

Figure 4.3: (Left): Low-mass star evolution on the main sequence. (Right): The same but higher-mass stars. From [Salaris and Cassisi \[2006\]](#).

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4.3 Evolution on the main sequence

4.3.1 Low-mass stars

- The time of arrival on the main sequence is known as the ZAMS - zero-age main sequence.
- “Where” it ends up depends only on mass and chemical mixture.
- The lower mass limit is roughly about $0.1M_{\odot}$.
- The upper mass limit is about $100M_{\odot}$.
- The mean molecular weight changes a lot. Consider fully ionized H in core at beginning at ZAMS (see Equation (2.23)):

$$\mu = \frac{4}{3 + 5X - Z} \simeq 0.61. \quad (4.21)$$

- As all of it gets converted to He, we then have

$$\mu = \frac{4}{3 + 5(0) - Z} \simeq 1.3. \quad (4.22)$$

It more than doubles!

- This change (increase) in mean molecular weight causes changes in other things. Note that the opacity is reduced, as He is less opaque than H.
- The number of free particles also decreases, as does the pressure.
- A low-mass star slightly contracts its core and heats up. Firstly, ρ increases by the core contracting. As this happens gravitational energy gets released according to the Virial theorem, which partly goes to increasing the thermal energy of the core - increased T .

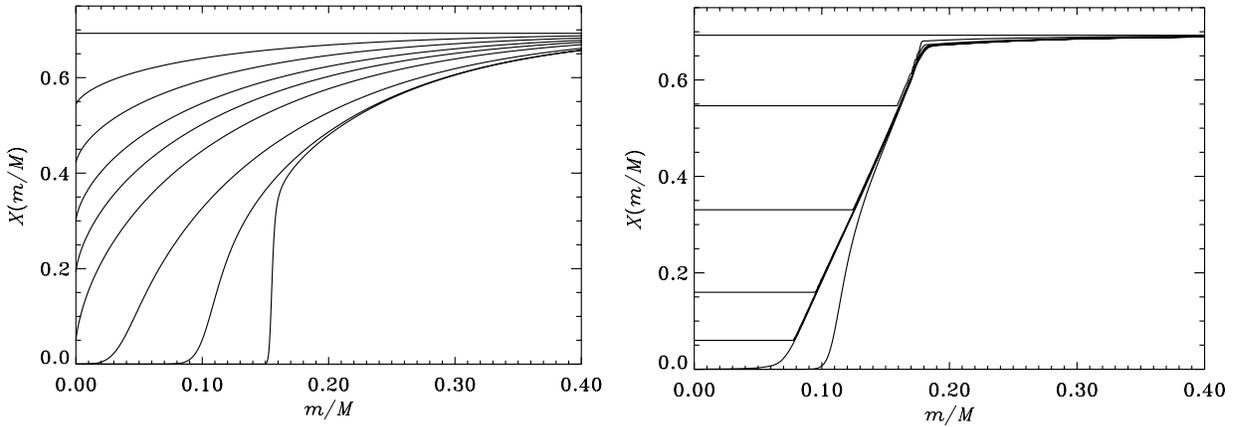


Figure 4.4: (Left): Hydrogen profiles showing the gradual exhaustion of hydrogen in a $1M_{\odot}$ star. The homogeneous initial model consists of a mixture with a hydrogen abundance by mass of 0.699. X as a function of the mass fraction m/M_{\odot} is plotted for nine models which correspond to ages of 0, 2.0, 3.6, 5.0, 6.2, 7.5, 9.6, 11.0 and 11.6 times 10^9 years, after the onset of hydrogen burning. The model at 5.0×10^9 years corresponds roughly to the present Sun, whereas the last two models are in the shell hydrogen burning phase. (Right): The same but for a $2.5M_{\odot}$ star. The lines show the hydrogen profiles for models of age 0, 1.5, 3.1, 4.0, 4.4, 4.6, and 4.8 times 10^8 years. From ?.

