Figure 5.13: Interior quantities (scaled) for a $5M_\odot$ model. The radius here is $44R_\odot$. This point corresponds to approximately point 12 in Figure 5.3, where we have double shell burning with an expanding region in between. From Iben [1966].

November 16 ....

5.3.4 Horizontal branch evolution

- In general, the H-burning shell decreases in efficiency and the He-burning core increases.

- If the luminosity from the shell is greater than that of the core, the star goes blueward on the HR diagram.

- When the core starts to dominate, the star moves redward.

- One starts to see "loops" that depend strongly on the parameters.

- There are also internal "breathing" phases from convective overshoot that influence the observable properties, but we won't discuss these.

- Also note the *Instability Strip* crosses the horizontal branch, where stars pulsate in long periods (RR Lyrae stars, more later).

- Very massive stars ignite carbon very quickly (more later).

- Consider again a $5M_\odot$ model, as most stars behave similarly now.

- Still refer to Figure 5.3 and Figure 5.5.

- The star is going to go left-right-left.

- There is core He burning and still shell H burning (E-F).

- The convective envelope is now gone after point E.

- More than half of the luminosity still comes from shell burning.

- Central He burning lasts about $1 - 2 \times 10^7$ yr.
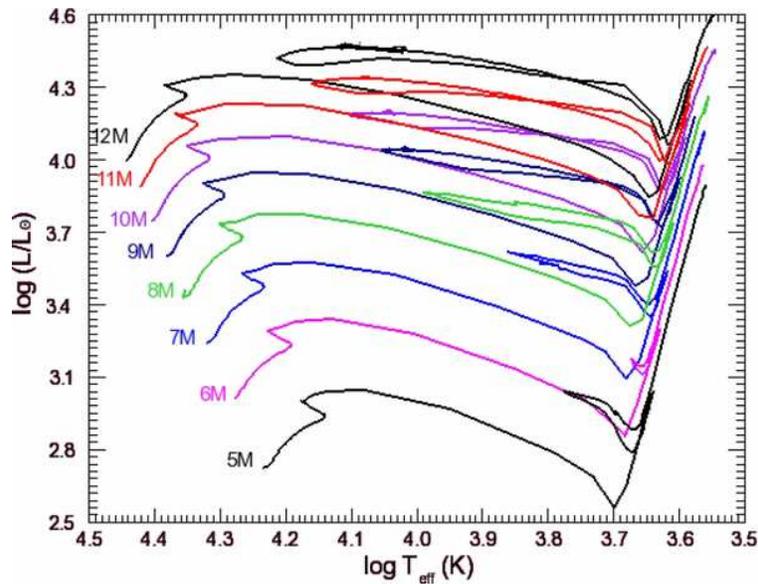
- The move from E to F represents a "loop."

Figure 5.14: Evolutionary tracks for intermediate-mass stars and higher, showing the extent of the blue loops.

- It's similar to the B to C movement, except that was for after H burning.

- He burning is getting stronger, and the core is expanding/envelope shrinking.

- At point F, the central He abundance is 0.5.

- The 3α energy fraction is about 20 percent.

- The number of loops, and how far "blue" they go, depends on stellar mass.

    - The more massive a star, the longer the loop to the blue.
    - That's because the H-burning shell contributes a significantly greater amount of energy in intermediate-mass stars than for low-mass stars.
    - Lower-mass stars don't have significant loops.
    - Increasing He abundance extends the blue loop, as does lowering the metallicity. However, models show nonlinear behaviors here.
    - Increasing core convective efficiency reduces the extension of the blue loop.
    - These loops are slow because of nuclear burning and therefore can be expected to be observed (and are: Cepheids!)
    - See Figure 5.14.

5.3.5   Asymptotic giant branch

- At point F or point 10 (in Figure 5.3 and Figure 5.5), He is less and less available for fusion.

- As C and O builds up in the core, the mean molecular weight increases.

- The core contracts and increases in temperature (as before, in the Hertzsprung Gap).

- The shell-burning law kicks in and the envelope expands, star moves to the red.

- The star starts moving redwards as the H-burning shell loses its dominance.

- Regardless of the mass, with low He abundances, the stars move back toward the Hayashi track.

- As He burning in the core exhausts, a new shell of He burning takes over (point G).

- The mass of the CO core increases.

- The contracting releases gravitational energy and some gets converted to thermal energy and it reignites He.

- The core is too cold for C or O to burn (neutrinos are cooling it!).

- Now at point G there are 2 shells! (or point 11 in Fig. 5.3)

- The core is contracting and heating up, and the space in between shells expands, and the outer part of the star contracts, moving to higher $T_{\rm eff}$. (G-H) or (11-12)

- This is the shell-burning law in triple!

- Figure 5.13 shows the interior state for a $5M_\odot$ model at about point 12.

- The luminosity is interesting and notable: The luminosity at the innermost region is from gravitational contraction of the core.

- Between the 2 shells, the luminosity is decreasing slightly as that region expands (does work against gravity).

- In the outer part above the H shell, the envelope is also contracting, releasing gravitational energy.

- Remember that $dL/dr = 0$ unless energy is being generated, so here we'd really have to account for $\varepsilon_{\rm nuc} + \varepsilon_{\rm grav}$.

- Does C now burn?

