2×10^{-24}	33.81	7.9×10^9
$D^2(p,\gamma)He^3$	5.493		2.5×10^{-4}	7.9×10^{-6}	37.21	4.4×10^{-8}
$He^3(He^3,2p)He^4$	12.859		5.0×10^3		122.77	2.4×10^5
$He^3(\alpha,\gamma)Be^7$	1.586		4.7×10^{-1}	-2.8×10^{-4}	122.28	9.7×10^5
$Be^7(e^-,\nu)Li^7$	0.861	0.80				3.9×10^{-1}
$Li^7(p,\alpha)He^4$	17.347		1.2×10^2		84.73	1.8×10^{-5}
$Be^7(p,\gamma)B^8$	0.135		4.0×10^{-2}		102.65	6.6×10^1
$B^8(\beta^+\nu)Be^{8*}(\alpha)He^4$	18.074	7.2				3×10^{-8}

† Computed for $X = Y = 0.5$, $\rho = 100$, $T_6 = 15$ (sun).

Figure 1.3: The properties of the relevant chains of the proton-proton reaction. From Clayton [1983].

- Let's be redundant, and clear. Consider the energy available to the star (sometimes referred to as the effective energy) for a generic β decay process, whereby a proton converts to a neutron in some nucleus, producing a positron and neutrino as well. This energy can be written as (using our notation)

$$
\begin{aligned}
Q &= \Delta m_{\rm nuc}(Z+1) - \Delta m_{\rm nuc}(Z) - m_e c^2 + 2m_e c^2 - E_\nu, \\
&= \Delta m_{\rm nuc}(Z+1) - \Delta m_{\rm nuc}(Z) + m_e c^2 - E_\nu, \\
&= \Delta m_{\rm atom}(Z+1) - \Delta m_{\rm atom}(Z) - E_\nu,
\end{aligned}
$$

  where the Δm are the mass excesses (in energy units), Z is the proton number of the nucleus, E_ν is the energy carried off by the neutrino (not available to the star), the $-m_e c^2$ is the energy required to create the positron, and the $+2m_e c^2$ is the energy produced when the electron-positron annihiliation takes place (gamma radiation is typically produced). At one point we switched from using "nuclear" mass excesses to "atomic" ones (as the ones in the tables) which take into account electron contributions. That's why the electrons are "already counted" in the energy budget, at some level.

1.2.1 PP-I chain

- The proton-proton reaction is

$$
\begin{aligned}
{}^1{\rm H} + {}^1{\rm H} &\longrightarrow {}^2{\rm D} + {\rm e}^+ + \nu = 2(7.289) - 13.136 - 0.263 = 1.18\,{\rm MeV};\ (10^9\,{\rm yr}) \\
{}^2{\rm D} + {}^1{\rm H} &\longrightarrow {}^3{\rm He} + \gamma = 13.136 + 7.289 - 14.93 = 5.49\,{\rm MeV};\ (6\,{\rm s}), \\
{}^3{\rm He} + {}^3{\rm He} &\longrightarrow {}^4{\rm He} + {}^1{\rm H} + {}^1{\rm H} = 2(14.93) - 2.42 - 2(7.289) = 12.86\,{\rm MeV};\ (10^6\,{\rm yr})
\end{aligned}
$$

  Noting that the first two reactions have to happen twice to produce 2 ^{3}He nuclei, the total energy is 2(1.18)+2(5.49)+12.86 = 26.2 MeV, as we saw before.

- Another way to write this is

$$
{}^1{\rm H}({}^1{\rm H}, {\rm e}^+\nu_{\rm e}){}^2{\rm D}({}^1{\rm H}, \gamma){}^3{\rm He}({}^3{\rm He}, 2\,{}^1{\rm H}){}^4{\rm He}, \tag{1.51}
$$

  where everything to the left of the comma is an ingredient of the reaction, and everything to the right is a product.

- See the table in Figure 1.3 for quantities, particularly the neutrino loss contribution. The times given apply to a single nucleus in the stellar interior environment.

- Neutrino production is the main reason why we know these processes are taking place in stellar interiors.

- The onset of H burning through this channel can occur at about 5-10 MK.
- The first part of the chain is by far the slowest, because a proton is converting into a neutron through the weak force (beta decay), and this is quite rare.
- Note this chain can occur in a pure hydrogen gas.
- The first reaction in the chain goes the slowest and so the rate of energy generation is controlled by it.
- The deuterium burning reaction is very fast, and stars should destroy all of it. The large abundance on Earth is an interesting problem.

