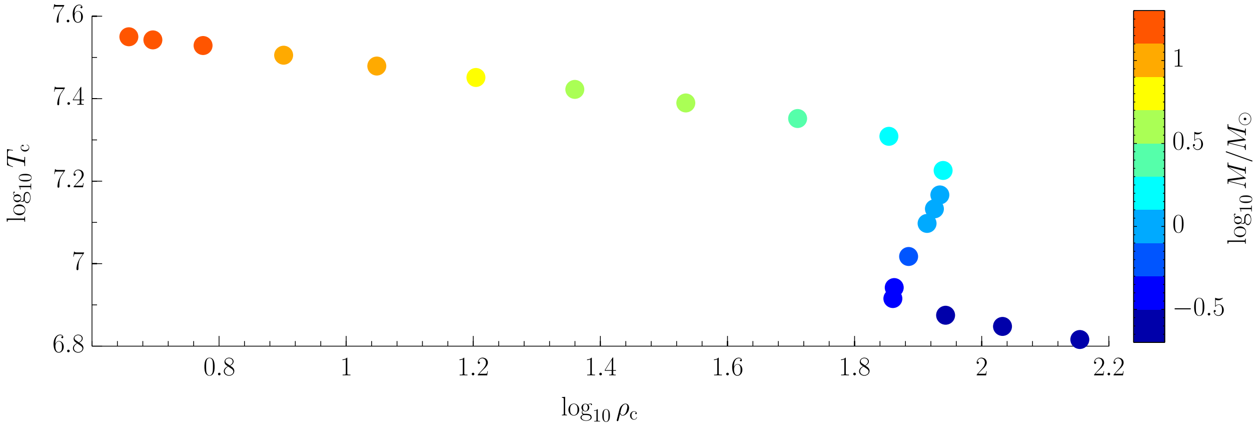

Figure 4.7: The central temperature and density for various `MESA` models of mass given by the colorscale. All models are just on the main sequence when $X_c = 0.68$.

November 7 ....

4.3.3 A note about very low mass stars

- Stars below about $0.3M_\odot$ are fully convective on the MS.
- They have large opacities due to low temperatures and very high densities.
- The densities are high because the stars need to contract to build up high enough temperatures for nuclear fusion.
- Only the PP-I chain can operate, so helium-3 is never really destroyed.
- But at the lower mass limit (high ρ), electron degeneracy kicks in.
- Conduction is very efficient here, which then cools the core below the minimum ignition temperatures.
- Some lithium or deuterium is burned, but these objects become brown dwarfs and cool down like white dwarfs.
- The difference between them and WDs is that they are fully mixed chemically, and their degenerate electrons do not move relativistically.

4.4 Summary of main-sequence properties

- In general, the star regulates its nuclear burning rate to maintain hydrostatic equilibrium.
- If the rate increases for some reason, the star expands, thereby decreasing its temperature and density, reestablishing equilibrium.
- Mass rules of thumb:
  - Below about 1.3 $M_\odot$ convective envelopes, above, radiative envelopes
  - Below about 1.2 $M_\odot$ PP chain, above, CNO cycle
  - Below about 1.5 $M_\odot$ late-type stars (F, G, K, M), above, early-type (O, B, A)
- Structure rules of thumb:

- Low-mass cores
  - PP chain is sufficient to balance gravity
  - Luminosity is not too steep, and energy flux is moderate
  - Radiation is enough to carry out the luminosity from the core
  - Core is radiative
  - Since the PP chain has a low temperature dependence, the region of burning is relatively a large mass fraction of the star, as in Figure 4.4, left.
- High-mass cores:
  - CNO cycle is necessary to balance gravity
  - High-temperature sensitivity ($\sim T^{20}$) means a very central energy generation region
  - Luminosity is very steep in core with a high flux
  - Temperature gradient is very steep, convection sets in
  - A convective core develops, which is very efficient
  - So efficient that here it equals the adiabatic gradient
  - Core mixing due to convection removes gradients in composition
  - The core temperature and density for different masses is shown in Figure 4.7.
  - Note the strong variations at a little above 1 solar mass stars, where convective cores start to appear.
- Low-mass envelopes:
  - Opacity is rather large because of hydrogen and helium ionization zones and corresponding bound-free transitions
  - Convection is needed to carry the radiative flux through the region; steep temperature gradient
  - Below about $0.3\,M_\odot$ the entire star is convective
- High-mass envelopes:
  - Hydrogen and helium ionized so rather low opacities
  - The radiative flux is carried out by radiation. Radiative envelope.
  - In very massive stars $> 10\,M_\odot$, some opacity peaks due to ionized iron and nickel can cause thin convection zones near the surface

- MS location rules of thumb (as we saw in homology relations):
  - Higher He content results in more luminous and hotter MS tracks.
  - The MS lifetime decreases with increasing He content.
  - Higher metallicity makes the star cooler due to increased opacity.
  - Alpha elements (O, Ne, Mg, Si, S, Ca, Ti, ...) that are enhanced in metal-poor stars produce fainter, cooler MS tracks.
  - Changing the mixing length, or convective efficiency, affects the MS.
  - There is no effect on the luminosity, but an increased effeciency sets up a lower thermal gradient.
  - This increases the effective temperature, and therefore the radius decreases.