- For stars lower than about $10M_\odot$, the carbon-oxygen core is degenerate.

- Thus, the contracting core does not heat up the gas and the high internal temperatures needed to ignite the core are not reached.

- This is clear from Fig. 5.12. Look at the $7M_\odot$ case.

- As the star reaches point H (or point 12) from G, the shell-burning law has the core contracting (increasing in mass), the inner shell expanding, and the outer envelope contracting and increasing in effective temperature.

- But now the region between the two shell-burning sources has expanded sufficiently so that the temperature in the outer H-burning shell drops and extinguishes.

- This is thought to occur in the range of $3 - 5M_\odot$ stars.

- Now there are only 2 distinct regions: the contracting core, and the now expanding envelope with He burning in between (shell-burning law).

- The star heads back to the right (toward K).

- The luminosity begins to increase as the CO core mass increases and contracts, and the star heads up the Hayashi track yet again.

- This is the Asymptotic Giant Branch (AGB).

- Since the outer shell burning ceased, the the luminosity drops a little and the star cools.

- A convective surface region develops and extends deeply, and "dredges up" processed material (at point K).

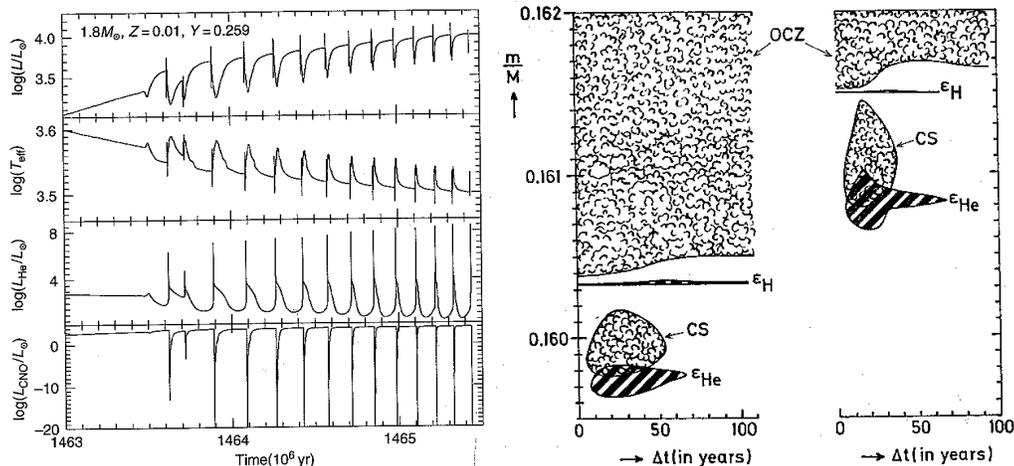- It reaches down to the regions where the H shell had been burning for a while.

Figure 5.15: Thermal pulses. The left is for a $1.8M_\odot$ model showing many pulses with time. The right is for a $5M_\odot$ model

- Low-mass stars don't typically have a second dredge up since their H shell burning continues strongly.

- This dredge up brings a lot of material to the surface and reduces the mass size of the H-exhausted region.

- This is one reason why very massive white dwarfs are not formed.

- An upper limit for carbon ignition in the core is about $8M_\odot$.

- The next phases are rather complex for stars below this limit .

5.3.6   Thermal pulses

- One sees that the growing He-burning shell approaches the bottom of the H-rich envelope.

- The He burning dies down a bit when it hits this region, contracts rapidly, and a H-shell reignites.

  1. As H burns, the He ashes fall onto the former shell burning region, and are compressed and heated.
  2. When the mass reaches about $10^{-3}M_\odot$ for a CO core mass of about $0.8M_\odot$, He ignites again.
  3. A runaway occurs, in that this ignition heats the overlying shell burning region and causes it to burn even more violently (note the temp. dependence of nuclear reactions).
  4. See Equation (5.12).
  5. The luminosity of the He burning reaches very high values, and this causes the layers above it to expand.
  6. The H-burning shell turns off.
  7. Because of the high luminosity, a convection zone develops.
  8. Eventually the convection helps expand the region, and the He burning drops strongly and cools.
  9. The convection zone disappears as the luminosity decreases.
  10. He burning continues, using up the He that the H-burning region produced before the flash.
  11. As this source reaches the new discontinuity, a new H-burning region is created as before.
  12. The He ash falls onto the He layer and the whole process happens again, now at a higher position.

- The thermal runaway is unusual because of the geometry.

- It can be shown using an ideal gas EOS (not much degeneracy here) that

$$\frac{d\rho}{\rho}\left(4\frac{s}{r} - \alpha\right) = \beta\frac{dT}{T}, \tag{5.12}$$

where s is the thickness of the shell located at r, and the constants are positive.

- Stable burning usually causes a decrease in temperature when expansion occurs (lower ρ), but for a small enough shell s, the expansion can cause a *increase* in temperature.

- The timescale between pulses can be approximated roughly by

$$\log\tau \approx 3 + 4.5(1 - M_c/M_\odot). \tag{5.13}$$

- For a core mass of $0.5M_\odot$, this gives about 10^5 years; but drops to about 10 years for near-critical mass stars.

- Some stars can go through hundreds of pulses before the H shell gets depleted.