PROBLEM 1.7: [5 pts]: The Sun's luminosity is $L_\odot = 3.9 \times 10^{33}\,\mathrm{erg\,s^{-1}}$. Assume that the energy for this luminosity is provided solely by the PP-I chain, and that neutrinos carry off 3% of the energy liberated. How many neutrinos are produced per second? What is the neutrino flux at Earth (# neutrinos per second per cm^2)?

PROBLEM 1.8: [5 pts]: Show that at a temperature of $T = 15$ MK the temperature exponent (Equation (1.33)) in the first reaction of the PP-I chain is $n \simeq 4$.

1.2.2 PP-II and PP-III chains

- After the second step of the PP-I chain, the ^{3}He that was produced has a choice, which is dictated mainly by temperature.
- Helium 4 can also be produced from hydrogen in 2 other ways.
- PP-II chain:
$$^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}(\mathrm{e}^-,\bar{\nu}_\mathrm{e})^7\mathrm{Li}(^1\mathrm{H},^4\mathrm{He})^4\mathrm{He} \tag{1.52}$$
- PP-III chain:
$$^7\mathrm{Be}(^1\mathrm{H},\gamma)^8\mathrm{B}(,\mathrm{e}^+\nu_\mathrm{e})^8\mathrm{Be}(,^4\mathrm{He})^4\mathrm{He} \tag{1.53}$$
- Note that the berrylium 7 nucleus has a choice to react with an electron (to form lithium 7) or a proton (to form berrylium 8).
- For temperatures above about 15 million K, helium 3 likes to react with helium 4 (rather than itself as in the last reaction of PP-I) and so the PP-I chain is not as dominant at hotter conditions.
- The average neutrino energy loss from PP-II in the berrylium electron capture is 0.8 MeV.
- The average neutrino energy loss from PP-III in the positron decay of boron is 7.2 MeV, which is large.
- These neutrinos coming from the Sun can be detected and have been critical to understanding fusion processes. They are the dominant ones observed in the water tank experiments and the cleaning fluid experiment: $\nu_\mathrm{e}(^{37}\mathrm{Cl},\mathrm{e}^-)^{37}\mathrm{Ar}$.
- The branching ratio among these three chains depends quite sensitively on interior conditions.

PROBLEM 1.9: [10 pts]: Compute the *full Q* values for the PP-II and PP-III chains (don't forget to include any necessary contributions from previous chains in the computation).

Table 5-2 The CNO reactions

Reaction	Q value, Mev	Average ν loss, Mev	S(E = 0), kev barns	dS/dE, barns	B
$C^{12}(p,\gamma)N^{13}$	1.944		1.40	4.26×10^{-3}	136.93
$N^{13}(\beta^+\nu)C^{13}$	2.221	0.710			
$C^{13}(p,\gamma)N^{14}$	7.550		5.50	1.34×10^{-2}	137.20
$N^{14}(p,\gamma)O^{15}$	7.293		2.75		152.31
$O^{15}(\beta^+,\nu)N^{15}$	2.761	1.00			
$N^{15}(p,\alpha)C^{12}$	4.965		5.34×10^4	8.22×10^2	152.54
$N^{15}(p,\gamma)O^{16}$	12.126		2.74×10^1	1.86×10^{-1}	152.54
$O^{16}(p,\gamma)F^{17}$	0.601		1.03×10^1	-2.81×10^{-2}	166.96
$F^{17}(\beta^+\nu)O^{17}$	2.762	0.94			
$O^{17}(p,\alpha)N^{14}$	1.193		Resonant reaction		167.15

Figure 1.4: The properties of the relevant chains of the CNO cycle. From Clayton [1983].