- Evolution (on main sequence) rules of thumb:
  - ZAMS to TAMS lifetimes are much shorter for high-mass stars
  - Tracks on HR diagram are vertical for low-mass, and diagonal for high-mass stars
  - Luminosity increases due to increase in molecular weight in core
  - Low-mass stars have abundance changes in core that is smooth

- High-mass stars have discontinuous changes due to convective mixing (although core shrinking can smooth out the discontinuities)
- Below about $0.3\,M_\odot$ the core is completely mixed and stars live long
- Near the end of main-sequence evolution:
  * Nearly isothermal core with zero luminosity
  * Core is very hot from high μ
  * At core boundary temperature is high enough for shell burning to occur
  * The high T and large volume of the burning region leads to high shell L
  * No thermal equilibrium, envelope expands

Unit 5

The Post Main Sequence

5.1 General considerations

- We can only really talk about stars greater than about $0.8M_\odot$, since less massive stars are still on the main sequence (of course, we can use theory to talk about lower-mass stars).
- As stars evolve on the main sequence they go **above** the ZAMS up and to the right or left depending on mass.
- Notice that this is only the case for chemically inhomogeneous models.
- If a star remained mixed and the mean molecular weight increased with time, it would evolve below the ZAMS for a given mass, as we saw in our homology relations
- When the central hydrogen content reaches about $X_c = 0.05$ (points 3 in Figure 5.1 and 2 in Figure 5.2) for stars above about $1.1M_\odot$, the opacity is dropping (increased He), and the envelope luminosity is greater than the energy generation in the core (not much H left!)
- The star shrinks on a Kelvin-Helmholtz time scale to make up for the excess luminosity, then the effective temperature increases a bit (see § 5.1.1).
- This causes the little wiggle (or "hook") on the HR diagram. Low-mass stars do not show this because they do not need to contract so much because the luminosity was never that great. See Figure 5.1 (points 3 to 4).
- Note the large differences for $1M_\odot$ stars or slightly more massive ones: the main difference is the convective core.
- The higher-mass cores deplete H over large regions, and thus the contraction is more drastic as to maintain nuclear burning at the right level.
- Nonetheless, as $X \to 0$ for all masses:
  - Core is filled with inert helium (too cool to burn, needs 10^8K)
  - But there is a large T_c and μ
  - Core is isothermal since $\varepsilon \to 0$ and then $\mathrm{d}T/\mathrm{d}r \to 0$ (see Equation (3.18)).
  - The temperature at the core boundary is high enough, however, to ignite leftover hydrogen
  - The contraction has pulled in H to hotter and denser regions (still the shell), so it ignites
  - The shell burns and adds helium to the core, whose mass increases and it contracts more, heating it up (eventually to ignite He)