- This must increase the pressure to account for the “heavier” material. Indeed, according to the equation of state, if P increases, and μ increases, then ρT must certainly increase.
- The nuclear energy generation increases and then so does the luminosity.
- This causes a slow increase of the star’s luminosity over the whole MS phase.
- Also, as the core contracts, the surface radius increases, slightly for low-mass stars, more rapidly for high-mass stars. This can affect the effective temperature (see below).
- See Figure 4.3 for the MS evolution.
- Figure 4.4 (left) shows the core hydrogen mass fraction as a function of time. The burning region extends out to a significant radial distance.
- How much has the solar luminosity changed over time? Calculations (including homology ones) show that the Sun’s luminosity compared to its ZAMS luminosity L_0 is $L \simeq 1.26L_0$. This means it was about 25% less luminous than it is today, which has/had implications for the Earth.
- All of the main properties of the Sun from its ZAMS point until today are shown in Figure 4.5.
- The important things to note are the increase in density and decrease in X . One would think the temperature increase too would really increase the nuclear energy generation rate ϵ_c .
- But as we saw (Equation (1.45)), it not only depends on T but also on X^2 , so it is somewhat halted by the decreasing hydrogen amount over time.

4.3.2 High-mass stars

- The main difference in these stars is the increased temperature in the core, as in Figure 4.7.
- Thus, the CNO cycle is the dominant luminosity source.
- This has the effect of concentrating the the luminosity production in the inner 10% of the mass for a $10M_{\odot}$ star, compared to about 30% for a $1M_{\odot}$ star.

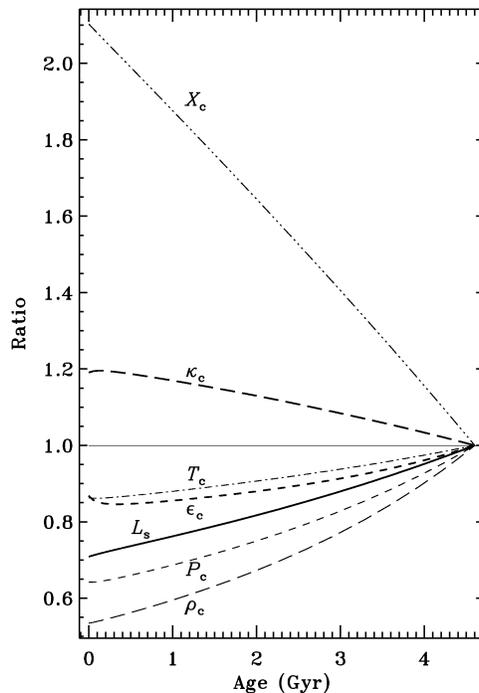


Figure 4.5: The changes in the solar properties at the center of a solar model. All variables are normalized with respect to the present Sun. From ?.

- The other effect is a steep temperature gradient in the inner regions due to the high flux. Thus, convection kicks in.
- This region becomes fully mixed chemically, as in Figure 4.4, right.
- The outer regions are radiative, since the ionization regions are very far out in the atmosphere compared to low-mass stars.
- As evolution occurs, the star gets brighter because of the strong dependence of L on μ , and μ .
- At the same time, the effective temperature shows a monotonic decrease, as in Figure 4.3, right. This is due to the increasing radius, which increases faster than the luminosity (especially compared to lower-mass stars).
- If the core of the star grows in size due to convective overshoot, it will also extend the MS lifetime and make the star brighter.
- Figure 4.4 (right) shows the core hydrogen mass, and note the shrinking convective core of the higher-mass star over time.
- One of the main reasons for this is the reduced opacity as H is converted into He and the electron scattering processes decreases.
- Also note in the higher-mass star in Figure 4.4 that since hydrogen burning is negligible at the edge of the convective core during the main-sequence phase, the hydrogen profile established during this phase reflects the decrease in the extent of the core. In contrast, the last model is in the hydrogen shell-burning phase, the helium core having grown substantially beyond the smallest extent of the convective core.
- Figure 4.6 shows a comparison of core mass size for a $1M_{\odot}$ and $20M_{\odot}$ model in both relative and absolute visualizations.