1.2.3 CNO cycle

- Another way to turn hydrogen into helium is by producing (and burning) carbon, nitrogren, and oxygen, as long as such species exist. This typically happens in higher-mass stars with higher internal temperatures.
- There are two possible ways this happens, each involving 6 reactions. This bi-cycle is written as

$$^{12}\mathrm{C}(^1\mathrm{H},\gamma)^{13}\mathrm{N}(,\mathrm{e}^+\nu_\mathrm{e})^{13}\mathrm{C}(^1\mathrm{H},\gamma)^{14}\mathrm{N}(^1\mathrm{H},\gamma)^{15}\mathrm{O}(,\mathrm{e}^+\nu_\mathrm{e})^{15}\mathrm{N}(^1\mathrm{H},^4\mathrm{He})^{12}\mathrm{C}, \tag{1.54}$$

$$^{14}\mathrm{N}(^1\mathrm{H},\gamma)^{15}\mathrm{O}(,\mathrm{e}^+\nu_\mathrm{e})^{15}\mathrm{N}(^1\mathrm{H},\gamma)^{16}\mathrm{O}(^1\mathrm{H},\gamma)^{17}\mathrm{F}(,\mathrm{e}^+\nu_\mathrm{e})^{17}\mathrm{O}(^1\mathrm{H},^4\mathrm{He})^{14}\mathrm{N}. \tag{1.55}$$

- The nucleon making 2 choices is ^{15}N upon interaction with a proton.
- The energy released is $Q = 25.02$ MeV.
- For Pop. I stars, CNO material makes up about 75% of metals, and O is about 70% of CNO.
- The slowest reaction and the one that determines the overall reaction rate is the proton capture $^{14}\mathrm{N}(^1\mathrm{H},\gamma)^{15}\mathrm{O}$. This reaction is highly temperature dependent (Problem 1.10).
- See Figure 1.5 for the crossover regime between PP and CNO channels.
- As this is the slowest reaction, the most abundant species in the CNO cycle is ^{14}N.
- Generally, through these channels, the final abundances of C and O decrease, while that of N increases.
- We'll come to He burning later in the post main-sequence unit

PROBLEM 1.10: [5 pts]: Show that at a temperature of $T = 15$ MK the temperature exponent in the slowest reaction of the CNO cycle is $n \simeq 20$.

COMPUTER PROBLEM 1.1: [20 pts]: Here you will look at the effects of "turning off" nuclear reactions at the main sequence to see how stellar evolution changes. `MESA` allows one full control of nuclear energy generation. We will explore how this changes the star right after the time it formed and is getting to the main sequence.
Hand in your answers to the questions below with figures. You may also prepare a document with all answers and figures and send as a .pdf (only).

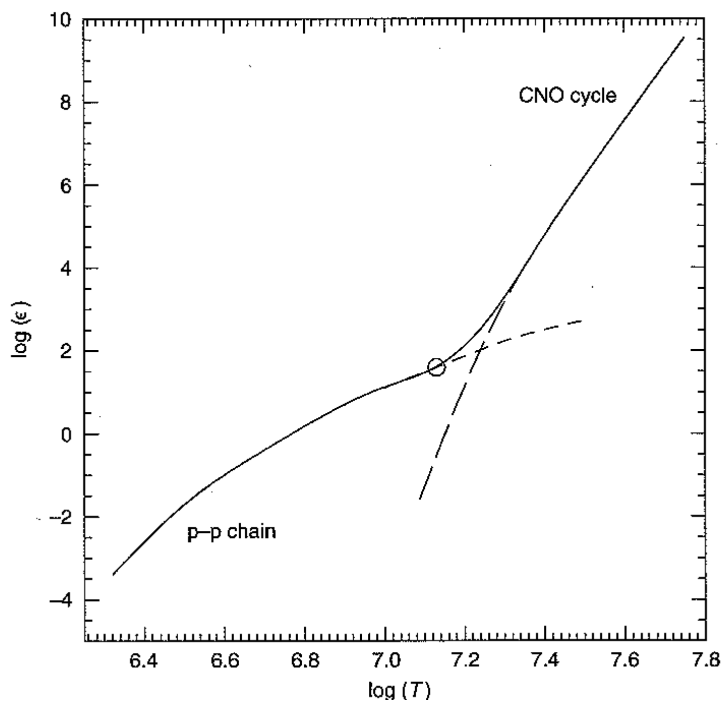

Figure 1.5: The nuclear energy rate as a function of temperature. The Sun is marked as a circle. From Salaris and Cassisi [2006].