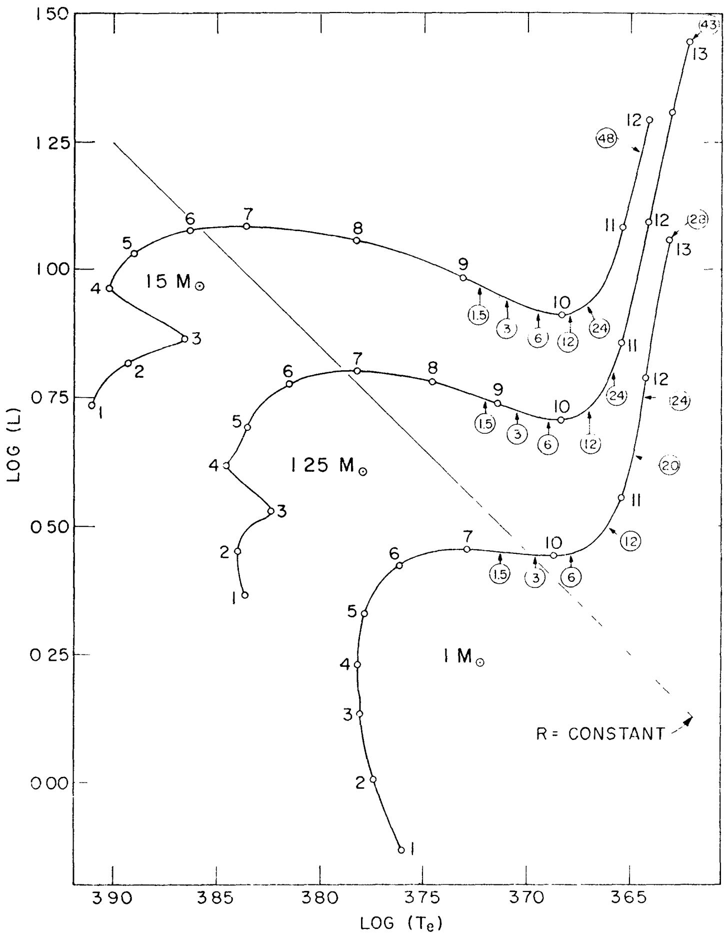

Figure 5.1: Evolutionary tracks for low-mass Pop. I stars. Basically, points 1-3 are the ZAMS to TAMS. From Iben [1967b].

- All of this emphasizes the **Shell-burning law**: When a region within a burning shell contracts, the region outside the shell expands; when the region inside the shell expands, the region outside the shell contracts.
- Despite many efforts, and the fact that numerical experiments show that this law is true, it is not obviously clear why it is the case.

5.1.1 Schönberg-Chandrasekhar Limit

- Let's look at what's happening in the core. Can it support the growing mass in the overlying layers from outer core burning?
- In 1942 Chandrasekhar and Schönberg looked at hydrostatic equilibrium for an isothermal He core and an ideal equation of state.
- Assume constant core temperature, and that the envelope provides a pressure $P_{\rm env}$
- Consider hydrostatic equilibrium and multiply both sides by $4\pi r^3$ and integrate in core (recall Equation (2.120)):

$$\int_0^{R_c} 4\pi r^3 \frac{\mathrm{d}P}{\mathrm{d}r}\mathrm{d}r = -\int_0^{R_c} \rho \frac{Gm}{r^2} 4\pi r^3 \mathrm{d}r = E_{g,c} \tag{5.1}$$

- Integrate by parts and use ideal gas law

$$4\pi R_c^3 P_c - 3\frac{M_c k_{\rm B} T_c}{\mu m_{\rm u}} = E_{g,c}. \tag{5.2}$$

- If we assume that the density is the mean core density $\rho \approx 3M_c/4\pi R_c^3$, then

$$E_{\rm g,c} \approx -\frac{3}{5}\frac{GM_c^2}{R_c}. \tag{5.3}$$

- Solving everything for P_c, we get

$$P_c = \frac{3}{4\pi R_c^3}\left(\frac{M_c k_{\rm B} T_c}{\mu m_{\rm u}} - \frac{1}{5}\frac{GM_c^2}{R_c}\right) \tag{5.4}$$

- The core pressure must match the envelope pressure for equilibrium, and must adjust its radius to do so.
- Can it always do so? Its maximum value is when

$$R_c = \frac{4}{15}\frac{GM_c \mu m_{\rm u}}{k_{\rm B} T_c}, \tag{5.5}$$

  which gives

$$P_c = \frac{10125}{1024 G^3 M_c^2}\left(\frac{k_{\rm B} T_c}{\mu_c m_{\rm u}}\right)^4. \tag{5.6}$$

- As you can see, as the core mass increases, the core pressure will drop and at some point may fall below the envelope pressure.
- The mass at which this happens is the Schönberg-Chandrasekhar limit.
- We know from hydrostatic equilibrium that $P_{\rm env} \propto M^2/R^4$.
- From homology, we can find that $P_{\rm env} \propto T_c^4/M^2$
- So the pressure at the surface of the core is independent of the core size.
- Using the right coefficients, it is then easy to show that

$$\frac{M_c}{M} \approx 0.37\left(\frac{\mu_{\rm env}}{\mu_c}\right)^2. \tag{5.7}$$

- If $\mu_{\rm env} = 0.6$ and $\mu_c = 1.3$ (solar composition), then the limit is roughly

$$\frac{M_c}{M} \approx 0.08. \tag{5.8}$$

- Above this limit, which will likely occur for stars greater than $3M_\odot$:
  - the isothermal core contracts rapidly
  - the density increases, the temperature increases and nuclear reactions speed up in the shell
  - This pushes in both directions and mass is lost in the shell, and burning is in a thin shell
  - Even though the energy rate increases, the luminosity decreases a bit because of the mass loss in the shell
  - Since the timescale is faster than the nuclear one, the stars become redder very quickly
  - This leads to observational Hertzsprung gap (points 4 to 5 in Figure 5.2)
  - Not many stars have time to be "observed"
- For low-mass stars ($\leq 1.3M_\odot$), the helium core is somewhat degenerate and higher pressures are present, so this limit is not applicable and the approach to the RGB is slower.
- For higher-mass stars, the contraction happens very quickly and isothermal cores never actually have time to set in.