- In high-mass stars, these cannot be observed because they are buried within the massive envelope.

- In low-mass AGB stars, the effects of the pulses can be seen chemically.

  - In the pulses when H burning is turned off, the surface convection zone moves inward, and a third dredge up can occur.

  - This can bring up carbon and heavy s elements (Sr, Y, Zr, Ba, La, Ce, Pr, Nd). Carbon-rich stars can be "produced."

  - Stars below about $1.5M_\odot$ will likely not go through a third dredge up, as their envelope mass isn't large enough.

  - For more massive stars, large amounts of lithium can be produced (from berrylium 7) and brought to the surface.

5.4 Last stages of evolution: low-mass stars

Here we discuss the late stages of evolution after the thermal pulses to the white dwarf.

5.4.1 Production of s elements

- As mentioned earlier, AGB stars are spectroscopically enriched in s-process elements.

- The s refers to *slow* neutron captures (compared to β decay).

- About half of the elements heavier than Fe are created by this process.

- You need neutrons to make these heavier elements, where the neutron eventually decays into a proton to make a stabler isotope.

- The neutrons densities needed are lowish, of order $10v$ per unit volume.

- Since at this stage there is a large abundance of N-14 in the intershell region, it gets converted to Ne by

$$^{14}\text{N}(\alpha,\gamma)^{18}\text{F}(\beta^+,\nu)^{18}\text{O}(\alpha,\gamma)^{22}\text{Ne}. \tag{5.14}$$

- If the temperatures at the base of the intershell region get to a few hundreds of MK, then $^{22}\text{Ne}(\alpha,\text{n})^{25}\text{Mg}$ can provide a source of neutrons.

- For stars of initial mass $< 3M_\odot$, it probably does not get hot enough for this reaction to occur.

- There may be some channels through carbon 13 that provide a high neutron density for low-mass AGB stars.

- However, this is still a very active area of research due to many physical uncertainties.

5.4.2   Planetary Nebula

- AGB stars of low mass continute to brighten as their (outer) H shell approaches the surface.

- The thermal pulse number is set by the mass of the H envelope and the mass of the CO core.

- If the CO core exceeds about $1.4M_\odot$, non-degenerate, non-explosive carbon burning sets in and the AGB phase is over.

- Due to mass-loss processes during the AGB pulses, no star is really able to reach that core mass, because the H-burning shell stops when it's at about $10^{-3}M_\odot$ below the surface.

- The mass loss can be quite large, even $10^{-5}\,\mathrm{M_\odot\,yr^{-1}}$

- These superwinds that are created speed off at $10\,\mathrm{km\,s^{-1}}$

- The mass loss could be explained by pulsations (Mira variables).

- The gas compression, and subsequent cooling and formation of molecules and dust grains, can trap the outgoing radiation and get carried away.

- Anyway, however it happens, when the pulses stop, and the star evolves to hotter effective temperatures.

- The maximal luminosity depends on the star's initial mass (and that of its envelope) and how much mass it has lost.

- The H shell burning region approaches the surface and the effective temperature increases.

- An even faster stellar wind is produced, up to $2000\,\mathrm{km\,s^{-1}}$, which bumps up against the previous shell ejecta - this produces interesting features in the planetary nebula.

- The shell is dusty and optically thick (masers).

- The envelope is irradiated by UV radiation from remaining hot central star (core).

- The gas gets ionized and recombines quickly, giving distinct emission lines when the stellar remnant reaches about $30,000$K.

- A thin H-burning shell continues until the bluest point on the evolutionary track.

- Then, the H-rich envelope and He-rich layer contract quickly. A few scenarios are now possible:

  1. All nuclear burning shuts off and the star cools as a WD.
  2. The heating of the He from contraction leads to a thermal runaway, and the star goes back near to the AGB (born again). Then does the same stuff and cools as a WD.
  3. The heating of the envelope causes a H-burning runaway and the star is a *self-induced nova*. The process can be dynamic and blow off all H layers to become a DB white dwarf. Or, the process can be quiescent and it will burn H and start to cool down, possibly leading to another nova event.

5.4.3 White Dwarfs

- Recall all the discussion in Sec. 2.6.4.

- Degenerate matter obeys polytropic relations $P \sim \rho^\gamma$.

- For non-relativistic particles, $\gamma = 5/3$.

- For relativistic particles, $\gamma = 4/3$.

- The cores of evolved degenerate stars like white dwarfs are dominated by electron pressure rather than ion pressure.

- That's because $\mu_e \approx 2$ and $\mu_{\rm ion} \approx 12$, and $P \propto \mu^{-1}$.

- Why are white dwarfs special?

- Let's simply consider approximations to equilibrium with averages over the star, so

$$\frac{P}{M} = \frac{GM}{4\pi R^4}. \tag{5.15}$$

- For the polytrope, replacing density by its average value

$$P \sim \left(\frac{M}{R^3}\right)^\gamma. \tag{5.16}$$

- The "pressure" term f_p from equilibrium and the EOS, and the "gravity" term f_g from equilibrium, are

$$f_p \sim \frac{M^{\gamma-1}}{R^{3\gamma}}; \qquad f_g \sim \frac{M}{R^4}. \tag{5.17}$$

- Their ratio must be 1 for equilibrium

$$f = \frac{f_g}{f_p} \sim M^{2-\gamma} R^{3\gamma-4}. \tag{5.18}$$

- This is $M^{1/3}R$ for $\gamma = 5/3$, and $M^{2/3}$ for $\gamma = 4/3$.

