September 7 . . . .

2.3.2 Mean molecular weight

- Before we start applying this machinery, let's take a brief detour here, since the mean molecular weight μ is important to understand.
- Stellar interiors have a mixture of atoms of different elements and various ionizations.
- Consider the mean mass $\overline{m}$ per particle

$$\overline{m} = \frac{\sum_j n_{j,\mathrm{I}} m_{j,\mathrm{I}} + n_\mathrm{e} m_\mathrm{e}}{\sum_j n_{j,\mathrm{I}} + n_\mathrm{e}} \approx \frac{\sum_j n_{j,\mathrm{I}} m_{j,\mathrm{I}}}{\sum_j n_{j,\mathrm{I}} + n_\mathrm{e}}, \tag{2.14}$$

where $n_{j,\mathrm{I}}$ is the ion number density of ion j, $m_{j,\mathrm{I}}$ is its mass, and n_e and m_e are the numbers and mass of the electron (and then we ignore the electron mass).

- The mass of the jth ion is approximately its number of protons and neutrons (A_j) times the amu, or $m_{j,\mathrm{I}} = A_j m_\mathrm{u}$.
- So then we define

$$\mu = \frac{\overline{m}}{m_\mathrm{u}} = \frac{\sum_j n_{j,\mathrm{I}} A_j}{\sum_j n_{j,\mathrm{I}} + n_\mathrm{e}}. \tag{2.15}$$

This can be interpreted as the average mass per particle (ion, electron, etc.) in units of the amu.

- Note that the total particle number density in the gas is

$$n = n_e + n_\mathrm{I} = n_e + \sum_j n_{j,\mathrm{I}} = \sum_j (1 + Z_j) n_{j,\mathrm{I}}, \tag{2.16}$$

since one ionized atom contributes 1 nucleus plus Z_j electrons. The total $n_\mathrm{e} = \sum_j n_{j,\mathrm{I}} Z_j$, where Z_j is the "charge" of each nucleus.

- In general though, the electron density (or level of ionization) is complicated and derived from the Saha equation. Such an equation gives ionization fractions y_i such that the electron number density would be $n_\mathrm{e} = \sum_j n_{j,\mathrm{I}} y_j Z_j$ (see later).
- But to be more useful, it's easier to express the number densities in terms of mass fractions X_i, where $\sum_i X_i = 1$.
- The number densities we looked at earlier are for some species i are

$$n_i = \frac{\rho}{m_\mathrm{u}} \frac{X_i}{A_i}. \tag{2.17}$$

Think of this as the mass per unit volume of species i (ρX_i), over the mass of 1 ion of species i ($m_\mathrm{u} A_i$).

- So using this, we now have

$$\mu = \frac{\sum_i \frac{\rho}{m_\mathrm{u}} X_i}{\sum_i \frac{\rho X_i}{m_\mathrm{u} A_i} + n_e}, \tag{2.18}$$

or

$$\mu = \frac{\sum_i \frac{\rho}{m_\mathrm{u}} X_i}{\sum_i \frac{\rho X_i}{m_\mathrm{u} A_i}(1 + Z_i)}. \tag{2.19}$$