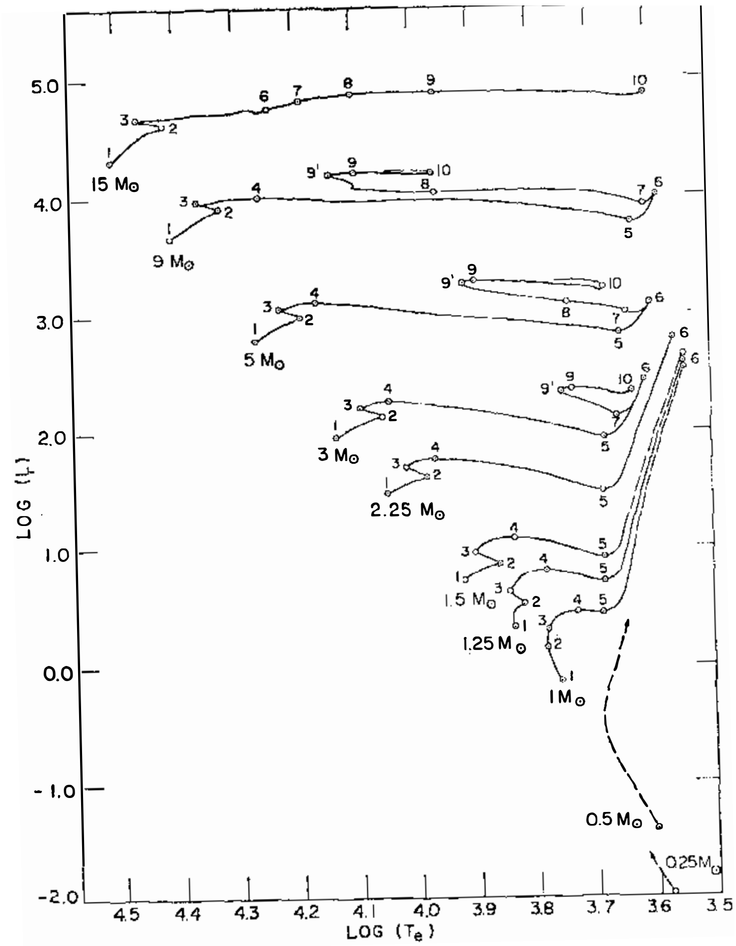

Figure 5.2: Paths in the HR diagram for a range of masses and solar metallicity. From Iben [1967a].

5.1.2 The subgiant branch

- To summarize the above once again, in general, the move across the H-R diagram to the right defines the subgiant branch (SGB).
- The envelope now has to adjust to a new source of energy, the thick burning shell
- The luminosity is larger as the burning takes place at a higher temperature than it was in the core
- With a large luminosity the shell has a difficult time radiating it (it will eventually become convective)
- But right now it absorbs the luminosity, heats up, and expands
- The Virial theorem shows some of the energy goes into expansion, not all of it makes it to the surface
- The effective temperature decreases and the stars move to the right on the HRD diagram, approaching the base of the red-giant branch
- The slope of the luminosity in this move across the HR diagram depends on mass
- This should happen over the timescale of shell burning, a nuclear timescale ...
- But other influences may affect it, as discussed below