- So consider a star less than some critical mass $M < M_{\rm crit}$ and non-relativistic electrons. The star can get into an equilibrium by just adjusting R so that $f = 1$.

- If we increase M so that $f > 1$ (more gravity), R must decrease to regain equilibrium (hence more massive WDs are smaller)

- Now consider relativistic electrons.

- We can only get equilibrium by setting the mass to a certain value $M = M_{\rm crit}$.

- If $M < M_{\rm crit}$, $f < 1$, and the pressure term is dominant and so the star expands so that the electrons become non relativistic.

- But if $M > M_{\rm crit}$ and $f > 1$, the gravity term forces the star to contract. But this does not help, because F is independent of R!

- The star collapses without "finding" an equilibrium.

- Clearly, $M_{\rm crit}$ is some limit.

- So again, consider a total degenerate equation of state then (recall Eq. 2.53)

$$P \approx \frac{R}{\mu_e}\rho T + K_\gamma \left(\frac{\rho}{\mu_e}\right)^\gamma. \tag{5.19}$$

- γ depends on density and relativistic effects, being $\gamma = 5/3$ for $\rho \ll 10^6$ and $\gamma = 4/3$ for $\rho \gg 10^6$.

- Using polytropic relationships once can derive a critical mass that governs the future behavior of the core of these dense stars

$$M_{\mathrm{crit}} = \left(\frac{K_{4/3}}{fG}\right)^{3/2} \mu_e^{-2},\tag{5.20}$$

  where f is the ratio of the mean density to the central density.

- The cricital mass is then identified as the Chandrasekhar mass (Equation (2.171)):

$$\frac{M_{\mathrm{Ch}}}{M_{\odot}} = \frac{5.836}{\mu_e^2} = 1.456\left(\frac{2}{\mu_e}\right)^2.\tag{5.21}$$

- It's also important to see how the central temperature and density depend on this critical mass:

$$\frac{\rho_c}{\mu_e} = \frac{1}{8}\left(\frac{K_{4/3}}{K_{5/3}}\right)^3\left(\frac{M_c}{M_{\mathrm{crit}}}\right)^2 \approx 2.4 \times 10^5 \, \mathrm{g\,cm^{-3}} \left(\frac{M_c}{M_{\mathrm{crit}}}\right)^2,\tag{5.22}$$

$$T_c = \frac{1}{R}\frac{K_{4/3}^2}{K_{5/3}}\left(\frac{M_c}{M_{\mathrm{crit}}}\right)^{4/3} \approx 0.5 \times 10^9 \left(\frac{M_c}{M_{\mathrm{crit}}}\right)^{4/3} \, \mathrm{K}.\tag{5.23}$$

- For core masses below critical, maximum temperatures cannot exceed about 500 million K.

- In white dwarfs it is believed that the electrons are relativistic in the central part, but non-relativistic in the outer part.

- This changes the above results quantitatively, but not qualitatively.

- The mass of the core compared to the critical (Chandrasekhar) mass can be distinguised by 4 cases:

Case 1: If $M_c < M_{\mathrm{crit}} \approx M_{\mathrm{Ch}}$ and if there is no significant envelope (from mass loss or just a small original mass), so that M_c will NOT approach M_{Ch} during shell burning, then the core becomes degenerate, will cool, and the star becomes a white dwarf. T_c peaks. If it is a member of a binary system, then it can accrete enough mass to ignite carbon, which will detonate He and destroy the star in a runaway, producing at Type I supernova

Case 2: Initially if $M_c < M_{\mathrm{crit}}$ but there remains an envelope such that shell burning M_c can grow to M_{Ch}, the core becomes degenerate and cools. However, ρ_c increases with M_c and carbon will be ignited. This will happen for $4 \lesssim M/M_{\odot} < 8$, and the stars will likely become white dwarfs as well, but C-O white dwarfs.

Case 3: If $M_{\mathrm{crit}} < M \lesssim 40 M_{\odot}$, degeneracy does not happen. The core can heat up even more (because of no degeneracy) and further nuclear reactions can occur. Eventually the core collapses leading to formation of a neutron star and ejection of the envelope. This is a Type II supernova. See next section.

Case 4: If $M_c \geq 40 M_{\odot}$, the core also will burn C non degenerately. Black hole. See next section.

- In reality, almost all stars born with about $8 M_{\odot}$ or less will lose much mass and will not reach the interior conditions to ignite carbon.

- Most WDs are observed at $0.6 M\odot$ with very little variation.

- The higher-mass stars will lose all of their envelope and become CO WDs.

5.4.4 Futher WD properties

- It can be shown through simple Virial arguments that white dwarfs cool (approximately) according to the Mestel law

$$\Delta t \propto \left(\frac{L}{M}\right)^{-5/7} \approx \frac{4.5 \times 10^7}{\mu_i} \left(\frac{LM_\odot}{L_\odot M}\right)^{-5/7} \quad \text{[year].} \tag{5.24}$$

- The Δt represents the time for some change in luminosity.