What to do

1. Copy the `WORK_DIR` to wherever you will be running MESA, rename it to something sensible.
2. Edit the `inlist_project` file. In `&star_job`, make sure `pgstar_flag=.true.` and we don't need to create a pre-main sequence track yet, so you can add `create_pre_main_sequence_model=.false.` In the `&controls` section, we want the `initial_mass=1.0`. Run the model until about 10 billion years, so set `max_age=1d10`. Most importantly, for this first run, turn off the nuclear reaction rates by setting `eps_nuc_factor=0` and `dxdt_nuc_factor=0`.
3. You shouldn't need to run more than about 1000 models (timesteps) to reach that age based on the default dt. All the data gets saved in `LOGS/`.
4. Now copy a new working directory and maybe copy the inlist you just used into it and change the following: Turn on the reactions by setting that variable to 1. We need a stopping criterion, because 10 billion years would take us to the terminal age main sequence, and so we'll use the onset of hyrdrogen burning and set an abundance criterion. So in `&controls` add `xa_central_lower_limit_species(1)='h1'` and then `xa_central_lower_limit(1)=0.69` (the default initial H abundance is 0.7). So right after a little bit of central hydrogen is burned (depleted), the simulation will stop.

Questions

1. How old was the star with nuclear burning when the hydrogen abundance dropped below 0.7? Is that reasonable?
2. Plot a proper HR diagram with the "tracks" of both stars on it (luminosity vs effective temperature, logarithmically). Try to give some indication of age on the plot. You may have to "zoom" in to the appropriate area with a second plot.
3. Describe the two tracks qualitatively.
4. Explain why or how the star with no nuclear burning gets so much hotter than the star with nuclear burning. What is physically happening? Then show a plot that should confirm your explanation that

uses interior parameters. What happens for the star with no burning at later times, explain its track on the HR diagram? (Look around for the appropriate quantities to plot in the evolution variables, it's up to you).

1.3 List of things not discussed

These are things that belong in this unit that won't be covered or will be later on.

- Equilibrium abundances. How the species change with time as material gets depleted or created. There are good discussions of this in other texts.
- Heavier element burning chains. Helium burning up to silicon burning will be discussed in later sections when post-main sequence evolution is covered.
- Gravity also provides an energy source, and will be discussed soon.

What You Should Know From This Chapter for Upcoming Material

- Understand how to compute the temperature dependence of nuclear reactions (given the specific formula)
- Know how to Taylor expand a function to a given order so that you end up with an approximate new function that can then be integrated analytically (as in Problem 1.5).

Table 4-1 Atomic mass excesses†

Z	*Element*	*A*	*M − A, Mev*
0	*n*	1	8.07144
1	H	1	7.28899
	D	2	13.13591
	T	3	14.94995
	H	4	28.22000
		5	31.09000
2	He	3	14.93134
		4	2.42475
		5	11.45400
		6	17.59820
		7	26.03000
		8	32.00000
3	Li	5	11.67900
		6	14.08840
		7	14.90730
		8	20.94620
		9	24.96500
4	Be	6	18.37560
		7	15.76890
		8	4.94420
		9	11.35050
		10	12.60700
		11	20.18100
5	B	7	27.99000
		8	22.92310
		9	12.41860
		10	12.05220
		11	8.66768
		12	13.37020
		13	16.56160
6	C	9	28.99000
		10	15.65800
		11	10.64840
		12	0
		13	3.12460
		14	3.01982
		15	9.87320
7	N	12	17.36400
		13	5.34520
		14	2.86373
		15	0.10040
		16	5.68510
		17	7.87100
8	O	14	8.00800
		15	2.85990
		16	−4.73655
		17	−0.80770
		18	−0.78243
		19	3.33270
		20	3.79900
9	F	16	10.90400
		17	1.95190
		18	0.87240
		19	−1.48600
		20	−0.01190
		21	−0.04600
10	Ne	18	5.31930
		19	1.75200
		20	−7.04150
		21	−5.72990
		22	−8.02490
		23	−5.14830
		24	−5.94900
11	Na	20	8.28000
		21	−2.18500
		22	−5.18220
		23	−9.52830
		24	−8.41840
		25	−9.35600
		26	−7.69000
12	Mg	22	−0.14000
		23	−5.47240
		24	−13.93330
		25	−13.19070
		26	−16.21420
		27	−14.58260
		28	−15.02000
13	Al	24	0.1000
		25	−8.9310
		26	−12.2108
		27	−17.1961
		28	−16.8554
		29	−18.2180
		30	−17.1500
14	Si	26	−7.1320
		27	−12.3860
		28	−21.4899
		29	−21.8936
		30	−24.4394
		31	−22.9620
		32	−24.2000
15	P	28	−7.6600
		29	−16.9450
		30	−20.1970
		31	−24.4376
		32	−24.3027

Figure 1.6: Mass excesses from Clayton [1983].