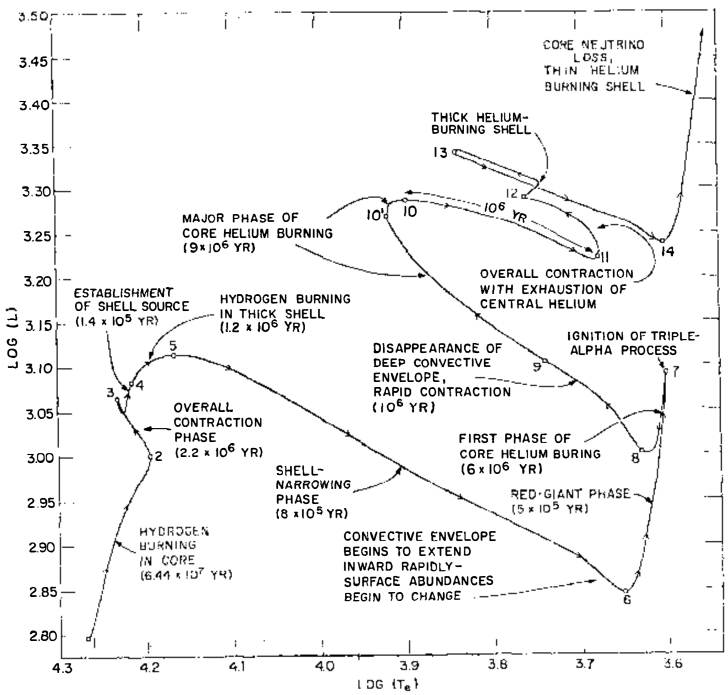

Figure 5.3: The path of a metal-rich $5M_\odot$ stellar model. From Iben [1967a].

5.2 Towards and up the RGB

5.2.1 High-mass stars

- Last time we were rather general, now let's get specific
- Let's start with about a $5M_\odot$ star.
- See Figure 5.3.
- Recap: hydrogen burning in points 1-2 via the CNO cycle at the center of a convective core
- The core mass fraction of energy burning drops from about 0.2 to 0.08 to zero following contraction (after point 3) because of the convective core.
- The balance between pressure forces can only be maintained (because of increasing molecular weight) by increased heating and increased density (contraction), leading to increased luminosity.
- Between 3 and 4, most nuclear processes shift to a thick shell where H is abundant.
- As the core contracts and H is pulled in to higher density and temperature regions, the ignition is somewhat explosive, and matter is pushed away in both direction from the thick shell.
- The radius increases, and some of the nuclear energy is used to expand the envelope, and so not all the luminosity "reaches the surface," and drops after point 3.
- The timing of when the shell burning takes over from the core burning as the dominant source is not completely understood and leads to uncertainties in observations.
- Between 4 and 5, energy is produced in a shell of about 5% of the star's mass, increasing to close to 10% at point 5 (also see Figure 5.5).

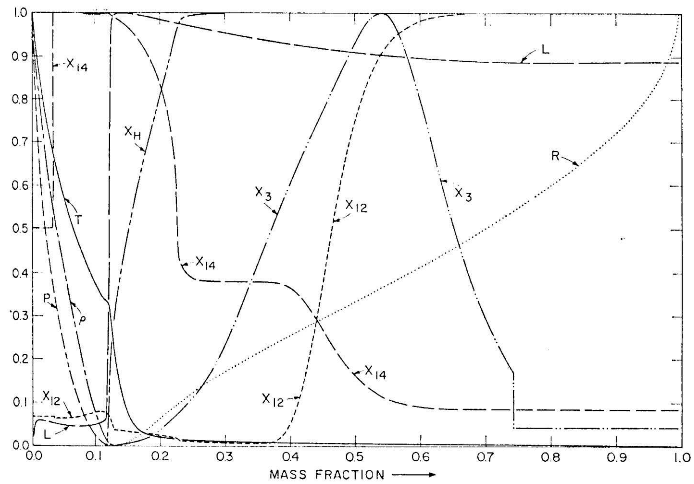

Figure 5.4: The $5M_\odot$ stellar model profiles roughly at point 6 in Fig. 5.3. All quantities are scaled to their maximum value. The surface radius of the model is 51 $M_\odot$. From Iben [1966].

- After point 5, the core contracts rapidly, energy generation increases, and another mildly explosive shell event takes place, expanding the envelope rapidly.
- The mass in the shell decreases to about 0.5% at point 6 from 3.5% at point 5, decreasing the overall luminosity in the process
- All of this is due to the C-S limit and the reaction of the whole star to it.
- It also happens very quickly.
- Approaching point 6, opacities increase because of the cooling, and a convection zone near the surface appears as Fig. 5.5 also shows this occuring.
- Mixing occurs near the surface.
- H^- is the dominant opacity source.
- The star is now about $45R_\odot$.
- As the effective temperature decreases beyond 6, the luminosity begins to increase as convection allows the luminosity to escape more readily.
- The shell burning power increases, but at the same time the mass in the shell decreases.
- Figure 5.4 shows the internal structure of the star before it ascends the RGB.
- The variation in the luminosity is interesting, as you can see that most of it is generated in a thin shell at about 0.12 M.
- But note that near the center there is a small amount due to initial He burning that is beginning to take place.
- The subtle decrease in L in the envelope is due to expansion of the outer envelope where no nuclear sources reside and gravitational energy is "lost."
- Note the core is no longer isothermal as a temperature gradient is required to carry out the luminosity supplied by He burning.