- So higher mass WDs cool more slowly, due to more storage of energy.

- Increasing the ionic mean molecular weight decreases the evolutionary time, since there are fewer ions in that case.

- Roughly, for a WD to reach 1/1000th of the solar luminosity, it would take 1 billion years.

- A more precise cooling law would treat the ions more properly, since the steady decreae in temperature causes Coulomb interactions to become more important.

- The specific heats ratio grows and the ions form a lattice: this is *crystallization.*

- The crystallization obviously affects the EOS, and therefore the cooling times.

- The envelopes of WDs canonically predicta He layer of $\sim 10^{-2} M_{\text{tot}}$ above the CO core, surrounded by a H envelope of mass $\sim 10^{-4} M_{\text{tot}}$.

- Mass loss causes these notions to change.

- Metals are rarely observed due to atomic diffusion processes.

- Typically there are about 4 H WDs for every 1 non-H WD, but this varies widely with effective temperature, as evolutionary processes are still ongoing.

- The outer layers determine the opacity, and hence the cooling times.

- As WDs cool, they can develop convection zones as well.

- WDs can also be He core stars from an RGB progenitor that lost its envelope.

- There are also O-Ne WDs from higher-mass progenitors.

5.4.5 Type Ia supernovae

- Type I do not have visible hydrogen in its spectrum.

- The progenitor of these events are CO WDs that have accreted mass from a companion to exceed the critical mass .

- The companion is likely on the RGB or AGB where mass can be transferred effectively.

- In one scenario two similar stars evolve through a common envelope, and the two CO WDs merge through angular momentum loss from gravitational waves.

- The resultant object is higher than the critical mass.

- This is the *double-degenerate* channel.

- No observational evidence of these massive systems has yet to be shown.

- In the *single-degenerate* case, the events that happen depend strongly on the mass-accretion rates onto the low-mass star.

- For low rates, H burning on top of He layers produces an electron degenerate scenario, which can undergo explosive flashes.

- These are typically the classical *Novae*.

- Building up enough mass this way to reach the critical limit takes longer than a Hubble time, however.

- At moderate rates it can be shown that an explosion would occur for *sub-Chandrasekhar* mass objects.

- If enough He is accreted (either through H-burning above or from an He-rich companion), violent He-ignition can occur which can detonate the CO core.

- The explosion models must give about 10^{51}erg, as well as the production of heavy elements, such as lots of nickel.

- The explosive event is a shock wave whose speed depends on densities and abrupt thermal changes.

- Typical SNIa light curves rise rapidly and after about 20 days the light fades monotonically, with slighly different behavior in different bands.

- Almost all of them have a maximum absolute magnitude of -20 which declines linearly within 15 days after maximum.

- Thus, they are excellent standard candles.

5.5   Last stages of evolution: high-mass stars

5.5.1   Nuclear burning

- Now we consider stars greater than $15M_\odot$.

- If we only focus on the core, the processes in the late stages follow the simple causes and effects of

$$\ldots \to \text{nuclear burning} \to \text{fuel exhaustion} \to \text{core contraction} \to \text{core heating} \to \ldots \qquad (5.25)$$

- At 500 million K, carbon burning can take place in the core, converting into Mg, Ne, and Na.

- Neutrinos from pair annihilation contribute a substantial amount to the luminosity (energy loss), even more than the nuclear burning.

- So the star contracts to make up the loss with gravitational energy.

- The convective C core is less massive for more massive stars, as neutrinos cause a considerable amount of mass loss.

- Eventually the burning moves to a shell of C and s-elements are produced.

- The core is now about 70% O, 25% Ne, and the rest Mg.

- Ne burning sets in .

- Oxygen burning can set in at 1.5 billion K.

- Eventually, the photons released in such reactions have such high energies that they can photo-disassociate surrounding nuclei.

- A large number of neutrons begin to be produced.

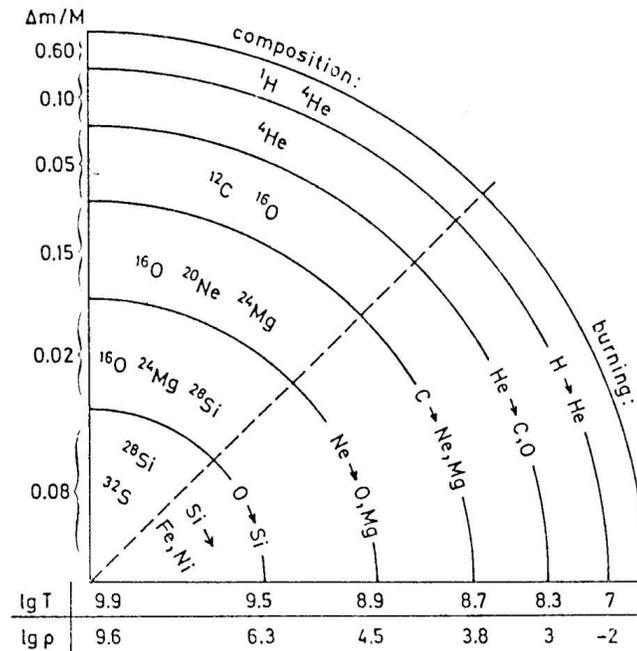- Silicon burning sets in through interactions with alpha particles.