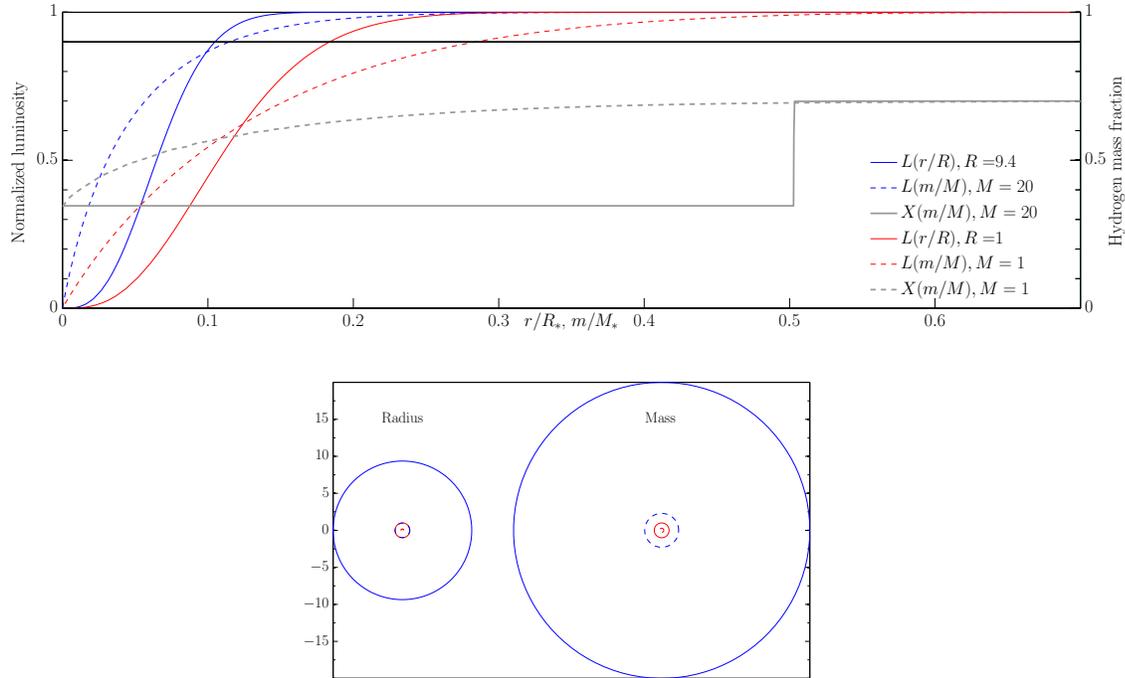


Figure 4.6: Comparison of core sizes for 2 models ($1M_{\odot}$ and $20M_{\odot}$) both with $X_c \approx 0.35$ on the main sequence. The top panel shows the normalized luminosity as a function of normalized radius and mass. A horizontal line at $0.9L_{\max}$ will be used to define the approximate core boundary. The blue lines are for the massive model, and red for the less massive one. The low-mass model has a core that is fractionally larger than the core in the high-mass model. The gray solid and dashed lines are the hydrogen mass fraction, given on the right y axis. In the bottom panel, the surface (solid lines) and core (dashed lines) boundaries are shown to scale in absolute masses and radii. On the left for the $9.4R_{\odot}$ model (blue), we see the core boundary in this space is almost exactly the same size of the entire low-mass star (red). In terms of mass, the core size of the massive star is larger than one solar mass.

- Note in this figure that even though the luminosity saturates at a high value very close to the center, the convective region of the core is quite extensive.
- The size of the convective core increases as the mass of the star increases too, due to the higher central temperatures.
- We can estimate the main-sequence lifetime of a star. If ε_{H} is the energy per unit mass per unit time of hydrogen burning, we know that

$$\tau_{\text{MS}} \propto \frac{q_c \varepsilon_{\text{H}} M}{L}, \quad (4.23)$$

where q_c is some fraction of the stellar hydrogen mass that actually participates in nuclear burning.

- If ε_{H} and q_c are roughly independent of total stellar mass, and we assume that $L \propto M^{\gamma}$ as we showed before, then

$$\tau_{\text{MS}} \propto M^{-(\gamma-1)}, \quad (4.24)$$

where we found $\gamma \approx 3-5$, and the relation is written to emphasize that the exponent is always negative, and main-sequence lifetime is inversely related to mass.