Figure 5.16: Illustration of the "onion-skin" structure in the interior of a highly evolved massive star. From Kippenhahn and Weigert [1990].

- Then it moves into the shell.

- The process stops at the iron group, and the last reaction is ^{52}Fe capturing an alpha particle to make nickel.

- Further reactions would **require** energy to proceed. All reactions now are balanced by their inverse reaction.

- Figure 5.16 shows how the core builds up its layers this way.

- For a $15M_\odot$ star, the burning of each successive element is rapid:

$$H(10^7); He(10^6); C(10^3); Ne(10^1); O(10^1); Si(10^{-1}), \tag{5.26}$$

  where the times are in years.

- Since all this happens so quickly, the surface is basically "frozen in" and the star does not move from right to left until it explodes.

5.5.2 Type II supernova - core collapse

- The core of the massive star is hot, $T_9 \approx 10$, and electrons are relativistic.

- Simply, the ratio of specific heats, γ, drops below $4/3$ and the star is in an unstable configuration.

- After silicon burning, electrons are captured by protons

$$p^+ + e^- \longrightarrow n + \nu_e, \tag{5.27}$$

  producing a lot of electron neutrinos and neutrons.

- Neutrinos carry away energy, cool the core, and pressure drops.

- Photo-disintegtration produces many free α particles.

- The loss of free electrons also reduces the pressure.

- The core collapses from overlying weight on the scale of a few seconds.

- This collapse halts when the neutron degeneracy pressure kicks in ($\rho \approx 10^{15}\,\mathrm{g\,cm^{-3}}$) - note that this is nuclear matter density!

- Energy release of about 10^{53}erg from change in gravitational energy $GM^2/\Delta R$.

- This is as much light as a galaxy shines at for decades.

- In one scenario, most of the light is not released however, but goes into the kinetic energy of a shock.

- This propagates outward into the outer core region that is still collapsing.

- Naively, this might blow off the outer layers, but that does not happen.

- In the more accepted scenario, the increased core density causes it to be optically thick to neutrinos.

- They begin to deposit their energy into the material.

- This causes the outward shock that blows off the star's layers.

- This is the core-collapse Type II event.

- Heavy nuclei are created through neutron capture (s and r processes)

- Just to note, the neutrino cross section is extremely small, and its mean free path is

$$\ell_\nu = \frac{1}{n\sigma_\nu} \approx \frac{1}{\mu_e A} \left(\frac{\rho}{\mu_e}\right)^{-5/3} 1.7 \times 10^{25}\,\mathrm{cm}. \tag{5.28}$$

- $\mu_e = 2$, $A = 100$, and a density of about 10^{10}, $\ell_\nu \approx 10^7$cm, which is contained withing the collapsing core

- So neutrinos do not escape without interaction.

- About 1% of the energy goes into the outward motion, and 1% of that gets released as photons

- So only about 10^{49} erg of energy gets radiated over a few months

- We observe these supernova because hydrogen lines are present.

- The collapse occured when a H-rich envelope still existed.

- If the initial star was $M \geq 25M_\odot$, the remnant is likely a neutron star.

- For higher masses, it is too much to be supported by neutron degeneracy pressure.

- The object then becomes a **black hole**.

- If the initial star was over $100M_\odot$, or a core He mass of about $40M_\odot$, a different scenario might take place.

- After He burning the thermal environment produces electron-positron pairs, reducing the specific heat so that $\gamma < 4/3$.

- The star immediately starts to collapse and subsequent burning is not enough to halt the collapse.

- The star produces a black hole in a *pair-instability* supernova.

5.5.3 Neutron star

- Masses between 1.2 and $2.5 M_\odot$ and $R \approx 10$km

- The mass-radius relationship for neutron stars in each case is (derived from our polytropic equations):

$$M = \left(\frac{15.12\,\text{km}}{R}\right)^3 M_\odot; \text{ non} - \text{relativisitic} \tag{5.29}$$

$$M = 5.73 M_\odot \equiv M_{\text{Ch}}^{\text{NS}}; \text{ relativisitic} \tag{5.30}$$

- However, tne maximum mass of a neutron star depends on the existence of a general-relativistic instability (interactions between nucleons)

- This likely takes place well before the M_{Ch} for a neutron star, hence the $\sim 3 M_\odot$ limit

- Most neutron star observations are in a very narrow range of masses $M_{\text{ns}} = 1.35 \pm 0.04 M_\odot$.

- The neutron-degenerate material creates mostly an isothermal environment.

- They cool down faster than WDs.

5.5.4 Black hole

- For Type II remnant $> 2.5 M_\odot$, or a progenitor $> 25 M_\odot$, a black hole is produced.

- The radius, determined from the escape velocity, is:

$$R = 2\frac{GM}{c^2} = 2.95 \times 10^5 \frac{M}{M_\odot} \text{ [cm]}. \tag{5.31}$$

5.6 Instability Strip and Pulsations

5.6.1 Background

- We first consider the types of stars that display pulsations.

- Figure 5.17 shows where many of these types of stars lie on the H-R diagram.

- The Cepheid instability strip is the main thing to pay attention to, which spans from high luminosity down to the main sequence.

- Here, there are Cepheids, RR Lyrae, and δ-Scuti stars, etc.

- The instability strip is very narrow, and we've learned that it's hard to "catch" stars in the Hertzsprung Gap.

- The higher luminosity stars must therefore be more massive, going through their "loops," which occur on a much longer timescale.

- These stars have pulsations excited by an opacity mechanism, explained later.

- Typically only one mode is observed for the more luminous stars (Cepheids and RR Lyrae), a radial mode.

- For the stars near the main sequence, such as the δ Scuti, γ Dor, slowly pulsating B stars (SPB) and β Cephei, multiple modes are observed.
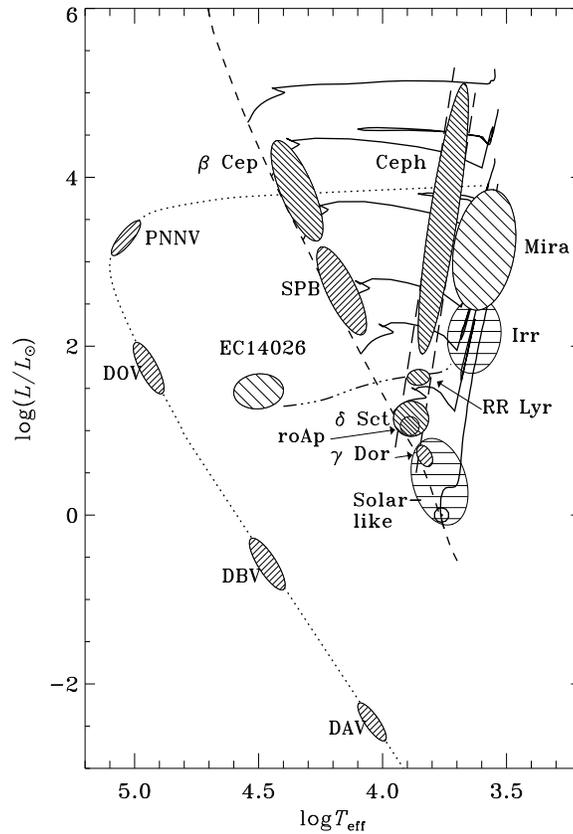
Figure 5.17: Schematic Hertzsprung-Russell diagram illustrating the location of several classes of pulsating stars. The dashed line shows the zero-age main sequence, the continuous curves are selected evolution tracks, at masses 1, 2, 3, 4, 7, 12 and $20 M_\odot$, the dot-dashed line is the horizontal branch and the dotted curve is the white-dwarf cooling curve. From Christensen-Dalsgaard [2003].

- δ Scutis have masses around $1.5 - 2.5 M_\odot$ (convective cores, F stars), and have periods of an hour or so.

- This makes their observation a bit difficult.

- They are expected to have acoustic pulsations.

- The Mira variables are related to the AGB stage of stars.

- The "EC14026" stars are subdwarf B variables (SdB), believed to also pulsate from an opacity mechanism (related to iron).

- These interesting stars are on the horizontal branch as core helium burning is proceeding.

- However, at 35,000K they are much bluer than most HB stars.

- They have somehow lost most of their H envelope beforehand.

- The "Irr" irregular pulsators show strange amplitude variations, but are now known as red giant solar-like pulsators.

- White dwarfs pulsate all along the cooling phase (ZZ Ceti as they are sometimes known).

- They are pulsating with periods up to 10 min, much longer than their dynamical timescales.

- Likely pulsating with internal gravity modes.

- Finally, solar-like oscillations are expected in stars < 7000K, due to convective mechanisms operating near the surface.

- Pulsations can be observed by intensity fluctuations, doppler-velocity observations, and line-profile variations (for nonradial modes).

5.6.2 Pulsation mechanisms

- The adiabatic sound speed of stellar interiors is given as

$$c_s^2 = \frac{\gamma p}{\rho}, \tag{5.32}$$

where again

$$\gamma = \frac{\mathrm{d}\ln p}{\mathrm{d}\ln \rho}.$$

- The most fundamental period of pulsations is inversely proportional to the mean density of a star

$$\Pi \propto \overline{\rho}^{-1/2}. \tag{5.33}$$

- This can be obtained from hydrostatic equilibrium and some basic principles, but can more easily be seen just from the dynamical timescale, Eq. (2.3):

$$t_{\mathrm{dyn}} = \left(\frac{R^3}{GM}\right)^{1/2} \propto \overline{\rho}^{-1/2},$$

which is its direct counterpart.

- A rigorous treatment can provide an exact equation for Eq. (5.33).

- Radial modes are standing waves, and the fundamental mode has only a node at the center and the surface of the star

- After that, we have overtones of the fundamental mode, with nodes placed internally to the star

- Cepheids are observed to pulsate in fundamental radial modes

- Their magnitude, temperature, radius, and surface velocity change as a function of time

- The temperature varies by about 1000 K

- The radius by about 10%

- The star is brightest when it is *expanding* after its minimum radius was reached

- There is a phase lag between the radius and the luminosity

- We do know that as stars pulsate, energy is lost in each pulsation cycle, as most of the volume of the star damps it

- These pulsations can only continue of there is a driving mechanism that is feeding energy into the pulsation, at least as much that has been damped out

- The first proposition to explain such pulsations was that of a heat engine (convert thermal to mechanical energy)

- Pulsations are driven if positive work is done, and damped if negative work is done (on a layer, and throughout the star)

- Eddington postulated a valve mechanism, whereby a layer of a star could block energy that would then push the layer outward.

- After expansion, the blocked region would allow energy to pass through and the cycle would start again

- So, simplistically, the driving mechanism is this

  - The increase in opacity due to ionization halts the energy flow
  - The absorbed energy ionizes and eventually heats the gas and the pressure increases, pushing the local layer outward past its equilibrium point (expansion)
  - This ionization process reduces the opacity since now the density decreases (expansion) and the temperature does not decrease too rapidly because of partial recombination ($\kappa \propto \rho T^{-3.5}$)
  - Radiation can now flow freely, the gas cools and can no longer support the overlying weight
  - The star contracts and compresses and the density increases
  - The opacity is thus raised again and the whole process stars over

- This is the κ mechanism

- The other main driving mechanism is stochastic driving due to convection near the surface

- These modes are typically stable, but there is a lot of acoustic energy in outer convection zones such that this noise is transferred to the energy of global oscillations

5.6.3   Ionization zones

- The ionization zones being considered are hydrogen and the first and second ionization levels of helium

- These occur at about 10,000-20,000K

- To fully ionize helium requires about 40,000K

- For hot stars, with a $T_{\text{eff}} \geq 7500$K, these ionization zones are near the surface

- There is not much mass here and oscillations cannot be driven strongly

- In cooler stars with $T_{\text{eff}} \lesssim 5500$K, these zones occur deeper

- However, convection is occuring in these envelopes

- Convection is efficient and does not allow a sufficient "blocking" of the radiation or energy flux

- Pulsations may not therefore occur from this mechanism, and the red edge of the instability strip is defined

- Most of the driving is due to ionizing He zones from modeling

- The H ionization zones are interesting and since they lie above the He zones, they "carry" the emergent luminosity outward

- Thus the phase lag in the luminosity-minimum radius relations

- Note that these ionization zones alter the value of Γ_1, and lower the adiabatic gradient.

- These regions can often be unstable to convection which can also drive pulsations through convective flux

- For β Cephei stars, it is Fe-group elements doing the driving

5.6.4 Asteroseismology

- The surface of a star oscillates with displacements in the radial and horizontal directions as:

$$\boldsymbol{\delta r} = \xi_r \hat{\boldsymbol{r}} + \boldsymbol{\xi}_{\mathrm{h}},$$

- The diplacements are solutions to the equations of motion expressed as

$$\xi_r(r, \theta, \phi, t) = \sqrt{4\pi}\, \tilde{\xi}_r(r) Y_\ell^m(\theta, \phi) \exp(-i\omega t) \tag{5.34}$$

and

$$\boldsymbol{\xi}_{\mathrm{h}} = \sqrt{4\pi}\tilde{\xi}_{\mathrm{h}}(r) \left(\frac{\partial Y_\ell^m}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{\sin\theta}\frac{\partial Y_\ell^m}{\partial \phi}\hat{\boldsymbol{\phi}} \right) \exp(-i\omega t). \tag{5.35}$$

- The r dependence terms are the (as of yet undetermined) eigenfunctions of the star, while ω is the angular eigenfrequency of a mode

- The Y_ℓ^m are spherical harmonics

- Three "quantum" numbers describe each mode: n gives the overtone, or number of nodes in the radial direction

- ℓ is the degree and specifies the number of surface nodes

- m is the azimuthal order and $|m|$ gives the number of nodal lines of longitude

- The equations of motion are derived from hydrodynamics (continuity, momentum, energy, Poisson, etc.) and approximations to them

- Namely 2: adiabatic small-amplitude (linear) changes

- After the approximations and plugging the spherically symmetric expansions into them, a set of coupled differential equations are derived

- These are then solved for the eigenfrequencies and eigenfunctions

- The pulsations can be classified in various ways

- **Radial modes** are the simplest with $\ell = 0$, these are the breathing modes.

- The star expands and contracts, heats and cools. The fundamental mode is what is foundin Cepheid and RR Lyrae stars.

- The center is a node and the surface is an antinode

- The first overtone $n = 2$ has 1 node away from the center.

- The motions above and below that region move in antiphase

- When stars exhibit both fundamental and first overtone pulsations, the ratio of their frequencies can be used to determine interior properties

- Unlike musical instruments, their ratios vary, which at least tells us that stars do not have constant temperatures or uniform sound speed

- **Nonradial modes** have $\ell > 0$ and $n \geq 1$, thus, a surface node

- These modes are degenerate in m as there are $2\ell + 1$ modes with the same frequency

- Rotation and possibly other effects can break this degeneracy

- The simplest is the dipole $\ell = 1$ mode

- The equator is a node, so the northern hemisphere will expand while the southern contracts, and vice versa

- Depending on n there are internal radial nodes as well

- $\ell = 2$ are quadrupolar, while $\ell = 3$ are octupolar modes

- Beyond this, except for the Sun, we cannot detect higher-degree modes, as the surface averaging partially cancels neighboring regions of hot/cold (intensity) or up/down (Doppler velocity)

- Nodal regions beyond about $\ell = 3$ are practically unattainable in disc-integrated observations (including for the Sun)

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